Modellen-sectie

A ONE-DIMENSIONAL STOCHASTIC UNFOLDING MODEL WITH AN APPLICATION TO PARTY PREFERENCES IN THE NETHERLANDS.*

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1. Introduction.**

This paper is confined to the elaboration of a one-dimensional preference model and its application to party preferences. The model is given in section 2. Goodness of fit criteria to test the model are elaborated in section 3. After a description of our data on party preferences in section 4, the results of the application of the model to these data will be presented in section 5.

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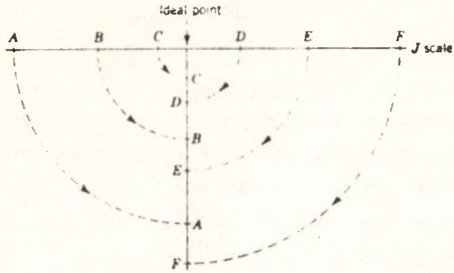
2. The model

Coombs (1950, 1964) developed the first one-dimensional model for preference data. Our model can be seen as a stochastic elaboration of this model. In this model each stimulus may be represented by a point on an underlying dimension. Each subject has an ideal which can also be represented by a point on that dimension. In our application the stimuli are political parties and a subject's ideal represents an, in general non-existing, ideal political party. Because both stimuli and subjects are represented as points on the underlying dimension, this common dimension is called a J-scale. Coombs' deterministic model assumes that each individual's preference order of the stimuli from most to least preferred corresponds to the rank order of the absolute distances of the stimulus points from the subject's ideal point, the nearest being the most preferred. The individual's preference ordering is called an I-scale and may be thought of as the J-scale folded at the ideal point with only the rank order of the stimuli given in the order of increasing distance from the ideal point (Coombs, 1964,80). Figure 1 is a graphical representation of the model. On the underlying dimension (the J-scale) the positions of six stimuli are given (A,B,C,D,E,F) and the ideal point of one subject. The preference order from high to low for that subject will be CDBEAF. The data consist of a set of I scales (preference orders) of a number of subjects or pairwise preferences from which such I scales (in case of transitivity) can be constructed. The analytical problem is how to unfold these I scales to recover the J scale.

The positions of the stimuli (political parties) and the ideal points (subjects) can be seen as unknown scale values on the continuum. The scale value of stimulus A will be denoted λ_A; the scale value of

Figure 1 An I scale preference ordering CDBEAF, obtained by folding a J scale



Source: Coombs, 1964, 80.

the ideal of subject i will be denoted θ_i. In the deterministic model a stimulus A is preferred to B by subject i if and only if

$$\theta_i < \tfrac{1}{2}(\lambda_A + \lambda_B);\ \lambda_A < \lambda_B \tag{1}$$

The quantity $\frac{1}{2}(\lambda_A + \lambda_B)$ in equation (1) corresponds to the unknown scale value of the midpoint of the scale values of A and B. We denote this scale value as δ_{AB}:

$$\delta_{AB} = \tfrac{1}{2}(\lambda_A + \lambda_B);\ \lambda_A < \lambda_B \tag{2}$$

Define the manifest function x_{AB} with

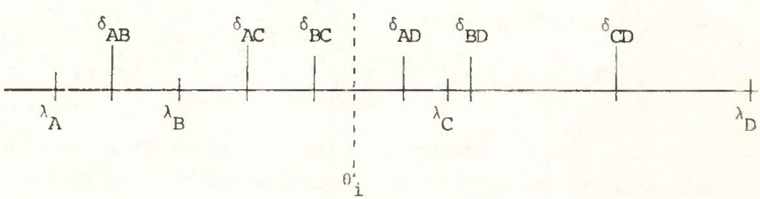
$$x_{AB} = \begin{cases} 1, \text{ when B is preferred to A} \\ \\ o, \text{ otherwise} \end{cases} \tag{3}$$

With preference orders of <u>four stimuli</u> A,B,C,D for each of the subjects, we here introduce with the function x_{AB} <u>six new dichotomous variables or items</u> for each of the <u>midpoints</u> AB,AC,AD,BC,BD and CD (in general with n stimuli we have ½n(n-1) midpoints and functions x_{AB}).
Items always refer to midpoints, stimuli to the political parties themselves.
<u>According to the deterministic unfolding model</u>:

$$\begin{aligned} x_{AB} &= 1 \quad \text{if and only if} \quad \theta_i > \delta_{AB} \quad \text{and} \\ x_{AB} &= 0 \quad \text{if and only if} \quad \theta_i < \delta_{AB} \end{aligned} \tag{4}$$

Figure 2 illustrates the meaning of x_{AB}. A subject with scale value θ_i in Figure 2 will have the score 1 on x_{AB}, x_{AC} and x_{BC} because of

<u>Figure 2</u> Scale values of one subject, four stimuli and this midpoints of the stimuli on a J scale

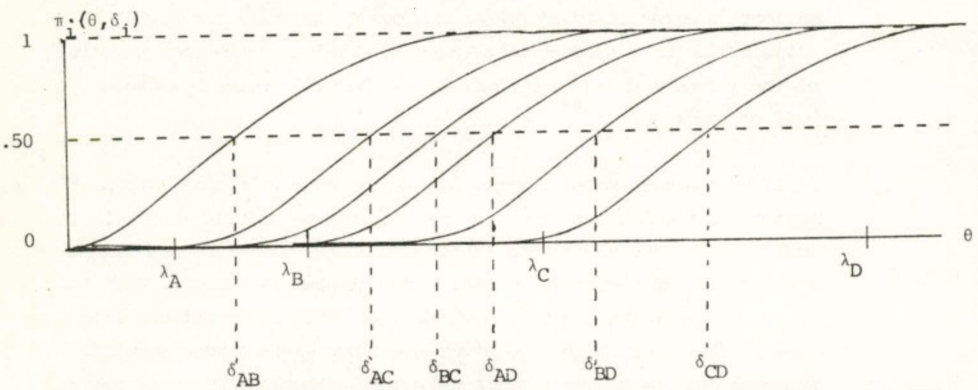


$\theta_i > \delta_{BC} > \delta_{AC} > \delta_{AB}$ and the score 0 on x_{AD}, x_{BD} and x_{CD} because of $\theta_i < \delta_{AD} < \delta_{BD} < \delta_{CD}$. The reader can easily verify that <u>the manifest</u>

functions x_{AB} on the midpoints form a deterministic cumulative scale (a Guttman scale), as has been explicitly mentioned by Dawes (1972)

Mokken (1970) developed a stochastic cumulative model for dichotomous data by introducting a trace function $\pi(\theta,\delta)$ of a one-dimensional subject parameter θ and a one-dimensional item difficulty parameter δ. Application of this model on the midpoints of the stimuli seems to be straightforward and enables us to formulate a stochastic version of the unfolding model of Coombs. This model is given in Figure 3 for the same four stimuli as we considered in Figure 2. In this model

Figure 3 Stochastic unfolding model for the four stimuli



the probability of a positive response ($\pi(\theta,\delta)$) on a item (e.g x_{AB}) increases with increasing subject values θ. In other words: the probability that B will be preferred above A will increase with increasing θ values. The trace lines therefore have the property of monotone homogeneity. Moreover, the trace lines of the different items (midpoints) are not allowed to intersect. For example, we see in Figure 3 that the probability to prefer C above A for a subject with a given value is always smaller than the probability to prefer B above A. The set of trace lines therefore have the property of double monotony or holomorphism. In other words: each trace line increases with increasing value θ; in addition to this monotony in θ, we have a monotony in δ: the probability $\pi(\theta,\delta)$ is a function that decreases with increasing values of δ.

Let us formulate the model once again in terms of our application: party preferences. The different parties have an unknown value on a one-dimensional continuum ($\lambda_A, \lambda_B, \lambda_C, \lambda_D$). Each subject has an unknown ideal point on this continuum, representing his ideal party (θ). In the deterministic model a subject will prefer party B above A if his ideal is closer to B than to A (in Figure 2 and 3 right of the midpoint δ_{AB}). Due to the fact that a subject will not exactly perceive the positions of the parties on the continuum and his own ideal position, we do not assume this to be the case in our stochastic model, but we assume that a subject is more likely to prefer B above A ($\lambda_A < \lambda_B$) the higher his value θ is. For a subject with the same θ value as the unknown value of the midpoint ($\theta=\delta_{AB}$) we assume that he will prefer B above A half of the time.

If we compare our stochastic unfolding model with other stochastic preference models, our model is the only non-parametric model. In the models of Bechtel (1968) and Zinnes and Griggs (1974) a normal distribution of the subjects over the continuum is assumed. This is not the case in the model of Sixtl (1973). This model differs from ours in that respect that he gives the trace lines a more specific function than we do: we require only double monotony of the trace lines, whereas he specifies a logistic trace line. In fact, as our model can be seen as an application and adaptation of the stochastic cumulative scaling model of Mokken (1970) for preference data, Sixtl's model is such an application of the Rasch model (Rasch, 1960 ; Fischer, 1974).

Although the stochastic unfolding model can be formulated as the stochastic cumulative model of Mokken on the midpoints of the stimuli, certain derivations from the model do not hold anymore, because of the fact, that the "responses" on the items (the midpoints) are not completely independent in case of rank orders. Moreover, certain extra restrictions hold on the order of the midpoints that can be used as additional tests of the model. These aspects will be considered in the next section .

3. Model tests and scale procedures

3.1 The order of the midpoints and its restrictions

For a holomorphic set of items the fraction of the population that gives the positive reponse on an item i (e.g the fraction in the population that prefers B above A) is directly related to the item difficulties δ_i. This fraction is denoted the population difficulty π_i. The population difficulties π_i reflect inversely the order of the item difficulties:

$$\delta_i > \delta_j \Rightarrow \pi_i < \pi_j \tag{5}$$

The population difficulties can be estimated consistently with reasonable precision by the sample difficulties p_i for large N (number of subjects). Therefore the order of the π_i (and hence that of δ_i) can also be estimated consistently and with good precision. For sample data therefore the numbering and ordering of the items (midpoints) can usually be based on the estimated order of their population difficulties (Mokken, 1970, 179):

$$p_i \leqslant p_j \Rightarrow i < j \tag{6}$$

The order of the midpoints AB, ... CD on the continuum can therefore be estimated on the basis of the sample fractions p_{AB}, ... p_{CD} that prefer the second stimulus above the first. However, on the basis of our model we can formulate other restrictions on the order of the midpoints. Given the order of the stimuli $\lambda_A < \lambda_B < \lambda_C < \lambda_D$ on the continuum the order of the midpoints is also fixed with the exception of the order of the midpoints BC and AD. If the midpoint BC

Table 1. Restrictions to the order of the population difficulties, given the order ABCD on the continuum ($\lambda_A < \lambda_B < \lambda_C < \lambda_D$)

$$\begin{array}{cccccccc}
 & A & & B & & C & & D \\
A & - & & \pi_{AB} & > & \pi_{AC} & > & \pi_{AD} \\
 & & & & & \vee & & \vee \\
B & & & - & & \pi_{BC} & > & \pi_{BD} \\
 & & & & & & & \vee \\
C & & & & & - & & \pi_{CD} \\
D & & & & & & & -
\end{array}$$

precedes AD($\delta_{BC} < \delta_{AD}$) the distance between A and B is smaller than that between C and D. If on the other hand AD precedes BC($\delta_{AD} < \delta_{BC}$) the distance between C and D is smaller than that between A and B. The order of these two midpoints contain metric information about the distances between certain stimuli.

The order of the stimuli restricts therefore the order of certain population difficulties π_i of the items. These restrictions are given in table 1 on the basis of the order ABCD on the continuum ($\lambda_A < \lambda_B < \lambda_C < \lambda_D$). For example, given the order ABCD of the stimuli, the midpoint AB should precede that of AC; therfore π_{AB} should be larger than π_{AC} because of (5). If we insert the population difficulties in a matrix of stimuli against stimuli, in the order of their scale values on the continuum, in the upper-diagonal part of the table the rows should decrease monotonely from left to right, the columns decrease monotonely from top to bottom. As we see in table 1 only the order between π_{AD} and π_{BC} is free. Of course, the sample difficulties p_i should roughly follow the same restrictions as the population difficulties π_i.

These restrictions, that have been formulated already for the deterministic model by Greenberg (1965), can be used in two different ways:

1) as a first check of the goodness of fit of the model, if the order of the stimuli is known (e.g on theoretical grounds).

No large disturbances of the monotony in table 1 are allowed on the basis of the sample difficulties p_i.

2) As a procedure to derive the order of the stimuli on the continuum by permutation of the matrix given in table 1. If permutation does not lead to the required monotony, it can be used as a first indication that the model does not fit. A straight forward algoritm for such a permutation is given by Lingoes and Coombs (1975).

On the basis of the order of the midpoints scale values of the stimuli can be computed by application of the 'equal delta solution' that has been given by Coombs (1964) (see also McClelland and Coombs, 1975; Van der Ven, 1977). In this algorithm the metric information that is contained in the order of the midpoints, is taken into account. For each four stimuli on the continuum this metric information is contained in the order of the midpoints of the two extreme stimuli in comparison with that of the two inside located stimuli: if the order of the stimuli is ABCD and BC precedes AD ($\delta_{BC} < \delta_{AD}$) the distance between A and B is smaller than that between C and D. If $\delta_{AD} < \delta_{BC}$, the distance between C and D is smaller than that between A and B.

The summation score of the subjects (the number of midpoints on which the subject gives the positive response) can be used to estimate the location of the subjects on the continuum. We can then apply the procedure of Coombs to give the subjects a scale value (see Coombs,1964 ; Van der Ven,1977). We will not consider these procedures here, as our first interest is the development of a stochastic model and goodness of fit criteria over the midpoints of the stimuli.

3.2 Positive correlation

Mokken derived that in case of monotone homogeneity of the trace lines, responses to item pairs are positively correlated. Let us consider for this aspect a (2x2) table between two items i and j (i<j, because $\delta_i > \delta_j$). It is given in table 2.

Table 2 Cross tabulation of two items i and j

		j +	j −		
i	+	$\pi_{ij}(1,1)$	$\pi_{ij}(1,0)$	π_i	$\pi_i < \pi_j$
i	−	$\pi_{ij}(0,1)$	$\pi_{ij}(0,0)$	$1 - \pi_i$	
		π_j	$1-\pi_j$	1	

$\pi_{ij}(1,1)$ denotes the fraction in the population that responds positively to both item i and j. Positive correlation implies that

$$\pi_{ij}(1,1) > \pi_i \cdot \pi_j \tag{7}$$

As a measure of positive correlation Mokken used ϕ/ϕ_{max} which he denoted H_{ij}:

$$H_{ij} = \frac{\pi_{ij}(1,1)-\pi_i\pi_j}{\pi_i(1-\pi_j)} = 1 - \frac{\pi_{ij}(1,0)}{\pi_i(1-\pi_j)} = 1 - \frac{E_{ij}}{EO_{ij}} \tag{8}$$

in which E_{ij} denotes the fraction $\pi_{ij}(1,0)$, the fraction in the 'error' cell and EO_{ij} the expected fraction in the 'error' cell ($\pi_i(1-\pi_j)$ if $\pi_i<\pi_j$) in case of statistical independence, given the population difficulties π_i and π_j.

This coefficient can directly be used if we consider two midpoints that have no stimulus in common, because the responses on these items (midpoints) are independent from one another. In case of the four stimuli of Figure 3 (A,B,C,D) this coefficient can therefore be used for the following three pairs of midpoints: (AB,CD),(AC,BD) and (BC,AD). For pairs of midpoints that have a stimulus in common (e.g (AB,BC)) this coefficient cannot be used because of the fact that the responses are not independent if the subjects have given a rank order of the stimuli. We shall therefore now investigate these pairs of midpoints more closely and define a new coefficient of homogeneity for triples of stimuli.

Let us consider a set of three stimuli A, B and C. Let their order on the continuum be $\lambda_A < \lambda_B < \lambda_C$. The order of the midpoints is now completely fixed, being

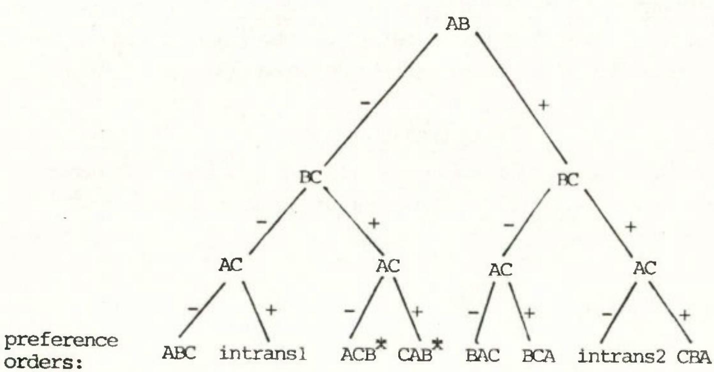
$$\delta_{AB} < \delta_{AC} < \delta_{BC} \Rightarrow \pi_{AB} > \pi_{AC} > \pi_{BC} \tag{9}$$

If we ask pairwise preferences, Figure 4 gives all possible combinations of the responses on the three items x_{AB}, x_{AC} and x_{BC}, resulting in 8 different response patterns. In 6 response patterns we are able to deduce a preference order, in 2 response patterns we are not, because of intransitivities in the preferences. These two intransitive response patterns cannot occur in case we directly ask a preference order instead of pairwise preferences. Two of the 6 transitive response patterns do not fit into a one-dimensional model with $\lambda_A < \lambda_B < \lambda_C$: ACB and CAB. For that reason we have given an asterix to these preference orders. In table 3 we have inserted the preference orders in the three cross tabulations between these midpoints. On the basis of (9) and according to the convention of (6) item BC is labelled as item i, AC as j and AB as k ($\pi_{BC} < \pi_{AC} < \pi_{AB} \Rightarrow i < j < k$). The upper left cells in the three tables are the 'error' cells on the basis of which E_{ij} in (8) is defined. We should now clearly distinguish two different cases, namely whether our data consist of:

1) preference orders, which preclude intransitive response patterns; and
2) pairwise preferences, which allows all 8 response patterns.

We shall now consider these two cases successively.

Figure 4 Possible combinations of responses to the midpoints of three stimuli ($\lambda_A < \lambda_B < \lambda_C$) on the basis of pairwise preferences.



*preference order does not fit in one-dimensional model with $\lambda_A < \lambda_B < \lambda_C$

Table 3 Preference orders inserted in the cross-tabulations between the midpoinst AB,AC and BC

		item j (AC) +	item j (AC) −		
item i(BC)	+	CBA CAB*	ACB* intrans2	π_i	$\pi_i < \pi_j$
	−	BCA intrans1	ABC BAC	$1-\pi_i$	
		π_j	$1-\pi_j$	1	

		item k (AB) +	item k (AB) −		
item i (BC)	+	CBA intrans2	ACB* CAB*	π_i	$\pi_i < \pi_k$
	−	BAC BCA	ABC intrans1	$1-\pi_i$	
		π_k	$1-\pi_k$	1	

		item k (AB) +	item k (AB) −		
item j(AC)	+	CBA BCA	CAB* intrans1	π_j	$\pi_j < \pi_k$
	−	BAC intrans2	ABC ACB*	$1-\pi_j$	
		π_k	$1-\pi_k$	1	

1. Preference orders. In case of preference orders the intransitive response patterns can not occur. In testing our model we are therefore only interested in the frequency with which the response patterns ACB* and CAB* occur: the fit of our model is better, the less these response patterns occur and in each case we want them to occur less often than we might expect in case of random preference ordering, but taking into account the relative popularity of the different stimuli. Their frequency of occurence is given in the error cell of the cross tabulation between the items i(BC) and k(AC), the two outside midpoints. Only this cross-tabulation is sufficient and should therefore be used to test the goodness of fit of a triple of stimuli. The expected value in the error cell, however, is not $\pi_i(1-\pi_k)$ because of the fact that intransitive responses are precluded. Given the difficulties of the midpoints the expected value in this error cell will therefore be taken in case of statistical independence under the additional condition of transitive responses. In Figure 4 we have seen that two response patterns result in intransitivities. The probability of the first intransitive response in case of random responses, given the difficulties of the midpoints is:

$$\mathcal{E}(x_i = 0 \cap x_j = 1 \cap x_k = 0/\pi_i,\pi_j,\pi_k) = (1-\pi_i).\pi_j .(1-\pi_k) \qquad (10)$$

The probability of the second one under the same conditions is:

$$\mathcal{E}(x_i = 1 \cap x_j = 0 \cap x_k = 1/\pi_i,\pi_j,\pi_k) = \pi_i.(1-\pi_j).\pi_k \qquad (11)$$

The probability of an intransitive answer is therefore the sum of the two probabilities:

$$\begin{aligned} &\mathcal{E}(010 \cup 101/\pi_i,\pi_j,\pi_k) = \\ &= (1-\pi_i).\pi_j.(1-\pi_k) + \pi_i(1-\pi_j).\pi_k = \\ &= \pi_i(\pi_k-\pi_j) + \pi_j(1-\pi_k) \end{aligned} \qquad (12)$$

The expected value in the error cell of the midpoints i(BC) and k(AB) in case of random response given the difficulties and transitivity is therefore:

$$EO_{ABC} = \mathcal{E}(ACB^{*} \cup CAB^{*} \,/\, \pi_i, \pi_j, \pi_k, \text{ transitivity}) =$$

$$= \frac{\pi_i(1-\pi_k)}{1-\pi_i(\pi_k-\pi_j) - \pi_j(1-\pi_k)} \qquad (13)$$

The coefficient of homogeneity for a triple of stimuli, H_{ABC}, can now be defined as:

$$H_{ABC} = 1- \frac{E_{BC,AB}}{EO_{ABC}} \qquad (14)$$

in which EO_{ABC} is given in (13) and $E_{BC,AB}$ is the observed fraction of the population in the error cell of items i(BC) and k(AB).

Although this coefficient is a coefficient for triples of stimuli, it should be stressed that it is still defined on the basis of comparison of pairs of midpoints (items).

2. Pairwise preferences. In case of pairwise preferences all response patterns can occur, also the intransitive ones. Two different strategies can be followed in this case, of which we advocate the second one:

a) Application of the Mokken scaling procedures on the midpoints without any changes. For each of the cross-tabulations in table 3 H_{ij}, as given in (8), can be computed and Loevinger's coefficient of scalability H can be taken as a goodness of fit for the three stimuli as a whole. If the subjects give only transitive answers, however, our test on the scalability of the items i(BC) and k(AB) will be rather severe. Although transitivity is not imposed in this case, internal consistency checks by the subjects may well result in the same restrictions as we considered in case of rank orders. This implies that the answers of the subjects on the pairwise preferences may not be considered to be independent, as is assumed in the Mokken model. In our data on party preferences we discovered that indeed the H_{ik}'s were consistently lower than the H_{ij}'s and H_{jk}'s (see table 3) within all triples of stimuli. We therefore propose another strategy in which the check on transitivity is seperated from that on the error response patterns ACB^* and CAB^*:

b) First test whether the pairwise preferences are transitive; if so, check the scalability of the three stimuli by application of the coefficient of scalability H_{ABC} as given in (14) for preference orders.

Kendall (1948) has given a coefficient of consistence for paired comparison judgements that can directly be used here. In this coefficient the number of circular triads (d) is counted and related to the maximum-number of circular triads that can occur with n stimuli (d_{max}). This maximum number of circular triads is:

$$d_{max} = \frac{n^3 - n}{24} \quad \text{when n is odd}$$

and (15)

$$d_{max} = \frac{n^3 - 4n}{24} \quad \text{when n is even}$$

The coefficient of consistence, ζ, is then defined as:

$$\zeta = 1 - \frac{d}{d_{max}} \tag{16}$$

Kendall also developed a significance test for this coefficient to test whether the number of circular triads is significantly smaller than expected in case of random judgements.

Under the hypothesis of random judgements a χ^2 distribution has been derived which can be used for this test. (see also Edwards, 1954).

After elimination of subjects that fail to meet the criteria of transitivity, the data are analyzed in the same way as preference orders, as considered above.

<u>In summary in case of preference orders or after testing the</u>

transitivity of pairwise preferences two different coefficients of correlation are defined: one between midpoints that have no stimulus in common and one for triples of stimuli. The first one is given in (8), the second in (14).

On the basis of these coefficients we can now define a coefficient of scalability for a whole set of stimuli. This coefficient of scalability, H, is a weighted average of the coefficients for triples and midpoints without a stimulus in common:

$$H = 1 - \frac{2\sum_{ABC} E_{AB,BC} + \sum_{CD<AB} E_{CD,AB}}{2\sum_{ABC} EO_{ABC} + \sum_{CD<AB} EO_{CD,AB}} \tag{17}$$

in which $E_{AB,BC}$ and EO_{ABC} are given in (14) and $E_{CD,AB}$ and $EO_{CD,AB}$ in (8). The first summations in numerator and denominator are taken over all triples, the last summations over all pairs of midpoints with no stimulus in common. The errors in the triples are weighted double, because two pairs of midpoints are involved in each error within a triple. This can easily be seen in table 3. The response pattern ACB* is located in the error cells of item i with j and of item i with k; the response pattern CAB* is located in the error cells of item i with k and of item j with k. In each error between two midpoints with no stimulus in common only one pair of midpoints is involved.*

*) The statistical properties of the coefficients under different assumptions (the null-case and non-null case) are a subject of further research. The same applies for the investigation of the double monotony of the trace lines.

3.3 Scale procedures

On the basis of the model tests, given in sections 3.1 and 3.2, the following scale procedures have been developed until now:

a) Testing of scalability of a whole set of stimuli with a fixed order of the stimuli on the continuum. The program prints the matrix of difficulties, as given in table 1, which makes it possible to investigate the required monotony in this matrix. All H_{ABC}'s for triples and $H_{AB,CD}$'s for pairs of midpoints without stimuli in common are printed as well as the coefficient of scalability for the whole set of stimuli. Midpoints with fixed order due to restrictions from table 1 are taken in that order, irrespective their difficulties.

b) Testing of scalability of a whole set of stimuli with a free order of the stimuli on the continuum. Using the algorithm of Lingoes and Coombs(1975) the program first determines the order of the stimuli by permutation of the matrix with difficulties (see table 1). If none of the permutations result in a monotone matrix, no further testing occurs. Otherwise, that matrix, all H_{ABC}'s for triples and $H_{AB,CD}$'s for pairs of midpoints without stimuli in common are printed as well as the coefficient of scalability for the whole set of stimuli. Of course, in this case the order of the midpoints is determined solely on the basis of their difficulties.

c) Searching of a scale with a fixed order of the stimuli on the continuum. Again, as under point a, the order of the midpoints is partly determined on the basis of the restrictions from table 1, irrespective their difficulties. First the H_{ABC}'s of all triples are computed and printed. The procedure continues with the best scaling triple. The coefficients of scalability H for all 4-tuples that include that triple, are computed and printed. Again the best scaling 4-tuple is chosen as start set to search a scale of 5 stimuli etc.

d) Searching of a scale with a free order of the stimuli on the continuum. First all H_{ABC}'s of all triples are computed and printed, the order of the stimuli within a triple being determined on the basis of the difficulties of the midpoints. The best scaling triple is chosen as start to search a scale of 4 stimuli. For all possible fourth stimuli it is first investigated whether the matrix of difficulties is monotone or not. If not, they are rejected. For the remaining possible scales of four stimuli, the coefficients of scalability H are computed and printed. Again the best scaling 4-tuple is chosen as start set to search a scale of 5 stimuli etc.

If the data consists of pairwise preferences, first subjects are eliminated that do not fulfil the requirements of sufficient transitivity in their preference. The researcher can specify a maximum number of circular triads that he allows. The program prints the probability that at most such a number of circular triads will occur under the hypothesis of random preferences.

In the searching procedures the researcher can specify a startset of three or more stimuli with which the searching procedure starts. If the order of the stimuli is kept free, the search procedure is stopped at once, if the matrix of difficulties cannot be made monotone for these stimuli.*)

The scale procedures make it possible to search for a maximal subset of stimuli that can be represented in one-dimension. In this respect our approach is similar to that of Dijkstra (1978). In our model the J-scale is estimated in a completely different way, however; moreover he devises different goodness of fit criteria. In his approach estimation of the order of the stimuli and goodness of fit are not separated, making it possible that scales are simultanously accepted with different orders of certain stimuli (Dijkstra, 1978, 173).

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*) In consultation with the authors the computer program is being implemented by Charles E. Lewis (Social Faculty, University of Groningen).

4. The Data

To a random sample of the Dutch population of 18 years and older, (N = 692), pairwise preferences were asked for six Dutch political parties. The survey is part of a longitudinal project of the Department of Political Science at the University of Amsterdam. For each of the respondents the order of the 15 pairwise preferences was determined at random. Moreover, the order of the stimuli (political parties) within each pairwise preference question was determined at random for each of the respondents.* These measures were taken to prevent any possible perceptual or preferential effect from the order of the questions or the order of the stimuli within the questions (see Boon and Niemöller (1976) for a similar problem in the context of similarity analysis).

The following question was asked:

"Here is a list with pairs of political parties. Please, indicate for each pair of parties which of the two you prefer".

("Hier ziet U een lijst met telkens twee politieke partijen. Wilt U voor elke twee partijen aangeven aan welke partij U de voorkeur geeft? Omcirkelt U maar telkens de partij die U beter vindt dan de andere.")

Due to time limitations we were unable to ask pairwise preferences between all Dutch political parties. The following six parties were chosen: the socialist party (PvdA),

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* The survey was the 14th. wave of the longitudinal project of the "Werkgroep Kwartaalonderzoek". The survey was conducted from 14 till 30 March 1977 by N.V. v./h. Nederlandse Stichting voor Statistiek. We thank Otto Schmidt for his suggestions with respect to the formulation of the question and Wim van Hoboken for the randomizing of the pairwise preferences.

the communist party (CPN), the Christian Democratic party (CDA), the liberal party (VVD), the radical party (PPR) and a more orthodox protestant party (SGP). The PvdA, CDA and VVD are the three largest political parties. The other parties were chosen to incorporate both the left-right split between the parties and the religious - non religious split. In earlier research quite often three dimensions have been distinguished: left-right, religious-non religious and large-small parties. All dimensions are well represented in our choice of the six parties: the six parties cover the whole continuum from left to right (in order: CPN,PPR, PvdA, CDA, VVD and SGP); three parties have religious backgrounds (CDA, PPR, SGP),three do not(CPN, PvdA, VVD); and three parties are large (PvdA, CDA, VVD) and three small (CPN, PPR, SGP).

For each pair of parties presented, subjects could give the following responses to the question of preference:

o first party is preferred over second party
o second party is preferred over first party
o no difference in preference
o don't know; no response

5. The results

To determine a coefficient of homogeneity for our six political parties we have worked with those 309 respondents who gave a completely transitive response. The matrix with difficulties of prefernece of right party to left party can be found in table 4.

Table 4: Difficulties of preference of right party above left party (fractions of respondents preferring column party above row party) for 309 respondents with a completely transitive rank order.

	CPN	PPR	PvdA	CDA	VVD	SGP
CPN	-	.91	.95	.89	.78	.62
PPR		-	.78	.68	.55	.30
PvdA			-	.53	.39	.21
CDA				-	.31	.07
VVD					-	.30
SGP						-

We shall discuss the values for the coefficient of homogeneity first for triples of political parties, then for four tuples, five tuples and for all six parties. Of all twenty triples out of the six political parties, sixteen had a negative coefficient of homogeneity H_{ABC}. The only four triples with a positive H_{ABC} were:

CPN-PPR-VVD $H = .08$
CPN-PPR-SGP $H = .14$
PPR-CDA-VVD $H = .10$
PvdA-CDA-VVD $H = .21$

The best scales are found with either the three largest or the three smallest parties. Also, replacing PvdA by PPR (the largest party by the largest remaining party), or replacing SGP by VVD (the smallest by the smallest remaining party) gives a positive H_{ABC}. Therefore, only triples of parties that are homogeneous with respect to size can be reasonably represented along J-scales that may be interpreted as left-right scales.

All four tuples with some violation in the monotony of the matrix of difficulties have negative H_{ABCD}'s. On the other hand, not all four tuples with a perfect monotone pattern in the matrix of difficulties had positive H_{ABCD} coefficients. These four tuples with a perfect monotone pattern were:

CPN-PPR-CDA-VVD	H = .16
CPN-PPR-CDA-SGP	H = .11
CPN-PPR-VVD-SGP	H =-.10
CPN-PvdA-CDA-VVD	H = .24
CPN-PvdA-CDA-SGP	H = .17
PPR-PvdA-CDA-VVD	H = .11
PPR-PvdA-CDA-SGP	H =-.02

Note that the coefficient of homogeneity may increase when a fourth stimulus is taken into account, because now not only triples, but also midpoints from disjoint pairs of stimuli add to the cumulative structure. The cumulativity of midpoints from disjoint pairs of stimuli may be better than that of midpoints from stimuli in triples. Adding still more stimuli can only lower the coefficient of homogeneity for the best fitting k-tuple, however. The best fitting four tuples are those without PPR and the worst fitting four tuples are those that include both PPR and SGP. The coefficient of the four tuple of the four smallest parties is negative. Also, the four tuple CPN-PvdA-CDA-SGP has a higher coefficient of homogeneity that the four tuple of the four largest parties PPR-PvdA-CDA-VVD. This casts doubt upon the explanation given for triples, that homogeneity with respect to size is a prerequisite for a J-scale from left to right. The coefficients of scalability for all five tuples are:

PPR-PvdA-CDA-VVD-SGP	H = .01
CPN-PvdA-CDA-VVD-SGP	H = .01
CPN-PPR-CDA-VVD-SGP	H =-.03
CPN-PPR-PvdA-VVD-SGP	H = .03
CPN-PPR-PvdA-CDA-SGP	H = .09
CPN-PPR-PvdA-CDA-VVD	H = .13

The coefficient of scalability for all six stimuli together is: H = .07.

For the best fitting k-tuple, the coefficient of homogeneity changes in the following way:

PvdA-CDA-VVD	H = .21
CPN-PvdA-CDA-VVD	H = .24
CPN-PPR-PvdA-CDA-VVD	H = .13
CPN-PPR-PvdA-CDA-VVD-SGP	H = .07

As this is our first experience with this coefficient of homogeneity we are somewhat hesitant to prescribe lower boundaries for it. In our opinion these results indicate the existence of a weak stochastic J-scale of four political parties: CPN, PvdA, CDA, VVD. The two other parties, PPR and SGP, diminish the goodness of fit to such an extent, that we reject them as part of the scale. We interpret the resulting scale as an unidimensional J-scale of parties according to a left-right dimension. To interpret preference for PPR or SGP, apparently a second dimension, religious-non religious, is needed. To interpret the position of CDA, the largest religious party , however, such a second dimension is superfluous. CDA is best interpreted as holding a middle position on the left-right scale between PvdA and VVD. The only metric implication, resulting from this scale of four stimuli, indicates that the difference in scale values between CPN and PvdA is larger than the difference in scale values between CDA and VVD.

6. Conclusions

The approach we have taken to interpret preferences differs from the existing models of multidimensional analysis of preference in a different approach of <u>parsimony</u>. Whereas in the ordinary multidimensional analysis of preference parsimony is sought by finding <u>the smallest space</u> that represents <u>all</u> stimuli, in our approach, parsimony is sought by finding <u>the maximal subset</u> of stimuli that can be represented in <u>one dimension</u>. We may find that not all stimuli can be represented along one dimension, and that for a representation of all stimuli more dimensions are needed. But we do not specify any specific relations between these dimensions, like orthogonality. This condition is problematic in multi-dimensional scaling: According to Bronner & de Hoog (1978, o.c.Ch. 11), for a large minority of subjects the left-right dimension is not independent from the large-small, or power-full-not powerfull dimension. Moreover, in our approach a subject who has to indicate his preference within a pair of parts need not to take those dimensions into account on which the pair of parties is homogeneous. For instance: if PvdA and VVD are compared, the size of the party or the non-religiosity of the party need not to be taken into account although in multidimensional analysis of preferences these two parties are represented with different values for these two dimensions. When the party system is truely multidimensional that is, when for a comparison of all pairs of parties all dimensions are needed, then we should be able to find more than one subset of parties that can be represented along one dimension. In such a case, our approach, which we would denote 'multiple scaling' rather than 'multidimensional scaling' can serve as a first indication of the dimensionality of all stimuli. A similar approach is being developed presently for the analysis of similarity data (for an application to political issues, see Lipschits a.o., 1979).

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