Modeling Use Diffusion*

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ABSTRACT

Two differential equations modeling the diffusion of use of cultural forms are developed as alternatives to an earlier model based on exponential epochs of diffusion. Both new models are similar in that they incorporate imitation and desertion effects, but differ in the manner in which these effects operate over the course of the diffusion of use. The model which has a Gompertz curve as its integrated form fits data from 17 instances of use diffusion better than the model which has a logistic curve as its integrated form and, in addition, is more parsimonious than the original exponential epoch model. The relationship between the observed values of the parameters representing imitation and desertion is discussed in the context of the process of use diffusion.

Cultural diffusion has usually been thought of as the gradual spread of a cultural form among a population. Cultural forms can mean many things—information, a custom, a role, a technical or social invention, a type of violence, even a psychiatric disorder. Their cumulative spread, adoption, or incorporation into the lifeways of a people is a much studied, well documented social process (see Chapin; Coleman et al.; Davis, a,b; Dodd, a,b; Griliches; Hamblin and Miller; Hamblin et al.; Hernes; Ogburn, c; Pemberton, a,b; Rogers, a,b; Ryan and Gross). While many feel that cultural diffusion is one of the most theoretically and methodologically developed lines of research in sociology, progress comes by successive approximations as important issues emerge from the collective effort and are resolved through further theoretical analysis and empirical investigation. The issue here is whether diffusion as a quantitative use process is better modeled by Gompertz or logistic equations or as exponential epochs.

*The first author worked out the Gompertz model, including the imitation law.

Theory

CONCEPTUALIZATION

Much of the research on cultural diffusion has been concerned with the incidence of use of an invention by members of the population in question. Thus, Alice Davis (a,b) counted over time the number of tailor-made cigarettes and the pounds of plug chewing tobacco sold and presumably consumed, the number of automobiles registered per 1,000 population, and the number of horses and mules used on farms. (She considered these to be indicators of either the positive diffusion of technic ways and the negative diffusion of folkways.) Ogburn (c) measured over time the number of air passenger miles to gauge the use diffusion of the commercial air transportation system. Griliches counted over time the number of farm acres sown in hybrid seed to measure the diffusion of hybrid corn. In these instances the essential measurement was not of the number of current or once users, but of the total use of the invention in question. The essential, but unasked, theoretical question was the nature of the collective influence process which accounted for changes in the incidence of use over time.

MODELING USE DIFFUSION

Prior Work

Alice Davis (a,b) proposed logistic and Gompertz equations for describing the cumulative distributions of use data, and fit both to different data sets without concluding which was better. Ogburn (c) used another equation, an exponential, to describe the through time distribution of use rates of the United State registered airlines. While his data for the 1930–44 period were described well by the exponential, Hornell Hart brought the data up to date and claimed the logistic equation to be a better description. Griliches fit logistic equations to the cumulative number of acres planted in hybrid corn in 132 crop-reporting districts and obtained a median r^2 of .98. However, none of these authors developed a dynamic model of the underlying change process of use diffusion using differential equations.

Hamblin et al. defined use diffusion and fit logistic and exponential equations to the incidence of use, i.e., the amount of use per unit of time, for a number of inventions and found that exponential equations describe epochs of use rates quite accurately. They also developed a rationale for their findings, an exponential epoch model, which involves the diffusion of an invention through a number of "markets." The use diffusion in each market proceeds at a different exponential rate (because of different levels of reinforcement) and the different rates mean that markets saturate at different points in time. Thus, the use diffusion process starts at an aggregated rate which equals the sum of all the rates for the different markets and subsequently decreases each time a market saturates (and its rate then supposedly goes to zero). Thus, the use epochs appear as a series of straight lines when the data are plotted on semi-logarithmic coordinates and each subsequent line (epoch) has a smaller slope (i.e., proceeds at a lesser exponential rate). Hamblin et al. (83–87) develop this explanation with differential equations, which are omitted here in the interest of space.

Simple Models of Use Diffusion

One way to develop a complex model is to isolate the mechanism by which the empirical process in question proceeds. In this instance that search is not very difficult. Around the turn of the century, Tarde suggested imitation as the primary mechanism in cultural diffusion, but his suggestion has been more or less ignored by sociologists. Imitation involves vicarious learning—based either on the observation of or conversations about others' behavior and the reinforcing consequences—and attempts to copy the behavior to obtain similar consequences (Bandura). It is assumed that via vicarious learning and behavioral modeling other people generally acquire use patterns of an invention and expectations about the consequent benefits and costs. Also, as the invention is improved, the cost–benefit ratio gets better and users generally learn vicariously of these improvements. Finally, individuals also generally try to imitate or reproduce their own prior patterns of successful use.

Building on this imitation analysis, we make the rather strong assumption that each incident of use of an invention is both a behavioral model for future imitation and an imitation of prior behavioral models. This implies an essential equality, that the cumulative amount of use (U) equals the cumulative number of behavioral models (M) equals the cumulative number of imitations (I) or:

$$U = M = I. \quad (1)$$

If dt is defined a very small increment (differential) in time and dI is the corresponding differential in imitations, then it would appear from past research that the differential rate of imitations (dI/dt) is always some positive proportion (p) of past imitations (I) or:

$$dI/dt = pI \text{ or } dI/I = p\,dt,\ p > 0. \quad (2)$$

Apparently, this equation for imitation is a basic law of change. It may well turn out to be a fundamental premise for all models of cultural diffusion, and perhaps of other macro social processes. The imitation parameter (p) in this application is assumed to increase as the observed benefits increase relative to those observed for alternative or competing inventions.

Given the equalities in equation (1) it follows that:

$$dM/dt = pM \text{ or } dM/M = pdt,\ p > 0, \tag{3}$$

and

$$dU/dt = pU \text{ or } dU/U = pdt,\ p > 0. \tag{4}$$

Equation (4), involving the use terms (dU and U), is the one of interest because of the close correspondence between these terms and the data. However, the others involve the imitation equations essential to the derivation of (4).

Even so, equation (4) is incomplete because the use of an invention is affected not only by the observed relative benefits, but also by the observed relative costs. The observed relative costs are assumed to inhibit the use process, causing people to decrease their use or, to some extent, desert the invention in favor of less costly alternatives. These negative or desertion effects, as they will be called, are assumed to be cumulative. Because of this, relative differential change in use at any point in time (dU/U) is not only proportional (p) to the increment in time (dt), but it also varies inversely with the cumulated desertions (D), as in the following equation:

$$dU/U = pdt/D,\ p > 0. \tag{5}$$

The desertions should simply reflect competition which is probably proportional to the number of other relevant inventions in the same class of inventions and to lesser degree in other classes. Thus, for example, the airline passenger system not only competes with other intercity passenger carriers—automobiles, buses, trains and boats—but also with the telephone system, the postal system and the telegraph system (as alternative means of communication). It also competes with other consumer goods. People have limited time and resources, and inventions may be said to compete with one another for that time and for those resources.

It has been found (Hamblin et al.; Lehman; Ogburn, a) that the increment in inventions (dx) per increment in time (dt) is a proportion (k) of all inventions (x) which are recombined to form new alternative inventions, as follows:

$$dx/dt = kx,\ k > 0. \tag{6}$$

The innovation process defined by (6) results in the attractive alternatives which change the observed relative costs and, hence, the competition, through time. Because the changing relative costs result in desertions which are observed and talked about, observational and symoolic learning occur, as does behavioral modeling. Consequently, the imitation law applies, so the differential rate of desertion (dD/dt) is proportional (h) to the cumulated number of desertions (D), as follows:

$$dD/dt = hD \text{ or } dD/D = hdt,\ h > 0. \tag{7}$$

By integration, this equation may be solved for D as a function of time:

$$D = D_o e^{ht}. \tag{8}$$

where D_o is the initial amount of desertion at $t = 0$. By substituting $D_o e^{ht}$ for D in equation (5), one obtains the following model:

$$dU/U = pdt/D_o e^{ht} = mdt/e^{ht} = mb^t dt$$

or

$$dU/dt = mb^t U. \tag{9}$$

Since $m = q/D_o$, m will be refered to as a net rate of positive imitation or, simply, the imitation rate; the h parameter is the negative imitation rate, or simply, the desertion rate. The b parameter, which equals e^{-h}, is often referred to as the discount rate (cf. Hernes). Equation (9) is a differential equation or a dynamic model of use diffusion and, when solved by integration, it results in a Gompertz equation.

A second model results in a logistic equation and its development begins with the same assumptions used previously—i.e., that the primary mechanism in use diffusion is imitation and the imitation processes are represented as equations (1) through (5). However, the negative exponential desertion process is conceptualized another way. It is assumed that $(A\text{-}U)$, where A is the use asymptote, is another way to represent mathematically the cumulative desertions $(1/D)$. Therefore, $(A\text{-}U)$ is substituted for $(1/D)$ in (5), to obtain the following differential equation.

$$dU/dt = pU(A\text{-}U) \text{ or } dU/U = p(A\text{-}U)dt. \tag{10}$$

Both (9) and (10) describe a continuous change in dU/dt over time. In the case of (10), dU/dt increases until $U = A/2$, then decreases. When the parameters p and A are constant, dU/dt at any point in time depends solely on U, and its rise and fall is symmetrical around the point in time where $U = A/2$. In contrast, when the parameters m and b are constant, the process described in (9) indicates that DU/dt is a function of both U and t. The quantity dU/dt still increases and then decreases over time. However, because t is used as an exponent to b, the discount rate, dU/dt, is not necessarily symmetric around the point of inflection but usually increases more rapidly than it decreases.

Since both of these models incorporate continuous rather than discontinuous change and are otherwise more parsimonious than the exponential epoch model, it is pertinent to ask if either is an adequate description of the data. If one or both prove adequate, as Davis and Griliches assumed, then an empirical and theoretical comparison with the exponential epoch model is relevant.

Analysis

THE DATA

From the data sets included in Hamblin et al. (Chapters 5 and 6), fifteen were selected for analysis. Each measures the quantitative use of an invention over an extended number of years.

Ten data sets involve inventions whose use rates are still increasing. These use measures include: automobiles produced, 1904–69; automobile registrations, 1904–68; automobile gas consumption, 1916–68; truck production, 1904–69; and truck registrations, 1904–68 (Automobile Manufacturers Association). Also used are: airline passenger miles, 1926–67; helicopter passenger miles, 1956–67 (Civil Aeronautics Board); color TV sets in use, 1955–72 (Television Digest); intercity water passenger miles, 1930–65 (National Association of Motor Bus Operators; U.S. Bureau of the Census, a); and cigarette production, 1890–70 (Internal Revenue Service).

Seven other data sets involve inventions whose use rates have peaked and are now declining. These use measures are: number of monochrome TV sets in use, 1946–72 (Television Digest); railway passenger miles, 1890–31, 1932–70 (Interstate Commerce Commission; Association of American Railroads); intercity bus passenger miles, 1930–65 (Automobile Manufacturers Association; National Association of Motor Bus Operators); movie attendance, 1922-65 (U.S. Bureau of the Census, b, c); production of cut tobacco and snuff in pounds, 1900-70 (Internal Revenue Service); and horses and mules on farms, 1870-60 (U.S. Bureau of the Census, b, c).

The early segments of some of these data sets were discussed or analyzed by Chapin, Davis (a,b) and Ogburn (b,c). However, they are used here primarily because of their accuracy. Most are taken from the tax records kept under the supervision of professional auditors and are checked and cross-checked for errors and falsification. The others are estimates from representative samples taken yearly.

Finally, it might be noted that the railway passenger system in the United States went through two epochs of development and decline, the second evidently because of the depression and World War II which temporarily reversed the conditions of competition to give it an edge over competing passenger systems, especially automobiles.

ESTIMATION

As with previous cultural diffusion models, it is assumed here that use is more of a continuous than a discrete process, that the underlying dynamic is more appropriately described by differential than difference equations. Gray and von Broembsen made the opposite assumption, but with yearly data, the discrete model implies a step function with increments in use rate occurring on January 1, with no further change in rate through December 31. We do not think this is the way use diffusion works; the rates appear to be continuously changing.

In the differential equation model, dU represents the increase in use during a very small interval of time (dt), a small fraction of a year. Since data on the use of inventions are usually given in annual rates, they do not correspond to dU/dt and, therefore, do not give appropriate estimates of the model's parameters. However, there is a standard solution. Mathematical integration as an operation corresponds to the accumulation of incidents. It makes little difference whether the accumulation is hourly, daily, monthly or yearly, just so there are enough data points to use in fitting the integrated equation. So it is quite appropriate to solve a differential equation model to obtain the corresponding integrated equation and to use that equation to estimate the parameters and the comparative fit of the model.

Solving (9) for U_t one obtains:

$$U = U_0 e^{m\,b^t/1nb} / e^{m/1nb} = c\, e^{nb^t} = ca^{b^t}. \qquad (11)$$

where $c = U_0/e^{m/1nb}$, $n = m/1nb$, and $a = e^n$. The last part of (11) is the form ordinarily given for the Gompertz.

Solving (10) for U one obtains:

$$U = A/1 - be^{-pAt}. \qquad (12)$$

Estimates of the parameters and measures of fit were obtained using the SPSS nonlinear regression program on a CDC 6400 computer. The program minimizes the error function with several variables through successive iterations to obtain least-squares estimates of the parameters. The SPSS results were cross-checked using another nonlinear regression program, LSTSQ by Schoenberg, based on an iterative least-squares method developed by Fletcher and Powell. The estimates were quite similar.

The results are summarized in Table 1.

Table 1. PARAMETER ESTIMATES FOR THE GOMPERTZ MODEL AND FIT (r^2)FOR THE GOMPERTZ AND LOGISTIC MODELS

	Gompertz Parameters					r^2	
Data Set	m*	h*	c	-n	b	Gompertz	Logistic
Automobile production x 10^{-3}	.1067	.0160	.190 x 10^7	6.66	.984	.991	.988
Automobile registration x 10^{-3}	.1715	.0197	.138 x 10^8	8.71	.980	.998	.904
Automobile gas consumption x 10^{-9}	.1210	.0165	.325 x 10^5	7.33	.984	.999	.998
Truck production x 10^{-3}	.1807	.0274	.139 x 10^6	6.59	.973	.999	.937
Truck registration x 10^{-3}	.1687	.0208	.268 x 10^7	8.10	.979	.999	.908
Helicopter passenger miles x 10^{-6}	.4528	.0659	.325 x 10^4	6.87	.936	.998	.985
Airline passenger miles x 10^{-6}	.3531	.0266	.648 x 10^5	13.26	.974	.999	.977
Monochrome TV sets in use x 10^{-3}	.768	.101	.175 x 10^7	7.57	.903	.999	.873
Color TV sets in use x 10^{-3}	2.648	.129	.119 x 10^7	20.58	.879	.999	.984
Railway passenger miles (1890-1931) x 10^{-9}	.222	.0482	.214 x 10^4	6.07	.953	.999	.927
Railway passenger miles (1932-70) x 10^{-9}	.6536	.1253	.122 x 10^4	5.22	.882	.996	.990
Intercity bus passenger miles x 10^{-9}	.3692	.0727	.927 x 10^3	5.08	.930	.999	.899
Movie attendance x 10^{-6}	.2909	.0771	.312 x 10^4	3.77	.923	.999	.996
Cigarette production x 10^{-9}	.2644	.0293	.501 x 10^5	9.02	.971	.999	.991
Pounds of cut tobacco and snuff produced x 10^{-6}	.1579	.0447	.272 x 10^5	3.53	.956	.999	.760
Horses and Mules on Farms x 10^{-3}	.1646	.0369	.180 x 10^6	4.46	.964	.999	.835
Intercity water passenger miles x 10^{-9}	.1333	.0454	.119 x 10^3	2.94	.956	.989	.982

*Calculated from five significant digit estimates of -n and b obtained from SPSS nonlinear program, using the identities m = n ln b and h = -ln b.

Discussion

CHOOSING AMONG ALTERNATIVE MODELS

Although some of the seventeen empirical diffusion processes were barely underway (color TV), others about at peak (cigarettes) and others, almost over (horses and mules as tractors on farms), the Gompertz equation describes the cumulative use measured in all of these data sets exceedingly well (the median r^2 equals .999) and, in every case is somewhat better than the logistic (whose median is .977). Hence, the Gompertz model is clearly superior to the logistic.

Another purpose was a comparison with the exponential epochs model. One of the best examples of exponential epochs involves the air passenger miles data, where different exponential equations describe three differential epochs, or segments, for the 1926–1968 period. The exact equations, time segments and fits are as follows:

$U = 0.97e^{1.16t}$, 1926–1930, $r^2 = .98$.
$U = 93.18e^{0.24t}$, 1931–1945, $r^2 = .98$.
$U = 7{,}010.8e^{0.12t}$, 1946–1968, $r^2 = .98$.

Some might argue that this level of fit, since it involves rate rather than cumulated data, is more or less comparable to that obtained with the Gompertz. However, the number of parameters is six versus three. This means the Gompertz is also preferred in this comparison by virtue of its greater parsimony, even if the fit were equal. The excellent results for the Gompertz model also obviate the rather complex and, perhaps, untestable model and auxiliary theory developed to explain why exponential use epochs occur. In the light of these results, exponential epochs are more cogently explained as an artifact of an inappropriate mathematical analysis.

INTERPRETING THE PARAMETERS

Three use measures of automobiles are analyzed in Table 1. They show that alternative measures do not necessarily produce similar estimates of the imitation (m) and desertion (h) rates. In particular, automobile registration yields a 0.17 imitation versus 0.11 and 0.12 imitation rates for automobile production and automotive gasoline consumption, respectively, and a 0.020 desertion rate versus 0.016 and 0.016 desertion rates for automobile production and gasoline consumption. In such instances of conflict, one simply has to make an epistemic judgment as to which measure best gauges use. In our judgment, gasoline consumption is the most direct and accurate use measure, and the 0.12 imitation rate and .016 percent desertion rates are the best estimates for the use diffusion of automobiles in the United States.

Surprisingly, of all the imitation rates, the 12 percent rate for automobiles is the lowest. The other smaller rates are 0.13 for water passenger miles, 0.18 for the use of trucks, 0.16 for the use of horses and mules on farms, 0.16 for the use of chewing tobacco and snuff, 0.22 for the passenger use of railroads from 1890–1931, 0.26 for the use of tailored cigarettes, and 0.29 for movie attendance. The moderate imitation rates are 0.35 for the passenger use of airlines, 0.37 for the passenger use of intercity buses and 0.45 for the passenger use of helicopters. The large imitation rates are 0.65 for the passenger use of railroads, 1932–1970, 0.77 for monochrome TV sets in use and, of course, 2.65 for color TV sets in use. One might intuitively question some of these estimates as being inappropriate, for example, for railroad passenger use (0.65) versus airline passenger use (0.35) for approximately the same period. However, it should be remembered that use diffusion is the joint effect of the rates of imitation and desertion.

In general, the desertion rate h increases as m, the rate of imitation, increases. Thus the lowest desertion rates are 0.016 for automobiles, 0.027 for trucks, 0.027 for airliners, 0.029 for cigarettes, 0.037 for horses and mules, 0.045 for chewing tobacco, 0.045 for water passenger carriers, and 0.048 for railway passenger carriers—1890–1931. The high rates of desertion are 0.066 for helicopters, 0.073 for intercity buses, 0.077 for movies, 0.101 for monochrome TV sets, 0.125 for railway passenger carriers—1932–1972, and 0.129 for color TV sets.

The relationship between the rate of imitation and the rate of desertion is not perfect, as may be noted in Figure 1. However, with the exception of one outlier (color TV sets in use) most of the data points are reasonably close to the linear regression line. (Without that outlier, the linear r^2 is .730.) From prior theory, we should expect a relationship between the imitation and desertion rates. Both should increase with the relative reinforcement and, hence, the demand for whatever it is that is provided by the invention. However, the imitation rate depends on relative benefits and the desertion rate on relative costs which are not perfectly related, so an r^2 somewhat less than 1.0 is expected.

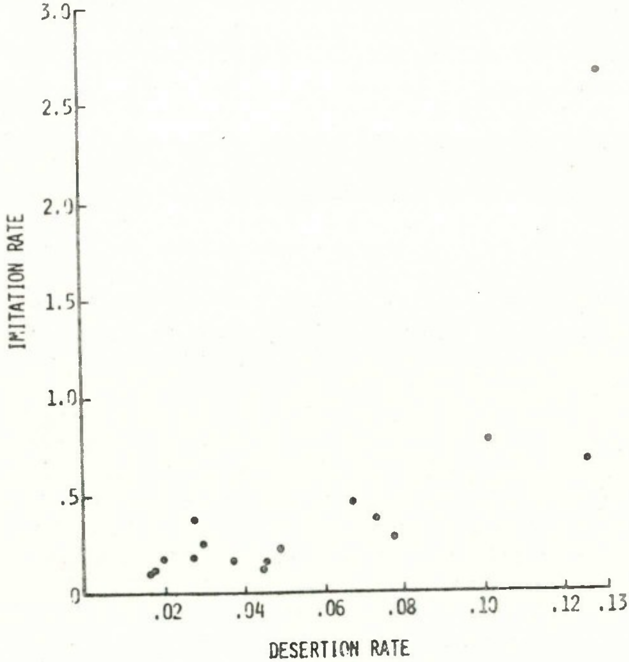

Figure 1. NET RATES OF IMITATION (m) AND DESERTION (h) FOR THE DATA SETS IN TABLE 1

IMPLICATIONS OF THE MODEL

It should be noted that the Gompertz model, as it is developed here, predicts that the use rate of all inventions will eventually become zero. That rather strong prediction is made on the assumption of a continuous innovation process which eventually makes all inventions so obsolete or costly that they will be discarded by everyone. Of the inventions whose data are analyzed here, only the use rates of horses and mules as tractors on farms approach zero. However, the use rates of a number of other inventions are declining (i.e., intercity buses, chewing tobacco and snuff, movies, railway passenger carriers and monochrome TV). The use rates of the other inventions—automobiles, trucks, airliners, helicopters and color TV's—are still increasing.

The basis of all the diffusion models seems to be a law of imitation (equation 2). This equation is generally counter-factual, somewhat like Newton's laws of motion which are true only in frictionless space, a condition that is seldom approached in nature, never reached. The imitation law would be true in the absence of changing relative costs and benefits, another condition that is only approached in nature, never reached. However, like Newton's laws of motion, the imitation law defines a component process which, depending on combination with counteracting processes, predicts and explains observable patterns of social change quite accurately. This law of imitation may not turn out to be as useful as Newton's three laws, but it could be very helpful in predicting and explaining dynamic macro patterns of behavior in human society.

Diffusion models portray society as a huge learning system where individuals are continually behaving and making decisions through time but not independently of one another. They watch one another and talk with one another about one another's behavior and the experienced consequences. Inventions are continuously being developed, tried and evaluated, and depending on benefits and costs relative to others, they are gradually incorporated into the life ways of the population or gradually dropped. Everyone makes his own decisions, not just on the basis of his own individual experiences, but to a large extent on the basis of the observed or talked about experiences of others. Thus, the collective process involves some direct learning but mostly observational and symbolic learning. Diffusion models account for or explain culture—more or less standard patterns like the use of the automobile in the United States—as well as cultural change—the waxing and waning of use patterns, like the increasing use of color TV and the decreasing use of monochrome TV.

The predictions of the model are not assumed to hold if substantial social crises such as a depression or a war of survival essentially change the competitive conditions. They do not always have an effect, but when they do, crises generally slow diffusion rates down until they are over, at which point the rates go back to the long-term trend. Pemberton (b) was the first to note this effect, and a number of other examples are pictured in Hamblin et al.

A more dramatic effect was observed for railroad passenger carriers as a result of the depression and World War II. Before the depression, competition from the automobile had almost overwhelmed the railway passenger carriers; their use rates had declined to about one half of what they had been in 1916, the peak year of the first cycle of railroad passenger travel. However, the depression priced automobile travel out of many people's reach, and the gasoline, tire and automobile shortage during World War II further increased the railroad's advantages. The result was a new use epoch for the railroad passenger carriers which peaked in 1944 and declined into the mid 1970s.

Finally, this diffusion model should be of interest to economists because what is here called "use" reflects what economists call "demand." Use is usually thought of as a function of both supply and demand. However, supply almost always runs ahead of demand in market economies, so use almost always just reflects demand at present prices.If so, the model then predicts and explains long-term demand curves, at least in market economies. Since the accuracy is rather great, the model can be used to analyze and project long-term markets for currently used inventions. Of course, the predictions, like others in science, are not fool proof. They are accurate only within the observed margin of error (which is small) and good short of crises which redefine the competitive situation. Therefore, this model should be quite useful for long-term socioeconomic forecasting and planning.

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