An optimal marketing strategy for porkers with differences in growth rates and dependent prices

J. Roemen* and J. de Klein

Abstract

In this paper we consider the question how a farmer should organise the delivery of groups of porkers, taking into account that each group consists of subgroups with different growth rates, that pork prices vary in time stochastically and interdependently and that the next fattening round can only be started when all animals in the current round have been delivered for slaughter. The feeding-regime is assumed to be given. So the central question is how a farmer should react upon the variability of the pork price. For the solution of this problem a Markov decision model is formulated. This model provides sets of critical pork price-pork age combinations during the periods in which the animals are slaughter-ripe. If the actual price-age combination in a week belongs to a specific set, the farmer should decide upon selling this subgroup (or combination of subgroups), whereas fattening should be continued otherwise. An example based on Dutch pork sector data is presented to clarify the theme of this paper.

* Department of Econometrics, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands. E-mail: J.H.J.Roemen@kub.nl.

1 Introduction

In this study we deal with the problem at what times a farmer should sell a group of animals to achieve a maximal profit, taking into account that the animals show different rates of growth, that the selling price varies with time, and that the next fattening round can only start when all animals in the current round have been delivered for slaughtering. For convenience the investigation of this general problem is restricted to the specific situation of fattening pigs.

(Dis)investment problems related to stocks of "living" commodities (for instances cattle or crops) have received much attention. Examples are the articles of Burt [2] Chavas, Klein and Crenshaw [3] Feinerman and Siegel [4], Kristensen [8], Rausser and Hochmann [10] or the publications mentioned under [1], [5], [6], [9] and [12]. Burt uses a dynamic programming formulation to derive decision rules that give feed rations as a function of animal weight and which provide the critical weight at which a group of animals should be sold and replaced with another group. He explicitly incorporates in this model the circumstance that it concerns competitive markets where selling prices generally spoken just cover costs incurred. Chavas, Kliebenstein and Crenshaw formulate an optimal control model to derive conditions for efficient production. These conditions treat simultaneously optimal input use and optimal replacement policy. Feinerman and Siegel present a farm-level feedlot optimisation model for calculating the optimal feeding schedule, market live weight and stock replacement decisions for a single animal over the planning horizon. Kristensen introduced the concept of the hierarchic Markov process in (animal) replacement models. It is a contribution to the solution of the dimensionality problem in Markov decision models. Rausser and Hochman formulate a dynamic programming model for the optimal marketing age of a commodity undergoing a growth process. The solution of this model gives critical values of the selling prices at each age below which the group of animals will be kept for another period and above which the animals will be sold in the current period.

In the papers mentioned above with the exception of [5] and [9] only homogeneous batches of animals are considered: just one growth function applies. Also, no attention is given to the situation where the selling prices in consecutive periods depend on each other, though Rausser and Hochman propose an extension of their model in this direction. In this paper we present an extension of the problem taking into account the existence of differences in growth within a group of animals. Also, the marketing strategy for the situation where the selling prices in consecutive periods depend on each other - that means that last period's price informs about this period's price - is incorporated in our model. However, in contrast to the papers cited, the feeding regime is assumed to be given, so no attention is paid to the determination of optimal feeding rations.

Nowadays, in many countries the production of pork mostly takes place on specialised farms. Such a farm has at its disposal a number of barns, which are divided into compartments. The number of animals that can be placed in a compartment varies from a few tens to well over one hundred, depending on the size of the compartment. Because of health considerations the so-called "all in-all out" system is usually employed for the production of pork. This means that a compartment is occupied by young pigs at one time, and that all these animals have to be delivered before the next fattening round in this compartment can start. The successive periods of fattening are separated by a short period in which the compartment is cleaned thoroughly. When the young pigs are placed in a compartment, they all have about the same weight. By a balanced feeding-regime the animals are then fattened during a certain period, that is, until they have reached a weight suitable for slaughtering. We assume that this feeding regime is given and will not be changed. This assumption is based on the following considerations. Worldwide, many experiments have been set up (and are going on) for determining the optimal quantity and composition qua nutritients of feed for porkers by state or sector financed agricultural research institutes. By comparing the results of these experiments these institutions develop feeding schemes which guarantee a good development of the porker at a reasonable cost and so can be advised to pork producers. Rather than trying to reach an optimal scheme by trial and error himself the producer will prefer to choose among the advised alternatives as their value is proven. To a once chosen regime the producer will, generally speaking, adhere, because changes in regime are often accompanied by disturbances in the development of the porker. Such a feed can be composed by the farmer himself, if he has at his disposal a mill, mixing equipment and the necessary ingredients. However, as these feeds are readily available in most countries in the quantities and compositions as demanded, the greater part of it is bought by the producers from specialised firms.

Now it turns out that during the period of fattening the animals in one compartment show a large variation in growth rates, that is, in the weight gain per kilogram feed intake. As a consequence, the animals reach a certain weight at different times. In modelling the development over time of the expected weight and feed intake of all animals in a compartment these differences have to be taken into account. For that purpose we conceive of the whole group of pigs as composed of a number of reasonably homogeneous subgroups, each of them characterised by just one growth and feed intake function. These functions describe the development over time of the average weight and the average feed intake within each subgroup. Appealing to the law of large numbers we assume that the expectation of the weight and feed intake of the whole group at every time can be well approximated by combining the subgroup functions. For our goal it suffices to distinguish two groups: fast growers and slow growers. As soon as an animal reaches a certain minimal weight, the farmer has the opportunity to sell it to the slaughterhouse. Because of the differences in weight and meat quality of the supplied animals, the slaughterhouses usually do not use one single price but a system of prices. The basic price is paid for an animal with a standard weight and meat quality. Deviations from this standard are taken into account by means of a system of bonuses and (penalty) discounts. In the present study we leave the aspect of quality out of consideration, and we assume that the price per kg is the same for all weights. This price, however, is not constant over time, but varies from period to period. In general, the price in an arbitrary period is found in a restricted interval around the price in the preceding period. Therefore we assume that the price of pork in an arbitrary period is a stochastic variable, which only depends on the price in the preceding period. The feed price and interest rate also show variability, but this variability is of a much smaller order of magnitude than that of the pork price. Therefore it is no important simplification to regard the feed price and interest rate as deterministic. The heterogeneity of the animals in one compartment together with the "all in-all out" system and the stochastic behaviour of the pork price raises the question whether it is profitable for the farmer to sell the animals in one compartment at different times or at one time.

2 A Markov Decision model

As an introduction we first consider the situation where all pigs have the same growth properties. These properties are assumed to remain constant through all cycles.

The decision problem of the producer in this situation can be formulated by a continuous time model. However, in view of the next section where one of the main elements of the model, the pork price, changes periodically and not continuously, it will be stated in discrete time.

At the beginning of a week, after a compartment has been cleaned up thoroughly at a cost of p_c and is again ready for use, it is filled with N young pigs bought at a price of p_a per animal. Of course, p_a is closely related to the price of fat porkers. At that moment the animals have completed an age of x_0 weeks, so fattening starts in their $(x_0 + 1)$ week of age. By a balanced feeding regime they are fattened till they have completed an age of (at most) $x_{\max}$ weeks. For the reasons given in the introduction this regime is fixed and cannot be changed by the farmer.

An animal of age x receives a feed ration of $u(x)$ kg. For the function $u(x)$ it is reasonable to assume that

$$u(k) - u(k-1) > 0,\ k = x_0 + 1, \ldots, x_{\max} \tag{2.1}$$

and that

$$u(k) - u(k-1) < u(k-1) - u(k-2), k = x_0 + 2, \ldots, x_{\max} \tag{2.2}$$

An ever growing part of the feed intake is needed for the maintenance of the animal. Feed is purchased at a price of p_u per kg.

Feed intake together with live weight determines live weight gain of an animal. We suppose that given the fixed feeding regime the weight development can be represented as a function of the age of the animal only. This weight will be denoted by $w(x), x = x_0, \ldots ; x_{\max}$. As to the growth function, $w(x)$, it is assumed that

$$w(k) - w(k-1) > 0,\ k = x_0 + 1, \ldots, x_{\max} \tag{2.3}$$

and that

$$w(k) - w(k-1) < w(k-1) - w(k-2), k = x_0 + 2, \ldots, x_{\max} \tag{2.4}$$

So weight increases from week to week, but at a decreasing rate. These increases are supposed to take place at the end of the weeks.

Only animals possessing a weight within a certain range, say 90-130 kg, can be sold. Animals having a weight outside this range do not satisfy the quality requirements imposed by the slaughterhouses and hence yield nothing. If the weight of an animal lies between the minimum and maximum allowed, we call it slaughterripe. The minimal and maximal weight of a slaughterripe animal will be denoted by $w_{\min}$ and $w_{\max}$ and the corresponding minimal and maximal slaughter age by $x_{\min}$ and $x_{\max}$.

The price per kg. of pork is determined by extraneous circumstances, so the producer cannot influence this price by the number of porkers supplied. In this section we assume that it is governed by a discrete probability distribution and that the prices in successive weeks are independent of each other and identically distributed. Thus the price of pork in week t, denoted by $\mathbf{Y}_t$, possesses the following distribution

$$q_j = P\{\mathbf{Y}_t = y_j\}, \tag{2.5}$$

where the possible realisations are numbered in ascending order, that is, $y_1 < y_2 < \cdots < y_m$.

By fattening pigs the farmer tries to earn an income. As an approximation to this goal we choose the maximalisation of the expected discounted net financial result from the ongoing and all future fattening rounds. This financial result depends on the decisions which the producer takes in the successive weeks of the succession of fattening rounds. In each week he decides whether to sell the animals or to postpone the sale and carry on fattening. In making his decision the farmer is guided by the weight of the animals and the price of pork. As long as the animals have not reached the minimal slaughter weight, the producer has no choice but to proceed with fattening. Neither has the farmer a choice, when the animals have such a weight that they, being fed for another week, will exceed the maximal slaughter weight. In this case the animals are sold immediately. In the remaining weeks the farmer can always choose from two alternatives. He can decide to sell the animals at the current, known, price or he can decide to dispose of the animals in one of the coming weeks at the price valid then, but currently unknown. We will denote the decision rule applied in week t by a_t. If $a_t = 1$, he sells and if $a_t = 0$, he continues fattening. The sequence of decision rules for the successive weeks is called a strategy and will be denoted by the symbol π. By terminating each fattening cycle at a suitable moment, that is, by choosing a suitable strategy, the producer can realise a maximal financial result.

For the determination of the optimal fattening strategy we make use of a Markov decision model. In accordance with the terminology of this method we introduce the stochastic proces $\{(\mathbf{X}_t, \mathbf{Y}_t), t = 0, 1, 2, \dots\}$, where $\mathbf{X}_t$ stands for the age of an animal in weeks and $\mathbf{Y}_t$ for the price of pork in week t. The age varies from x_0 up to $x_{\max}$ and the pork price from y_1 up to y_m. Now suppose that in an arbitrary week, week k, the system is in state (x, y) and the farmer chooses with some probability rule a. Then the farmer receives a reward $r(x, y; a)$ in that week and the system changes into a new state $(\hat{x}, \hat{y})$. The probability that such an event occurs is denoted by $p_{x,y;\hat{x},\hat{y}}(a)$. If the farmer chooses the strategy π, that is, in week t he chooses rule a_t, and the system is initially in state (x, y), then the expected total discounted return is given by

$$v_\pi(x, y) = E_\pi \left\{ \sum_{t=k}^{\infty} r(\mathbf{X}_t, \mathbf{Y}_t; a_t)\alpha^{t-k} \,|\mathbf{X}_k = x, \mathbf{Y}_k = y \right\}, \tag{2.6}$$

where α stands for the weekly rate of discount and E_π represents the conditional expectation, given that strategy π is chosen. Note that discounting the returns mitigates the difficulty of an infinite horizon. Now the problem faced by the farmer is to find a strategy π^* that maximizes $v_\pi(x,y)$, that is,

$$v_{\pi^*}(x,y) = \max_\pi v_\pi(x,y) \tag{2.7}$$

The optimal policy value function v_{π^*} satisfies the Bellman optimality equation (see Ross [11]):

$$v_{\pi^*}(x,y) = \max_a \left\{ r(x,y;a) + \alpha \sum_{(\hat{x},\hat{y})} p_{x,y;\hat{x},\hat{y}}(a) v_{\pi^*}(\hat{x},\hat{y}) \right\}, \tag{2.8}$$

where the summation is over all possible states $(\hat{x},\hat{y})$.

For the decision problem of the farmer the optimality equation (2.8) can be specified as follows.

After the termination of a fattening round and the cleaning of the compartment the next fattening round starts at the beginning of a week with the arrival of piglets, that have completed an age of x_0 weeks. That means that for $x = x_0 + 1$

$$v_{\pi^*}(x,y) = -Np_a - Nu(x)p_u + \alpha \sum_{j=1}^{m} q_j v_{\pi^*}(x+1, y_j). \tag{2.9}$$

As long as the animals are not yet slaughterripe, the farmer has no choice but to continue fattening. So for $x_0 + 1 < x \le x_{\min}$

$$v_{\pi^*}(x,y) = -Nu(x)p_u + \alpha \sum_{j=1}^{m} q_j v_{\pi^*}(x+1, y_j) \tag{2.10}$$

If the animals have gained sufficient weight, the farmer can decide to continue fattening or to sell and start a new fattening round. So for $x_{\min} < x \le x_{\max}$

$$\begin{aligned} v_{\pi^*}(x,y) = \max \Bigg\{ & -Nu(x)p_u + \alpha \sum_{j=1}^{m} q_j v_{\pi^*}(x+1, y_j), \\ & Nw(x-1)y - p_c + \alpha \sum_{j=1}^{m} q_j v_{\pi^*}(x_0+1, y_j) \Bigg\}, \end{aligned} \tag{2.11}$$

assuming that the decision to sell is taken and executed at the beginning of a week and that in that same week the cleaning will take place.

When the animals have completed an age of $x_{\max}$ weeks, the farmer must sell them, clean the compartment and start the next fattening round. So for $x = x_{\max} + 1$

$$v_{\pi^*}(x,y) = Nw(x-1)y - p_c + \alpha \sum_{j=1}^{m} q_j v_{\pi^*}(x_0+1, y_j) \tag{2.12}$$

From (2.11) it can be concluded that selling is optimal if

$$Nw(x-1)y - p_c + \alpha \sum_{j=1}^{m} q_j v_{\pi^*}(x_0+1, y_j) > -Np_u u(x) + \alpha \sum_{j=1}^{m} q_j v_{\pi^*}(x+1, y_j) \tag{2.13}$$

whereas the sale should be postponed if the opposite is true. If the equality sign holds for (2.13), selling and continuing fattening are equally profitable.

The implication of (2.13) is that for each slaughterripe age there exists a critical pork price. If, given the age of the animals (and so the weight) in a certain week, the pork price in that week is larger than the critical pork price, the animals should be sold immediately, whereas fattening should be continued in the opposite case.

These critical selling prices can also be obtained by the reasoning proposed by Rausser and Hochmann. They argue that postponement of the date of sale causes opportunity costs to arise. By these opportunity costs they mean the expected net revenue per week in the long run. Postponement means that the inflow of this net result is shifted from the current week to one of the weeks to come, so for at least one week. This argument is given shape by, next to the feeding expenses for another week, including this amount of missed net revenue as an expense in the week for which postponement is decided.

3 Incorporation of Heterogeneity and Dependent Prices

After these preparations we are ready to present the main theme of this paper. How should the farmer arrange the delivery of heterogeneous groups taking into account the dependency between pork prices from week to week and the "all in - all out" system?

By heterogeneity we understand here that the animals in a compartment grow with different speed, that is, they differ in weight gain per kg. feed intake. For convenience it is assumed that out of the total of N animals in a compartment, a number of N_1 grows relatively fast and a number of N_2 relatively slow. We suppose that soon after the start of a fattening cycle the farmer can indicate to which subgroup each individual animal belongs. A fast grower of age x receives a feeding ration of $u_1(x)$ and a slow grower $u_2(x)$. The weight of a fast grower of age x is denoted by $w_1(x)$ and that of a slow grower by $w_2(x)$. At the beginning of each cycle the animals, all x_0 weeks of age, have all the same weight, $w_1(x_0) = w_2(x_0)$

For the functions $u_1(x), u_2(x), w_1(x)$ and $w_2(x)$ we maintain the assumptions (2.1) till (2.4), i.e.:

$$u_i(k) - u_i(k-1) > 0, k = x_0 + 1, \ldots, x_{i,\max}, i = 1, 2 \tag{3.1}$$

$$u_i(k) - u_i(k-1) < u_i(k-1) - u_i(k-2), k = x_0 + 2, \ldots, x_{i,\max} \tag{3.2}$$

$$w_i(k) - w_i(k-1) > 0 \tag{3.3}$$

$$w_i(k) - w_i(k-1) < w_i(k-1) - w_i(k-2) \tag{3.4}$$

Heterogeneity will be understood as

$$\frac{w_1(k)}{\sum_{j=x_0}^{k} u_1(j)} > \frac{w_2(k)}{\sum_{j=x_0}^{k} u_2(j)} \tag{3.5}$$

The fast growers reach the minimal weight for which a positive pork price holds at the age of $x_{1,\min}$ weeks and they can be delivered at latest at the age of $x_{1,\max}$ weeks. With the delivery of the slow growers the farmer can only start at a later age, namely at the age of $x_{2,\min}$ ($> x_{1,\min}$). At latest these animals leave the farm at the age of $x_{2,\max}$ ($> x_{1,\max}$). In the sequel it is assumed that $x_{1,\max} \geq x_{2,\min}$ which is the most common configuration.

In the preceding section we assumed that the pork prices in successive weeks are independent and identically distributed random variables. The implication of this hypothesis is, that this week's price contains no information with respect to the price in the coming week. However, for prices for agricultural products off farm one often finds that this week's price deviates none or but little from the price in the foregoing week. This phenomenon also applies to the Dutch pork price, as appears from table 3.1 which gives the distribution of the (absolute) price mutations from week to week (in cents) for pork off farm in the Netherlands during the years 1987-1996. During that period the average pork price amounted to fl. 3.04 per kg.

Table 3.1 Distribution of the (absolute) pork price mutations from week to week in cents in the Netherlands during 1986-1996

Mutation (in cents)	0-3	4-7	8-11	12-15	16-19	20-31
Distribution (in %)	29	22	19	14	4	12

The circumstance that this week's price is informative as to next week's price can be incorporated by treating the pork prices in successive weeks as dependent. In modelling this dependency we choose a Markov process to describe the evolution of the pork price over time. For convenience we again assume that the price in any week possesses a discrete distribution. The possible realisations are given by $y_1, y_2, \ldots, y_m$ with $y_1 < y_2 < \cdots < y_m$. Further we assume that the transition probabilities are stationary in time. So, denoting the pork price in week t by $\mathbf{Y}_t$ as before, the process $\{\mathbf{Y}_t, t = 0, 1, \ldots\}$ constitutes a Markov chain with stationary transition probabilities q_{ij} :

$$P\{\mathbf{Y}_{t+1} = y_j | \mathbf{Y}_t = y_i\} = q_{ij} \tag{3.6}$$

Given a proportional relation between the prices for piglets and fat porkers and a pork price of y_i in period t, the price for young animals in that period now amounts to $p \cdot y_i$ with p the proportionality constant.

For the determination of the optimal delivery strategy for a heterogeneous group of porkers and dependent pork prices we again formulate a Markov decision model. To this end we introduce the stochastic process $\{(\mathbf{X}_t, \mathbf{C}_t, \mathbf{Y}_t), t = 0, 1, 2, \ldots\}$, where $\mathbf{X}_t$ stands for the age of an animal, $\mathbf{C}_t$ for the composition of the livestock and $\mathbf{Y}_t$ for the price of pork in week t. The composition indicates whether both groups are present ($\mathbf{C}_t = 12$) or whether only group 2 is present and group 1 is already sold ($\mathbf{C}_t = 2$). We denote the expected total discounted return by $v_\pi(x, c, y_i)$, given that the farmer chooses the delivery strategy π and that the system is initially in state (x, c, y_i). The weekly rate of discount is given by α. The farmer now attempts to find a strategy that maximizes $v_\pi(x, 12, y_i)$.

In deriving the optimality equations we distinguish the following situations (compare the similar derivation in the preceding section).

i. When a fattening round has come to its end and the compartment has been cleaned, the next fattening round starts with the arrival of young piglets of age x_0 at the beginning of a week. That means that for $x = x_0 + 1$ we have

$$v_{\pi^*}(x, 12, y_i) = -Npy_i - \{N_1 u_1(x) - N_2 u_2(x)\}\, p_u + \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x+1, 12, y_j) \tag{3.7}$$

ii. As long as the animals are not yet slaughterripe, the farmer can only continue fattening. So for $x_0 + 1 < x \leq x_{1,\min}$ we have

$$v_{\pi^*}(x, 12, y_i) = -N_1 u_1(x) p_u - N_2 u_2(x) p_u + \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x + 1, 12, y_j) \qquad (3.8)$$

iii. If the fast growers have gained sufficient weight, but the slow growers are still too light, the farmer can choose between two possibilities. He can decide to continue fattening both groups, or he can decide to sell the fast growers and to proceed with the fattening of only the slow growers. His choice of course depends on the value of the pork price. Assuming that this decision is taken and executed at the beginning of a week, the revenue from selling the fast growers is calculated by multiplying the weight at the end of the preceding week by this week's pork price. So for $x_{1,\min} < x \leq x_{2,\min}$ we have

$$\begin{aligned} v_{\pi^*}(x, 12, y_i) = \max \Bigg\{ & -(N_1 u_1(x) + N_2 u_2(x)) p_u + \alpha \sum_{j=1}^{m} q_{ij} \\ & v_{\pi^*}(x + 1, 12, y_j), N_1 w_1(x - 1) y_i - N_2 u_2(x) p_u + \\ & \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x + 1, 2, y_j) \Bigg\} \end{aligned} \qquad (3.9)$$

iv. If both groups are slaughterripe, the farmer can choose from three possibilities. He can proceed with fattening both groups, he can sell the fast growers and continue fattening the slow growers or he can sell both groups. So for $x_{2,\min} < x \leq x_{1,\max}$ we have

$$\begin{aligned} v_{\pi^*}(x, 12, y_i) = \max \{ & -(N_1 u_1(x) + N_2 u_2(x)) p_u + \\ & \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x + 1, 12, y_j), N_1 w_1(x - 1) y_i + \\ & -N_2 u_2(x) p_u + \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x + 1, 2, y_j), \\ & N_1 w_1(x - 1) y_i + N_2 w_2(x - 1) y_i - p_c + \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x_0 + 1, 12, y_j) \} \end{aligned} \qquad (3.10)$$

v. When the fast growers have completed their maximal age, they must be sold. The choices open to the farmer are then to sell the fast growers and to continue fattening the slow growers or to sell both groups. So for $x = x_{1,\max} + 1$ we have

$$\begin{aligned} v_{\pi^*}(x, 12, y_i) = \max \{ & N_1 w_1(x - 1) y_i - N_2 u_2(x) p_u \\ & + \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x + 1, 2, y_j), N_1 w_1(x - 1) y_i + \\ & + N_2 w_2(x - 1) y_i - p_c + \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x_0 + 1, 12, y_j) \} \end{aligned} \qquad (3.11)$$

Finally we have the optimality equations for the situations where the fast growers have already been sold.

vi. When the slow growers are not yet slaughterripe, the farmer has no choice but to continue fattening. So for $x_{1,\min}+1 < x \leq x_{2,\min}$ we have

$$v_{\pi^*}(x,2,y_i) = -N_2 u_2(x) p_u + \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x+1,2,y_j) \tag{3.12}$$

vii. When the slow growers are slaughterripe, the farmer can decide to proceed with fattening or he can decide to sell. So for $x_{2,\min} < x \leq x_{2,\max}$ we have

$$v_{\pi^*}(x,2,y_i) = \max\left\{ -N_2 u_2(x) p_u + \alpha \sum_{j=1}^{m} q_{ij} \right. \\ \left. v_{\pi^*}(x+1,2,y_j),\, N_2 w_2(x-1) y_i - p_c + \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x_0+1,12,y_j) \right\} \tag{3.13}$$

viii. At age $x = x_{2,\max}+1$ the farmer has no longer any choice: he must sell. We get

$$v_{\pi^*}(x,2,y_i) = N_2 w_2(x-1) y_i - p_c + \alpha \sum_{j=1}^{m} q_{ij} v_{\pi^*}(x_0+1,12,y_j) \tag{3.14}$$

The solution of this model provides us with three sets of what we call critical age-price combinations. The first set holds for the fast growers during the period $x_{1,\min}$ till $x_{1,\max}$ and will be denoted by $\gamma_1(x,y)$. If during this period the actual age-price combination in a week belongs to this set, then (at least) the fast growers should be sold immediately. However, when this combination falls outside this set, the fattening of this subgroup should be continued. The second set, $\gamma_{12}(x,y)$, holds for fast and slow growers together during the period $x_{2,\min}$ till $x_{1,\max}$, given that the fast growers have not been sold during the period $x_{1,\min}$ till $x_{2,\min}$. When the actual age-price combination in a week falls within this set, both subgroups should be sold, while in the opposite case fattening should be continued. When the fast growers have already been sold, the decision whether to sell the remaining animals or not is governed by the third critical set, $\gamma_2(x,y)$. This set applies to the period $x_{2,\min}$ till $x_{2,\max}$. For age-price combinations within this set this subgroup should be sold immediately, while fattening should be continued in the opposite case. As soon as the slow growers have been delivered, a new fattening round can be started after a thorough cleaning of the compartment. Because the starts of the fattering rounds and the feed intake and weight gain functions differ from farm to farm, it is to be expected that the sets γ_1, γ_2 en γ_3 vary from farm to farm also.

Several numerical methods are available for the computation of the critical sets-$\gamma_1(x,y), \gamma_{12}(x,y)$ and $\gamma_2(x,y)$. The Markov decision problem formulated above can be solved by the value iteration method, by the policy iteration method and by linear programming. A detailed discussion of these methods can be found in Ross [11] or Tijms [13]. Here the policy (or strategy) iteration method was used for finding an optimal stationary policy. First for given π the value of the elements $v_\pi(x,c,y)$ for all x, c and y is calculated. After that the optimal strategy π^* is sought: $v_{\pi^*}(x,c,y) = \max_\pi v_\pi(x,c,y)$.

A numerical example may help to clarify the idea of these critical sets. As a starting point for modelling growth and food intake we chose the following functions from among the many available alternatives

$$\begin{aligned} \overline{z_{1j}(x)} &= a_{j1} \exp - \left\{ b_{j1} \overline{w_j(x-1)} + \frac{c_{j1}}{\overline{w_j(x-1)}} \right\} j = 1, 2; x = x_0 + 1, \dots, x_{j,\max} \\ \overline{z_{2j}(x)} &= a_{j2} \exp - \left\{ b_{j2} \overline{w_j(x-1)} + \frac{c_{j2}}{\overline{w_j(x-1)}} \right\}, \end{aligned} \tag{3.15}$$

where $\overline{w_j(x-1)}$ stands for the average weight of an animal during week $x-1$ after birth, $\overline{z_{1j}(x)}$ for the average daily growth in week x, $\overline{z_{2j}(x)}$ for the average daily food intake in that week and $a_{j1}, a_{j2}, b_{j1}, b_{j2}, c_{j1}$ and c_{j2} for coefficients. Given the coefficients, the weight of a porker at an age of x weeks after birth, $w_j(x)$, and the food intake, $u_j(x)$, can easily be calculated. These relations are a variation on those used by Kanis for the description of the development of growth and food intake of individual porkers in his research concerning food intake and production traits of animals [7]. (Actually Kanis used $\overline{w_j(x)}$ instead of $\overline{w_j(x-1)}$.) To specify the coefficients in these functions we used frequency distributions of growth and food intake data collected during feeding regime experiments as published in [5]. As the representation of the subgroups we took the average of the top 50% of these frequency distributions for the fast growers and the bottom 50% for the slow ones. It should be remarked that these data cover only a part of the fattening cycle. In this way we arrived at the following specifications

$$
\begin{aligned}
\overline{z_{11}(x)} &= 2,569 \exp - \left\{ 0,0075\overline{w_1(x-1)} + \frac{40}{\overline{\overline{w_1(x-1)}}} \right\} \\
\overline{z_{12}(x)} &= 2,800 \exp - \left\{ 0,0110\overline{w_2(x-1)} + \frac{43}{\overline{\overline{w_2(x-1)}}} \right\} \\
\overline{z_{21}(x)} &= 6,600 \exp - \left\{ 0,0030\overline{w_1(x-1)} + \frac{46}{\overline{\overline{w_1(x-1)}}} \right\} \\
\overline{z_{22}(x)} &= 5,000 \exp - \left\{ 0,00275\overline{w_2(x-1)} + \frac{41}{\overline{\overline{w_2(x-1)}}} \right\}
\end{aligned}
\tag{3.16}
$$

Taking $N = 100$ we arbitrarily divided it up between $N_1 = 40$ and $N_2 = 60$. As an approximation of reality we chose $x_0 = 8$, $w(x_0) = 25$, $w_{\min} = 90$, $w_{\max} = 130$, $x_{1,\min} = 22$, $x_{1,\max} = 28$, $x_{2,\min} = 24$ and $x_{2,\max} = 33$.

For the determination of the pork prices and the transition probabilities to use in the example we took the weekly bid prices per kg. slaughter weight of standard quality off farm as quoted by some mayor Dutch slaughterhouses over the period 1987-1996. The highest price observed during this period was *fl.* 4,59 and the lowest *fl.* 2,16. The average price amounted to *fl.* 3,04 with a standard deviation of *fl.* 0,48. In view of the calculation of the transition probabilities in (3.6) price classes, each comprising a range of prices within a lower and an upper limit, were defined as states in the Markov chain. For ease of computation only 7 states are discerned. As a consequence the range within each state is rather wide. The average bid price was chosen as class middle of the mid price class. By dividing the number of transitions from class i to class j by the total of transitions from i the following Markov matrix was obtained

Table 3.2 The matrix P

$$\begin{array}{c|ccccccc} & j & & & & & & \\ i & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 0,68 & 0,32 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0,08 & 0,79 & 0,13 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0,11 & 0,79 & 0,10 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0,20 & 0,66 & 0,14 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0,18 & 0,63 & 0,19 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0,20 & 0,55 & 0,25 \\ 7 & 0 & 0 & 0 & 0 & 0,01 & 0,18 & 0,81 \end{array} \tag{3.17}$$

Range class 1: 217-241 class 4: 292-316 class 7: 367-391

Range class 2: 242-266 class 5: 317-341

Range class 3: 267-291 class 6: 342-366

Class 1 also encompasses some observations below 2.17, as class 7 does for a few observations above 3.91. The matrix (3.17) clearly exhibits a diagonal structure as was to be expected from table 3.1. Because the observations within each price class are distributed fairly homogeneously, we took the mid of each class as the representative price for that class. These representative prices were then considered as the possible realisations of a Markov chain that is governed by the matrix of transition probabilities (3.17).

For the remaining prices, p_c, p_u, p_a and α, we took the following values: $p_c = 750, p_u = 0,50, \alpha = 0,9975$ and, on the basis of a regression analysis,

$$p_{ai} = 32,8y_i, i = 1, \ldots, 7, \tag{3.18}$$

because the prices of young and fat porkers are closely correlated.

The result of the application of the Markov decision model to the example is given in the figures 3.1, 3.2 and 3.3. Figure 3.1 concerns the set $\gamma_1(x, y)$, figure 3.2. the set $\gamma_{12}(x, y)$ and figure 3.3 the set $\gamma_2(x, y)$. The elements of these sets are indicated by crosses (figure 3.1), stars (figure 3.2) and circles (figure 3.3). These elements were calculated by the strategy iteration method.

Figure 3.1. The set γ_1 for the matrix P

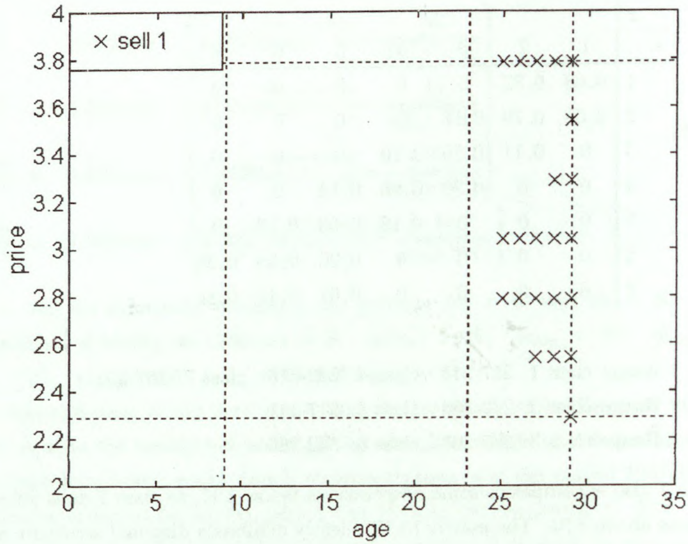


Figure 3.2. The set γ_{12} for the matrix P

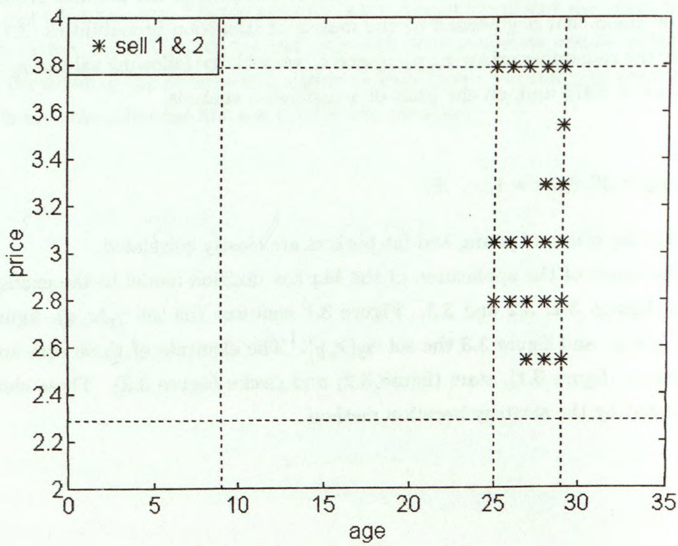


Figure 3.3. The set γ_2 for the matrix P

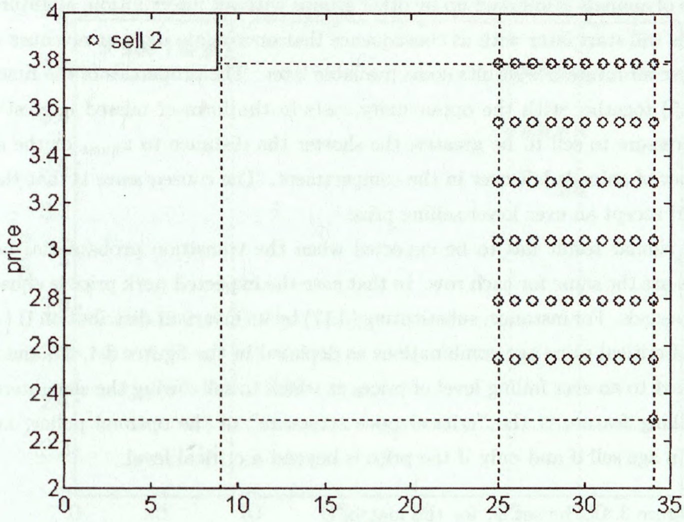


Figure 3.2, for instance, can be explained as follows. If in a certain week the animals reach an age of 25 weeks and the pork price for that week amounts to $fl.$ 3,79 (class 7), $fl.$ 3,04 (class 4) or $fl.$ 2,79 (class 3), then both subgroups should be sold in that week. However, if the pork price in that week falls into class 6 ($fl.$ 3,54), class 5 ($fl.$ 3,29), class 2 ($fl.$ 2,54) or class 1 ($fl.$ 2,29), then the animals should be kept for fattening them further. The figures 3.1 and 3.3 can be given a similar explanation. It should be noted that for this example the sets $\gamma_1(x,y)$ and $\gamma_{12}(x,y)$ differ in but one element.

At first sight a clear structure seems to be lacking in the figures 3.1 and 3.2. For instance, taking figure 3.2, if it pays to sell at age 25 at a price of $fl.$ 3,04, why shouldn't that hold at the higher price of $fl.$ 3,29? However, unclear though this may seem, the shape of the figures can very well be understood by the following reasoning.

When (part of) the animals are slaughterripe, the producer can decide to keep them for another week or to sell them irrespective of the level of the price. If he does not sell, but keeps them, this postponement has financial consequences. First, it means that the batch is fed for another week, so the total of feeding expenditures rises. Of course, this results in an increase in weight and so ceteris paribus in a higher amount of revenues in the future. However, it also means, that an amount of interest revenues is missed by not putting into a bank account the capital invested in the animals. Moreover, when this group of animals is followed up by other groups without interruption, all future fattening rounds will start later with as consequence that once again interest revenues are missed, because all future net results come available later. The properties of the functions (3.1) to (3.5) together with the opportunity costs in the form of missed interest now cause the pressure to sell to be greater, the shorter the distance to $x_{i,\max}$ or the smaller the number of animals left over in the compartment. The consequence is that the producer should accept an ever lower selling price.

A similar result has to be expected when the transition probabilities between the prices are the same for each row. In that case the expected pork price is equal for all future periods. For instance, substituting (3.17) by its invariant distribution $\Pi \left(= \lim_{n\to\infty} P^n\right)$, sets of critical price-age combinations as depicted in the figures 3.4, 3.5 and 3.6 result.

Next to an ever falling level of prices at which to sell during the slaughterripe period a striking feature is the "critical price structure" of the optimal policy, i.e. given a certain age sell if and only if the price is beyond a critical level.

Figure 3.4. The set γ_1 for the matrix Π

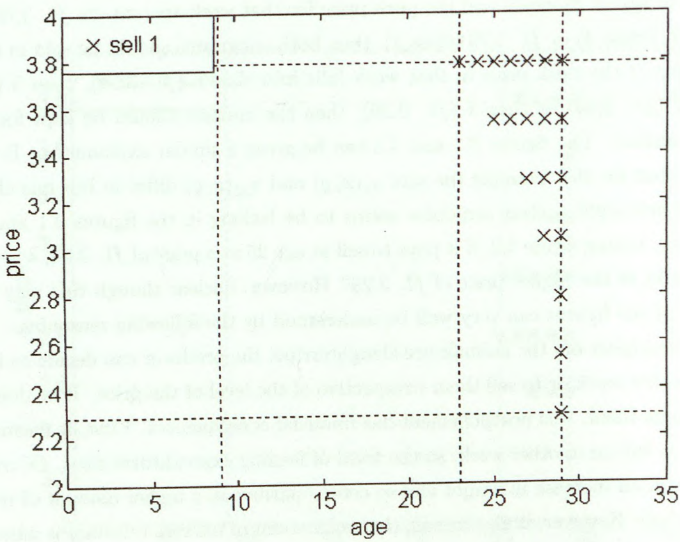


Figure 3.5. The set γ_{12} for the matrix Π

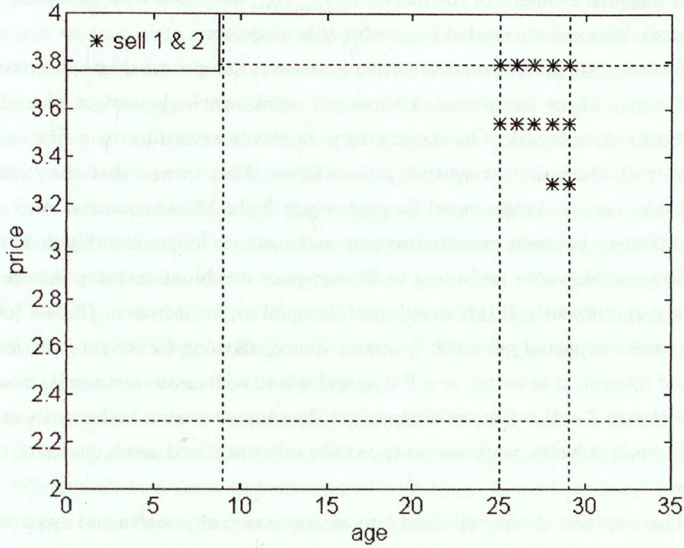
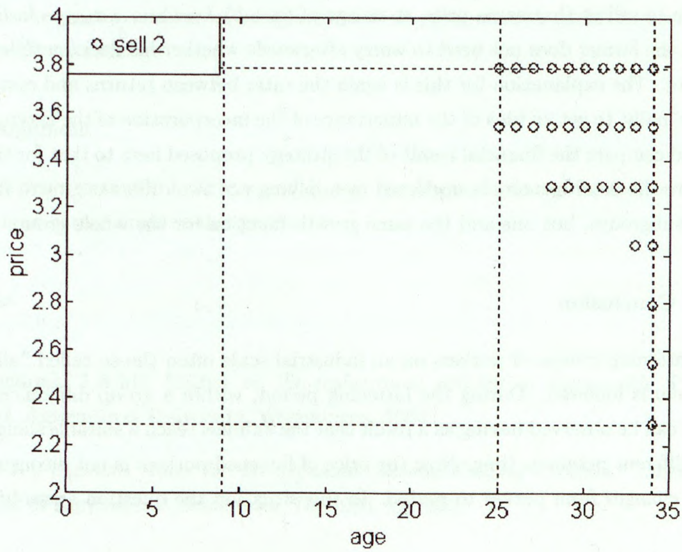


Figure 3.6. The set γ_2 for the matrix Π



We conjecture that such a critical price structure always holds when the sum of the upper diagonal elements of the matrix P, $\sum_{j=1}^{N} q_{ij}$, decreases with increasing i. However, as yet we have not succeeded in proving this conjecture.

However, the price of pork is neither a constant nor governed by a discrete probability distribution where the prices in successive weeks are independent of each other and identically distributed. On the contrary, it moves according to a first order Markov model with stationary transition probabilities. That means that the expected future pork prices are no longer equal for every state in the Markov matrix. As a consequence the difference between expected returns and costs no longer steadily declines with age, but in principle varies according to the age-price combination taken into consideration.

Stated differently, if this week's price is equal to, for instance, *fl.* 3.04 (class 4), then next week's expected price will be lower. Hence, allowing for weight gain, feed costs and missed interest, it is better to sell now and not to wait until next week. However, in the price classes 2 or 6 it is more likely to get the same or even a higher price in the coming week. So it is worth while to postpone the sale until next week or one of the weeks to come.

That explains the at first sight curious succession of positive and negative differences for a given age in the figures 3.1 and 3.2.

It should be noted that for a given pork price no such interruptions appear when varying the age. If it is worth while to sell at some price at age x, then it is also worth while to sell at that same price at an age of $(x+k), k = 1, \ldots, x_{\max} - k$. That means that the farmer does not need to worry afterwards whether his decision to sell was right or not. The explanation for this is again the ratio between returns and costs.

Finally, to get an idea of the importance of the incorporation of the heterogeneity one could compare the financial result of the strategy proposed here to that for the situation where the heterogeneity is neglected by applying not two different growth functions for the subgroups, but one and the same growth function for the whole group.

4. Conclusion

In fattening groups of porkers on an industrial scale often the so called "all in all-out" system is followed. During the fattening period, within a group differences in growth rate can be observed having as a result that the animals reach a suitable slaughter weight at different points in time. Now the price of fattened porkers is not a constant in time, but changes from period to period. In this situation the question arises how a farmer striving after a maximal financial result should react on these factors in exploiting his firm.

In this paper we formulate a decision model for this problem. Using this model the optimal delivery strategy for a heterogeneous group of porkers can be determined. The kernel of this optimal strategy is formed by sets of critical age-price combinations. Such a set applies to a subgroup (or a combination of subgroups) during the slaughter-ripe period. If the actual age-price combination for a subgroup (or combination) belongs to that set, then it is worthwile to sell that subgroup (or combination) in that week. If on the other hand the actual combination does not fall into that set, then the sale of the corresponding group should be postponed to a later date. Using an example an impression of the shape of these critical sets as function of the age of the animals and the occupation rate of the compartment was obtained. This shape can be understood on the basis of theoretical economic considerations.

In everyday's practice of fattening the phenomenon of heterogeneity is of course accounted for in selling animals. Generally speaking a (heterogeneous) group of porkers will not be delivered all at a time, but distributed over time. In such decisions considerations with respect to floor space and weight undoubtedly play a role, and possibly also the comparison of current and expected pork prices. As demonstrated above, next to space, both (different) weight and price development can be incorporated in a model for these decisions. Therefore a model as proposed here, possibly after an extension to encompass a greater number of subgroups and/or price classes, could be useful in selecting a delivery strategy.

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