GLIMMIX
A PROGRAM FOR MIXTURES OF GENERALIZED LINEAR REGRESSION MODELS, AND ITS APPLICATIONS IN MARKETING

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Abstract

Recently, a general methodology has been developed for simultaneous classification and prediction (Wedel and DeSarbo 1995). In the approach, the observations are grouped into an a-priori specified number of classes, and at the same time a generalized linear model, which relates the observations to a set of covariates, is estimated within each class. The methodology uses a maximum likelihood framework to estimate the parameters. A user-friendly computer program, called GLIMMIX, that implements the methodology is currently being developed. It enables the estimation of such mixture generalized linear models for data distributed according to several members of the exponential family. In this paper, the methodology and the design of the computer program are outlined, and a review of applications in marketing research are reviewed.

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1. Introduction

Mixture models have become popular statistical tools in classification problems. In mixture models it is assumed that a sample of observations arises from a (pre-specified) number of underlying populations, of unknown proportions. A specific form of the density of the observations in each of the underlying populations is assumed, and the finite mixture approach decomposes the sample into its mixture components (cf. Titterington, Smith and Makov 1985). Whereas in these classical approaches for finite mixtures the unconditional expectations of the underlying densities are estimated, Wedel and DeSarbo (1995) recently proposed a conditional mixture methodology, which enables the estimation of the expectations of the observations in each mixture component, conditional upon a set of covariates. The methodology, called GLIMMIX after the program implementing it, provides a probabilistic classification of a set of observations, and simultaneously fits a generalized linear model in each respective class, which relates the expectation of the observations to a set of covariates.

The methodology is formulated within the exponential family, including the most commonly used distributions such as the normal, the binomial, the poisson, the negative binomial, the inverse gaussian, the exponential and the gamma. This allows for a wide range of data to be analyzed. The exponential family is a very useful class of distributions, and the common properties of the distributions in this class enables them to be studied simultaneously, rather than as a collection of unrelated cases. The previous literature on conditional mixture models consists of a collection of such independent special cases. As such, this area of research, just as other area's of statistics, suffers from the constant danger of fragmenting into a host of special topics and applications (Dawid 1993, p.1). The GLIMMIX methodology is an attempt to alleviate this problem, by integrating most of the previous work in this field. In this paper I will outline the approach, describe the GLIMMIX program developed for estimation, and provide a review of marketing applications to date.

2. The GLIMMIX model

Here the GLIMMIX model will be briefly outlined; for details see Wedel and DeSarbo (1995). Assume that the objects j, $j=1,...,n$, on which $k=1,...,K$ independent measurements represented by a multivariate random variable $\mathbf{y}=((y_{jk}))$ are taken, arise from a superpopulation which is a mixture of a finite number I of populations in proportions $\pi_1,....,\pi_I$, where it is not known in advance from which class a particular observation arises. For the probabilities π_i it holds that $\pi_i>0$, and that their sum equals one. We assume that the conditional probability density function of $y_j=(y_{jk})$, given that y_j comes from class i, is one of the exponential family. Conditional upon class i the y_{jk} are i.i.d. with expectations μ_{ijk}. A linear predictor η_{ijk}, and some link function g(•) are specified, such that in class i:

$$\eta_{ijk} = g(\mu_{ijk}), \tag{1}$$

where the linear predictor is produced by P covariates $\mathbf{x}_1,...,\mathbf{x}_P$ (p=1,...,P) and the parameter vectors $\mathbf{\beta}_i = ((\beta_{ip}))$ in class i:

$$\eta_{ijk} = \sum_{p=1}^{P} x_{jkp}\,\beta_{ip}. \tag{2}$$

The commonly used link functions are the identity, the logit, log, and inverse links for respectively the normal, binomial, poisson and gamma distributions. These link functions are the so called canonical links for the respective distributions, which lead to desirable statistical properties (McCullagh and Nelder 1989). Thus, conditional upon class i, a generalized linear model is formulated, consisting of a specification of the distribution of the random variable, y_j, as one of the exponential family, a linear predictor, η_{ijk}, and a function g(•), which links the random and systematic components.

The unconditional probability density function of an observation y_j can now be expressed in the finite mixture form:

$$f_j(y_j\,|\,\phi) = \sum_{i=1}^{I} \pi_i f_{j|i}(y_j\,|\,\beta_i). \tag{3}$$

The purpose of the analysis is to estimate the parameters of the model, contained in the vector $\boldsymbol{\phi}$. To accomplish this, the likelihood function is maximized (note that due to the independence assumption the replications enter the likelihood through a product over K). This problem is solved using an EM-algorithm (Dempster, Laird and Rubin 1977). In the E-step the log-likelihood is replaced by its expectation, calculated on the basis of provisional estimates of the parameters. In the M-step, the expectation of the log-likelihood is maximized with respect to the parameters. For the latter purpose the iterative reweighted least squares procedure (McCullagh and Nelder 1989) is used. The computational advantages of this algorithm arise because for the exponential family sufficient statistics exist which are linear in the parameters (for the canonical links). The E- and M-steps, constituting the major iteration cycle, are alternated until no further improvement in the likelihood function is possible. The combination of the EM- and the iterative reweighted least squares algorithms provide a particularly powerful combination of statistical optimization algorithms. Convergence of both the major and minor iterations to at least a local optimum is ensured.

The parameter estimates, being based on maximum likelihood, are asymptotically normally distributed, given the number of classes. The asymptotic covariance matrix of the parameter estimates, conditional upon class i, is calculated from the inverse of the Fisher information matrix. Objects are assigned to classes on the basis of a-posteriori memberships, α_{ij}, calculated according to Bayes rule, using the final estimates of the parameters. In order to examine the separation of the classes an Entropy statistic, E_s, can be used (cf. Wedel and DeSarbo 1994). E_s varies from $E_s=0$, indicating complete overlap of the classess, to $E_s=1$, indicating perfect separation.

When these models are applied to real data, the actual number of classes, I, is unknown, and has to be inferred from the data. The standard generalized likelihood ratio tests for tests on I versus I-1 classes are not valid, and statistics such as CAIC or ICOMP (cf. Wedel and DeSarbo 1994) can be used to determine the appropriate value of I, where that value is chosen that minimizes these statistics.

3. The GLIMMIX program

The GLIMMIX program is a dedicated program for estimating mixtures of generalized linear models as outlined above, and is currently developed by the institute PROGAMMA (Groningen, The Netherlands). It runs on DOS-compatible personal computers, under Windows. Below, I describe its main options. The program that is currently being developed is a first version in which a restricted number of options is included.

3.1 The main program

GLIMMIX is menu-controlled, and can be run by activating 'managers', from a toolbar or button panel. Figure 1 displays the GLIMMIX toolbar and menu-structure. There are four buttons in the toolbar: The Files button, the Options button, the Windows button, and the Help button.

GLIMMIX uses so-called control files. A control file contains all the commands necessary to run the program for a specific application: all current options selected from the menu's (or the defaults) are stored in it, which makes it possible to more quickly run the program in subsequent analyses. The specification of the control-file consists of two-parts: the Data-specification and the Analysis-specification.

Figure 1. The GLIMMIX menu-structure.

	GLIMMIX,	Wedel & DeSarbo	1994
Files	Options	Windows	Help
CF-manager	(no) Toolbar	Enlarge	
DS-manager	(no) Confirm	Tile	
AS-manager	Directories	Cascade	
OP-manager		Arrange	
Printer			
eXit			

3.2 The Control-file (CF) manager

This option enables a user to select an existing, or open a new control-file. Options are available to copy, delete, save, or run selected control files. A control-file editor makes it possible to quickly modify an existing control file.

3.3 The Data-specification (DS) manager

When the DS manager is activated, an overview of existing datafiles is displayed. Again, datafiles can be opened, modified, copied, or deleted. An included spread-sheet program can be used to visually inspect a datafile. The DS manager includes a Variables manager, and a Cases manager.

3.3.1 The Variables (VR) manager

By activating the VR manager the user is displayed a list of the variables in the datafile. A variable may be selected from the list, and the user may subsequently activate a function from the buttonpanel. By activating the Definitions button, the variables can be assigned names for identification, and should be identified as being continuous or discrete (factors, with a limited number of levels). The levels of the factors can be named. Missing-value codes for the variables can be indicated using the Missings button. The dependent and independent variables used in the analysis need to be indicated, as well as variables identifying the consumers and replications for each consumer.

The Transform button allows one to preprocess the data before the analyses. Dependent and independent variables may be centered, scaled, multiplied by a constant, or square-root or log-transformed. The levels of discrete variables can be recoded, and dummy variables (effects- or dummy-coding) may be calculated from them. All transformed variables may either be computed temporarily and used only for the current analysis, or stored in the (new) datafile.

3.3.2 The Cases (CS) manager

The CS manager makes it possible to select a subset of the cases in the datafile for the analyses. First, a prespecified percentage of cases may be selected at random, where the remaining cases may be kept, e.g. for validation purposes. Second, a selection of cases can be made on the basis of a set of levels of categorical variables in the dataset. Third, cases can be selected manually, using the spread-sheet (obviously, this is only recommended for small datasets).

3.4 The Analysis-specification (AS) manager

The AS manager enables the user to specify the details for the analysis. A control file can be selected, opened, or copied. The convergence criterion for the EM-algorithm and the maximum number of iterations can be specified, or altered. The distribution assumed for the dependent variable can be chosen to be either normal, binomial, poisson, or gamma, and correspondingly, the link function is the identity, logit/probit, log or inverse function.

The number of segments needs to be specified. The user can specify a range, which is useful for the determination of the appropriate number of segments. The number of repeated analyses (for a given number of segments) can be specified. This option uses different random starts for the algorithm, and can be used to investigate the presence of local optima.

A final choice is to use a random or rational start for the algorithm. A random start is generated from random uniform numbers, scaled to satisfy the sum-constraint, a rational start is generated from the K-means clustering procedure, applied to the dependent and independent variables simultaneously. Alternatively, a rational starting partition can be read from a datafile.

3.5 The Output-specification (OP) manager

The output-option enables the user to select the output required from the program. The estimates of the coefficients, their asymptotic standard errors, t-values and correlation matrix can be printed. The program may provide the final log-likelihood, degrees of freedom, and AIC, CAIC, ICOMP and Entropy statistics. Computational details of the analysis can be provided, such as number of EM-iterations and amount of CPU used. Posterior probabilities of segment membership may be printed in the output file, or in a separate file, so that they can be used for further analysis (i.e. segment profiling). One can produce a variety of graphical output, such as bar-charts of the estimated coefficients in each segment and the segment sizes, a plot of the minimization of the likelihood during the iterative estimation procedure, etc.

3.6 Help, Checks and Defaults

GLIMMIX will include an extensive help-facility, that can be activated from the main toolbar, or for any specific action identified in the roll-down menu's. Information on the control and datafiles can be stored and retrieved. GLIMMIX will use dialog-boxes for the file-open, delete, save and save-as options, which display information on the current directory, its files, etc.

For most of the above options, defaults are included in the program to facilitate analysis. A number of checks are performed before and during the analyses, and in case of estimation problems, such as collinearity, diagnostics are printed.

4. Marketing Applications

Below I will review applications of mixture linear models to marketing problems. Table 1 provides an overview of the applications. Most of the applications provide special instances of the general GLIMMIX model (Wedel and DeSarbo 1995), outlined above. The applications pertain to a classification problem that is of great strategic importance in marketing: segmentation. The purpose of segmentation is to identify homogeneous groups of consumers, that can be more effectively targeted by marketing mix variables (price, advertising, promotions, distribution, new/modified products) in order to better satisfy their specific wants and needs, resulting in improved performance of the firm in question (Wedel 1993). The applications can be classified according to the type of data they are applied to: normal data, binomial data, multinomial data, count data. A final and important category of applications concerns conjoint analysis.

Table 1. GLIMMIX applications in marketing.

Authors	year	Application	
DeSarbo & Cron	1988	Trade Show performance	N[1]
Ramaswamy et al.	1993	Marketing mix effects	N
Helsen et al.	1993	Country segmentation	N
DeSarbo et al.	1992	Conjoint Analysis, Consumer products	N
Wedel & DeSarbo	1994/95	Conjoint Analysis, SERVQUAL	N
Kamakura & Russell	1989	Brand choice Analysis	M
Kamakura & Mazzon	1992	Value segmentation	M
Bucklin and Gupta	1992	Purchase incidence and brand choice	M
Gupta & Chintagunta	1993	Brand Choice & segment description	M
Böckenholt	1993	Brand Choice	DM
Kamakura et al.	1994	Conjoint analysis, & segment description	M
De Soete & DeSarbo	1990	Pick-any choices	B
Wedel & DeSarbo	1993	Paired comparison choices	B
Dillon & Kumar	1994	Paired comparison choices & segment description	B
Wedel & Leeflang	1994	Gabor Granger price experiments	B
Wedel et al.	1993	Direct Mail	P
Wedel & DeSarbo	1995	Coupon Usage	P
Bucklin et al.	1991	Purchase frequency analysis	TP
Ramaswamy et al.	1994	Purchase frequency analysis	NB
DeSarbo et al.	1992	Conjoint analysis	M

[1] The mixture distributions used: N=normal, M=multinomial, B=Binomial, DM=Dirichlet-Multinomial, P=Poisson, TP=truncated Poisson, NB=Negative Binomial.

4.1 Normal Data

One of the first mixture regression models was proposed by DeSarbo and Cron (1988). They assumed a dependent variable that is normally distributed, so that the M-step in their EM algorithm involves an ordinary multiple regression of a dependent on a set of independent variates. They applied their model to identify segments in the evaluation of the performance of trade shows, and simultaneously estimate the effects of characteristics of the trade shows within the segments. The same model was applied by Helsen, Jedidi and DeSarbo (1992) to a country- segmentation problem in international marketing. The purpose was to identify segments of countries with a similar pattern of diffusion of durable goods. Ramaswamy, DeSarbo, Reibstein and Robinson (1993) applied the mixture regression approach to cross-sectional-time series data. Here the purpose was to pool the time series in order to enable effective estimation of the effects of marketing mix variables on the market shares of brands. This model assumes a multivariate normal distribution of the dependent variable (market shares), and allows the time-series data within the pools (classes) to have an arbitrary covariance structure.

4.2 Binary Data

Pick-any experiments are experiments in which subjects are required to pick any number of objects/brands out of sets of size n. The data are binary. The choices are to be explained from the attributes of the brands. Subjects may be heterogeneous in the extent to which they consider the attributes important. De Soete and DeSarbo (1990) developed a mixture of binomial distributions (with probit link) for the analysis of these data. Using this model the annalist can identify a number of segments in the sample, and simultaneously estimate the importances of the brand attributes. The model was applied by the authors to explain consumers' pick-any/n choice data for a set of communication devices.

A related model was developed by Wedel and DeSarbo (1993). This model is a mixture of binomial distributions with a logit link function, applied to paired comparisons data. In paired comparisons data subjects are offered stimuli in all possible pairs, and are asked to choose one of each pair. These data are also commonly modelled by a binomial distribution. The model of Wedel and DeSarbo (1993), identifying the segments of subjects using different attributes in making their pairwise choices, was applied to analyse data on risk perception in the choices of automobiles. Dillon and Kumar (1993) extended this approach for paired comparisons data, and simultaneously profile the derived segments with consumer characteristics, using a so called concomitant variable mixture model.

The model proposed by Wedel and Leeflang (1994) is tailored to the analysis of pick-any data from Gabor-Granger price experiments. In such experiments, consumers are offered a certain brand at several prices, and are asked to indicate whether they intent to buy the brand at each of the prices. In order to describe the complex price-response functions within segments, the authors use flexible (first order) spline-functions, accommodating amongst others psychological price effects and the use of price as an indicator of quality. The model was applied to the analysis of a Gabor-Granger experiment for a brand in a category of products for personal care.

4.3 Multinomial data

Among the first mixture regression models applied in marketing was also the one provided by Kamakura and Russell (1989). They developed a model for the analysis of scanner panel data. These data are collected with scanning devices at the checkout counters of supermarkets, that register the barcodes that are used to identify products. Using these codes it is possible to identify the product, its price, and promotional activities such as coupons, displays, and features. Individual consumers are identified through credit cards, so that purchase data can be accumulated at the individual level. The dependent variable in the model of Kamakura and Russell is the choices made by consumers among a set of brands (of coffee in their application). Conditional upon the segment to which a consumer belongs, a multinomial distribution for these choices is derived from stochastic choice theory. The model that arises is a mixture of multinomial distributions, using a logit link. In the application, price was used as a covariate, and choice-patters in different segments were explained from the prices of the brands. The model simultaneously identifies the segments and the price-sensitivity (price-elasticities) of consumers within those segments.

Bucklin and Gupta (1992) extended this approach and allowed for a nested multinomial logit model within segments, which simultaneously allows for purchase incidence (the probability that the product category is bought) and brand choice (conditional upon a purchase of the category). Gupta and Chintagunta (1993) extend the Kamakura and Russell (1989) approach and simultaneously estimate the relationship of segment membership probabilities, using a concomitant variables mixture model. They also apply their model to scanner panel data. Other extensions were developed by Böckenholt (1993), which are specifically tailored to the analysis of longitudinal choice data, under a variety of different distributional assumptions including the Dirichlet-Multinomial. Kamakura and Mazzon (1991) used a mixture of multinomial distributions to derive segments on the basis of value-priority rank-choices by consumers in value surveys.

4.4 Count Data

Wedel, DeSarbo, Bult and Ramaswamy (1993) developed a mixture of poisson distributions (with log-link), that they used to analyse the frequency with which consumers purchase brands in response to direct mail offerings. The frequency of purchases was assumed to be described by a poisson distribution, and the model identified the effects of mailing- and consumer characteristics on the response, within segments. The model was applied to the analysis of a house-list of a direct marketing company. Such house-lists contain accumulated data on received mailings, orders, payments, etcetera, of the customers of the company, and present an essential instrument in direct marketing. As an additional feature, the posterior probabilities of the poisson mixture model can be used to select those consumers for future mailings, that have optimal probabilities of response. A similar model was applied to scanner panel data by Wedel and DeSarbo (1995) to investigate heterogeneity in the effects of determinants of the frequency of coupon-usage. They use scanner panel data to calibrate their model.

Bucklin, Gupta and Siddarth (1991) use a related approach, but assume that the poisson distribution is truncated at zero. Ramaswamy, Anderson and DeSarbo (1993) extend these models, and allow for within segment heterogeneity. They assume that the parameters of the poisson model within segments follow a gamma distribution, and develop a mixture of negative binomial distributions (with a log-link), that was used to analyse purchase frequencies derived from scanner data.

4.5 Conjoint Analysis

An important field of application of mixture generalized linear models that has attracted a substantial amount of interest, is conjoint analysis. In conjoint analysis synthetic product-profiles are designed on the basis of a set of predetermined attributes, which are varied at a number of levels, chosen by the researcher. Fractional factorial designs are used to produce a limited number of product profiles from the attributes. These profiles are offered to consumers, who are required to assign preference-scale values to them (metric conjoint), or to choose from a number of choice sets containing the profiles (conjoint choice experiments). The purpose of conjoint experiments is to derive the relative importances (part-worths) of the attributes from these consumer judgements.

To both types of data GLIMMIX models have been applied. DeSarbo, Wedel, Vriens and Ramaswamy (1992) developed a conjoint segmentation model for metric data. The preference values of a subject are assumed to follow a multivariate normal distribution, allowing for possible covariances among preference judgements for the profiles. The authors apply their model to a conjoint experiment on remote controls for cars, and simultaneously identify segments, and estimate the part-worths and covariance structure within these segments. Similar applications of a mixture regression model to conjoint experiments were provided by Wedel and DeSarbo (1994, 1995), who analyzed conjoint data on the measurement of the quality of services.

A multinomial mixture model for both conjoint choice and rank-order experiments was developed by Kamakura, Wedel and Agrawal (1994). As an additional feature, these authors provided a simultaneous profiling of the segments with consumer descriptor variables, using a concomitant variables mixture model. This model was applied to the analysis of a conjoint experiment on banking services.

DeSarbo, Ramaswamy and Chatterjee (1992) have proposed a model for constant-sum data collected in conjoint analysis, where consumers are asked to allocate a fixed number of points across the alternative profiles. They use a mixture of dirichlet distributions to describe these data (a log-link function is used). Their model, as the previous ones, simultaneously estimates segments, and identifies the part-worths of the conjoint attributes within the segments. It was applied to a conjoint study on industrial purchasing, where profiles were constructed on the basis of supplier selection criteria.

A recent Monte Carlo study on the relative performance of the mixture regression approach on synthetic data has revealed that it outperforms some eight other clustering models on parameter recovery (Vriens, Wedel, Wilms 1994).

5. Concluding remarks

Not all of the above GLIMMIX applications are accommodated at this stage in the GLIMMIX program that is being developed. The version that is being developed is a first version, in which choices with regard to the options that were to be included had to be made. The first version of the GLIMMIX program will accommodate normal, binomial, poisson and gamma distributions, and the corresponding canonical link functions. The program deals with (an equal number of) replicated measurements for each object, under the assumption of independence. Consequently, the binomial and poisson applications in Table 1 above are all accommodated in the GLIMMIX program; the applications based on a normal distribution are accommodated insofar they pertain to independent replications. The multinomial, dirichlet and negative binomial applications are currently not included in the GLIMMIX program. The program does not deal with concomitant variable mixture regression models, nor does it accommodate restrictions on the parameters, (e.g. equality restrictions across classess).

In the last five years, GLIMMIX models have become an important tool in marketing research, especially in the area of segmentation. Their application has however been limited by the lack of user friendly programs that perform the necessary computations. Programs are available, but they are testversions, written in different programming languages, and are not very easy to use. Moreover, for a specific application at hand the appropriate options (such as specific numbers of subjects, brands or attributes) may not be available in the existing program, and performing the analysis may require considerable time from the analyst in getting the available programs running. The purpose of the GLIMMIX program that is currently being developed, is to alleviate this problem. GLIMMIX integrates many of the existing mixture regression models into one unifying framework. Moreover, it provides user-friendly software to perform the calculations. The potential analytic power of GLIMMIX is not limited to marketing research, but extends to other fields of business and economic research, and research area's such as psychology, sociology, anthropology, political science, geography, food research and sensory research.

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