De Programmatuur-sectie

### Computer programs for Multidimensional Scaling

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For this issue of M.D.N. we originally intended to give an overview of the programs for Multidimensional Scaling (MDS), available at the Dutch universities.

The source to consult for this is the Netherlands Computer Program Information (N.C.P.I.) which describes the about 500 programs in the field of datamanagement and statistical analysis, available in 1974. It is obtainable from:

SISWO, Oude Zijds Achterburgwal 128, Amsterdam, or from Steinmetz Archief, Kleine Gartmanplantsoen 10, Amsterdam.

At this moment, however, not only the number of universities with MDS-programs has increased, but also the number of programs itself. New models are being developed, and older models are programmed in more "user friendly" ways. We therefore decided against an overview of all programs in favor of an overview of standardized programs and packages.

We cannot give an introduction to MDS here. From our list of references the reader could consult: Ahrens, Bezembinder, Coxon, Green & Carmone, Kruskal, Roskam and Van Raay, to mention only a few.

The programs and packages to be mentioned here are: I) the Edinburgh-Cardiff MDS project; II) POLYCON and III) KYST.

For ALSCAL and the HOMALS-series, the reader is referred to the article by Jan de Leeuw, elsewhere in this issue.

# I. The Edinburgh-Cardiff MDS-project

A number of the best known and most used programs were adapted by Coxon (formerly at Edinburgh, now at Cardiff) and his staff to a standardized SPSS-like input structure and a standardized output layout.

They are available from the Zentralarchiv für Empirische Sozialforschung, Universität zu Köln, Köln 41, Bachemerstrasse 40, B.R.D., for DM 300,-. Versions exist for IBM 360 and 370, ICL, Siemens and CDC-installations.

We thank profs. Coxon, Molenaar and Roskam for their comments on an earlier version of this article.

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We shall discuss them under three headings: a) programs to analyze similarity data; b) programs to analyze preference data; c) others. The latest extensions of the project, consisting of ADDIT (Roskam), SIMULES (Carroll & Chang) and HICLUS (Johnson), are not discussed here.

### a) programs to analyze similarities data

### MINISSA (authors: Roskam & Lingoes)

Starting from a lower triangle of similarities measures between pairs of elements, the program finds a best fitting representation of the elements in a space of r dimensions (r to be specified by the researcher - in general the smallest possible number -) such that the distances between the elements in the space is a monotone decreasing function of the similarity measures between the elements. The dimensions are interpreted as 'determining' the similarities, and the place of the elements in the space is interpreted according to the coordinate values of the elements on each of the dimensions. An example: we ask people to rate their similarity judgements between pairs of Dutch political parties on a nine point scale

(e.g.: PvdA-KVP=5; VVD-PSP=2; ARP-KVP=8).

With MINISSA we may find a best fitting representation of the political parties in two dimensions that we interpret as a left-right dimension and as a government-opposition dimension.

Our inductive generalization is that people judge similarity between political parties in terms of these two dimensions. We also get the coordinate values for each of the parties on these two dimensions, that might be used for other analyses.

MINISSA is the most used model in MDS.

See Bezenbinder (p. 330) for a number of references about applications. MINISSA is an acronym for Michigan-Israel-Nijmegen-Integrated-Smallest-Space-Analysis, as it integrates Kruskal's original MDSCAL program and Guttman (Israel) and Lingoes' (Michigan) programs Smallest Space Analysis (SSA).

Roskam (Nijmegen, Netherlands) has developed a number of other models and algorithms, based on the same principles as MINISSA. The names of these programs generally start with "MINI" (like 'MINIRSA'). A simplified version of MINISSA is available in Amsterdam and in Groningen and - with some extra options and the name MNSSAST - in Nijmegen.

#### MRSCAL (author: Roskam)

MRSCAL is an acronym for MetRic SCALing.

The difference between MINISSA and MRSCAL is that the relation postulated between the input similarities data and the distances of the solution is the more restrictive, linear instead of monotone function.

The principle upon which the algorithm is based is the same as of MINISSA, but it is more restrictive in the sense that it imposes stronger assumptions on the level of measurement of the data. It replaces Torgerson's metric MDS model, and solves the additive constant problem (see Torgerson).

The original version of MRSCAL is available in Nijmegen.

### PARAMAP (authors: Carroll & Chang)

PARAMAP is an acronym for PARAmetric MAPping.

This model differs from MINISSA and MRSCAL in that the function relating data to distances is <u>at best</u> locally monotone, and minimizes an index of continuity. It is designed to accommodate highly non-linear structures.

Although there are relatively few examples of its application, this model is well adapted to scaling profile data and to projecting a space down into a very small number of dimensions. See Shepard & Carroll and Green & Rao for some examples.

### INDSCAL (authors: Carroll & Chang)

INDSCAL is an acronym for <u>IN</u>dividual <u>Differences SCALing</u>. In contrast to MINISSA the input data do not consist of a single lower triangle of similarities between pairs of elements, but of a number of such matrices, one for each 'occasion' (= subject, point in time, subgroup, etc.).

With MINISSA, each lower triangle of data can be input to find a best possible representation of the elements in a smallest space, but generally, such solutions would be different for different subjects. The idea behind INDSCAL is that those individual or private spaces use the same dimensions, but that on each occasion different patterns of weights are attached to each dimension. Or: the private spaces can be derived from a master space or group space, by individually stretching or shrinking each dimension.

The output of an INDSCAL analysis includes: a) the configuration of the elements in the group space; b) weight factors  $w_{it}$ , for each 'occasion' i, for each dimension t, to adapt the group space to the private space according to that 'occasion'.

The INDSCAL model is already very popular and many applications already exist in the literature.

See Carroll and Carroll & Wish for some examples.

### IDIOSCAL (authors: Carroll & Chang)

IDIOSCAL is an acronym for Individual Differences In Orientation SCALing.

The idea behind it is generalized from INDSCAL.

Private configurations fitting each individual's similarities matrix are derived from a group space where not only stretching and shrinking of the dimensions by appropriate weight factors is allowed, but the group space may also be individually rotated first, before any weight factors are applied.

In this respect, IDIOSCAL is similar to phase I in PREFMAP (see below). I am only aware of one official publication of the IDIOSCAL-model: Carroll & Wish.

### b) programs to analyze preference data

### MINIRSA (author: Roskam)

MINIRSA is an acronym for MINI(see above)-Rectangular-Space-Analysis. It is used as a distance model for preference analysis (i.e. multidimensional unfolding). A rank order of preferences for a number of stimuli, given by a subject is interpreted as a rank order of distances from the ideal stimulus of the subject (his ideal point) to the stimulus point. Stimuli and subjects (or better: ideal points of each subject) are represented as points in an r-dimensional space. Generally the number of subjects is not the same as the number of stimuli, so the input data matrix assumes a rectangular shape (hence the term rectangular space analysis).

The dimensions are interpreted as 'determining' the preferences. The solution contains the coordinate values on the dimensions of both the stimulus and the subject points.

A simplified version of MINIRSA is available in Amsterdam and Groningen, and with the name MNRSAST, in Nijmegen.

# MDPREF (authors: Carroll & Chang)

MDPREF is an acronym for MultiDimensional PREFerence analysis. Like MINIRSA, this is also a model for preference analysis. There are three important differences, however:

- a. The program will analyze either a set of (0,1) pair comparison matrices, or a set of rankings;
- b) The subject is not represented in the model as an ideal point, but as an ideal vector, that makes a given angle with each of the dimensions. (This angle indicates the importance of these dimensions for the preference function of the subject.

The more a stimulus has of the specific combination of dimensions indicated by the direction of the vector - the more the stimulus is preferred).

In geometrical terms: the stimuli are represented as points in a space, and the subject is represented by the vector. The projections of the points on the vector are related to the preference values of the subject for the stimuli.

c. In contrast to MINIRSA, the relation between the projections and the preference values is linear instead of monotone. This means that an analytic, rather than an iterative, solution is possible.

For an example: see Carroll or Coxon.

## PREFMAP (authors: Carroll & Chang)

PREFMAP is an acronym for PREFerence MAPping.

It differs from both MINIRSA and MDPREF in the following respects:

 a) MINIRSA and MDPREF determine both the place of stimulus points and subject's ideal points or ideal vectors from the subject's ranking. In PREFMAP by contrast, the user provides a previously obtained configuration of stimulus points, and the program fits the subject's data in this space, according to a number of models. The nature of the preference function for each subject can now be determined in some more detail.

We can distinguish four different models, or types of preference function which can be tested successively as four phases of the program. In reverse order:

phase 4: the vector model. The preference function is interpreted as "the more the better". It ressembles MDPREF in this respect.

phase 3: the unweighted distance model. The preference function is interpreted as "the closer to the ideal the better", where all dimensions are equally important for the determination of the distances. It ressembles MINIRSA in this respect.

phase 2: the weighted distance model. The same preference function as in phase 3, but dimensions may differ in importance.

phase 1: the general distance model. The dimensions may be rotated differently for each subject and then be differentially weighted.

For an extended introduction and some examples to PREFMAP, see Carroll, Coxon and Van Schuur.

## c) others

# PROFIT (authors: Carroll & Chang)

PROFIT is an acronym for PROperty FITting.

It is a program meant to aid in the interpretation of configurations that are the result of an MDS- or factor analysis. For each of the elements, values on one or more 'properties' can be collected. In the case of political parties, such 'properties' might be: the number of members, voting or rating measures of how left or how right the party is, whether the party is a governmental or opposition party, etc. Each of these properties is then used as a dependent variable. The coordinate values of the elements on each of the dimensions are used as independent variables to 'predict' the property values, and the PROFIT program then estimates the direction (or vector) in the space in which the property most increases, by performing a multiple regression analysis. Very often, the original MDS-configuration can be rotated to this regression line as a permissable transformation. For some examples of PROFIT, see Coxon and Van Schuur (1975).

### UNICON (author: Roskam)

UNICON is an acronym for <u>UNI</u>-dimensional <u>CON</u>joint measurement for multi-facetted designs.

The model of conjoint measurement is related to the nonmetric models of MDS. For an introduction we refer the reader to Bezembinder (p. 79 ev). We borrow an example from him to illustrate UNICON.

According to Hull, in learning theory the performance (P) is equal to the product of learning (H), drive(D) and incentive(K):

### $\mathbf{P} = \mathbf{H}\mathbf{D}\mathbf{K}$

According to Spence, however:

P = H(D + K)

When testing these theories, we can speak of three facets (learning, drive and incentive), each with a given number of levels. These levels may be nominal or ordinal categories. For all combinations of facet-levels we can obtain a performance level, which we have to be able to rank order.

UNICON finds numerical values for all facet-levels, such that the numbers are strictly monotonely related to the performance levels, according to for instance either the theory of Hull or the theory of Spence.

That is: facets may be additively, subtractively or multiplicatively combined. Combinations of up to five facets are admissible. See Roskam (1974) for a more elaborate introduction.

### II. POLYCON: POLYnomial CONjoint analysis.

A second project to mention is the POLYCON package, developed by Forrest W. Young. In POLYCON, a large number of possible non-metric analysis are available as options in the package. Input structure follows an SPSS-like format. It is available at Leiden, and will be available soon in Groningen, and possibly on other CDCinstallations as well.

The POLYCON package can be obtained from Forrest W. Young, The L.L. Thurstone Psychometric Laboratory, Univ. of North Carolina, Chapel Hill, N.C., 27514 for \$ 125,-

This introduction to POLYCON is a summary of Young's article(1972). Young shows that a wide range of models can be derived as specifications of a more general model. These submodels include: a. Minkowski distance scaling;

- Multidimensional unfolding. The rectangular version of the Euclidean nonmetric multidimensional scaling submodel corresponds to the multidimensional unfolding model, proposed by Coombs;
- c. Monotone analysis of variance. Luce and Tukey have presented an additive model, which they refer to as the conjoint measurement model, and which can be regarded as an ordinal analog of two-way analysis of variance;
- d. Polynomial conjoint measurement;
- e. Nonmetric factor analysis;

f. Subjective expected utility. According to the subjective utility model (Savage,1954), when a subject chooses between two gambles he makes his choice by maximizing the subjective expected utility of the choices. The subjective expected utility of a gamble is equal to the sum, over the various choice objects, of the product of an outcome and its subjective probability of occurrence. Under certain conditions, this model is formally identical to the nonmetric analog of the factor analysis model.

g. Bradley-Terry-Luce choice model. This model is formally identical to a one dimensional Minkowski metric.
This model (Luce,1959) specifies the relation of choice probabilities to the scale values of the object when two choices are presented. The model states that

$$p(c,d) = \frac{v(c)}{v(c) + v(d)}$$

Where v(c) and v(d) represent the scale values. The ordinal version of this model can be written as:

# $p(c,d) > p(e,f) \leftrightarrow v(c) - v(d) > v(e) - v(f)$

#### III. KYST

KYST, acronym for Kruskal, Young, Shepard and Torgerson, is a very flexible program to do multidimensional scaling and unfolding, and succeeds the older MDSCAL program. It is available from Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974, USA.

I am only aware of one piece of documentation about it: Kruskal, Young & Seery (1973).

It is similar to POLYCON in its key-word input and in a number of the (distance-) models.

Judging from the literature on POLYCON and KYST, the major differences between the packages seem to be:

a. the impossibility of additive and multiplicative models in KYST

- b. the possibility of other relations between s and Dhat besides monotone regression. In KYST polynomial regression from linear up to the fourth degree is possible, with or without a constant term.
- c. more and better documented ways to generate an initial configuration in KYST than in POLYCON.
- d. KYST is perhaps more 'portable' (machine independent) and smaller in terms of core memory than is POLYCON.

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