

## Distribution optimization by means of break quantities: a case study

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### Abstract

In this paper we present a case study on physical distribution optimization in Western Europe. Finished goods can be delivered either from distribution centres or from production service centres. The lead time of the first way is shorter, but higher costs are involved. A selection between the two ways is based on the so-called *stockmix* and on the *break quantities*. The stockmix is the set of products stocked at the distribution centre, which for efficiency reasons is restricted. Orders smaller than the break quantity are delivered from the closest distribution centre provided the product ordered is present in its stockmix. Else they are delivered from the production service centre that makes the product. The problem is to decide on the stockmix and the break quantity for each distribution centre. The objective considered is the minimization of distribution and handling costs subject to service constraints. We present an heuristic optimization method together with a simulation method to determine associated distribution costs, handling costs and service performances.

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## 1. Introduction

The company considered produces self-adhesive materials in several varieties, which for transportation purposes are almost the same. Products are shipped to industrial customers both from plant service centres (PSC's) and distribution centres (DC's) in Western Europe. Service reliability is a key competitive-edge for the company and it receives full commitment from management. Therefore, regular studies are done to ensure fast delivery to customers against as low distribution costs as possible. One main tactical problem in this respect, is to decide which orders to deliver from a DC and which from a PSC. The lead time of the first way is shorter, as the product is delivered from stock at a DC, yet the second is cheaper, because of fewer handling costs. To create a consistent view towards clients, the company applies a so-called break quantity (BQ) concept. Orders smaller than this quantity are delivered from the nearest DC provided the products ordered are present in the DC's stockmix. Else they are delivered from the PSC that makes these products. The problem now is to determine both the stockmix and the break quantity for each of the six distribution centres, with the objective to minimize total distribution costs subject to constraints on service. This should not only be done for the present situation; the company needs a method to solve the problem repeatedly. Finally, the study should also indicate the value of such an approach over the present situation.

A simulation model was developed to evaluate various distribution options. First the effects of three different strategies concerning the stockmixes were studied, while assuming realistic BQ's. The strategies were: "stock the most ordered products," "stock the products most ordered in small quantities" and "stock the products most ordered in large quantities". The following step was to develop a method to determine for each DC a break quantity. Due to the complex cost structure and the presence of service constraints for the whole distribution network, it was not possible to decompose the problem into separate BQ determinations per DC. Our approach was therefore as follows. We first generated a large set of realistic BQ combinations. For each combination we simulated the distribution with all actual orders received over a fixed four-month period. As a result we obtained service levels and were able to calculate distribution costs for each BQ combination. Using these results we developed three methods to determine the BQ's. A straightforward method is to select the best BQ combination out of the generated set that satisfies all constraints. To enable optimization we fitted a simplified function to the total costs. The second method consisted of optimizing this function using a nonlinear programming (nlp) solver. The third method tried out, was to simplify the distribution costs from a PSC by assuming 80% truck loads. This lead to a simplified objective function that could be optimized by a nlp solver. The three methods were compared with each other with respect to performance and computational effort. Both the outcomes as well as the selected solution procedure were presented to the management.

The forementioned problem seems to be unique in literature. This may be the result of the cost structure and the products being quite similar which allows a break quantity concept. Furthermore, much more literature focuses on strategic issues, like locating DC's, than on tactical optimization as was needed here. Related studies worth mentioning are from Blumenfeld et al. (1987), Daganzo (1991) and Kasturi Rangan and Jaikumar (1991).

Blumenfeld et al. (1987) present a case study for the intra company distribution with General Motors. They also consider stock holding costs and show that their problem can be decomposed in determining optimal shipment size and route for each plant-DC-customer combination. They assume fixed prices for transport to customers. Their decomposition is

based on fixed shipment sizes for plant-DC routes which does not hold in our problem.

Daganzo (1991) also considers the option to distribute directly from a plant or from a distribution centre. He assumes fixed distribution costs per kg for each PSC or DC customer combination. These costs mainly depend on the location. Distribution via a DC involves higher handling costs. Such a formulation leads to a linear programming problem with variables indicating the amount supplied per DC to a customer. Due to the assumptions it has a 0-1 solution. Our case has a different cost structure. Capacity constraints further complicate our problem.

Kasturi Rangan and Jaikumar (1991) consider consumer rebates if direct transportation is accepted. The company now can choose the rebates such that its distribution costs are minimized. Again only a simplified cost structure is assumed.

The structure of this paper is as follows. In the following section the distribution problem is presented in more detail. In section 3 we start with a general analysis, next we consider various stockmixes and finally we present the three methods for determining the breakquantities. Section 4 presents the results concerning the methods.

## 2. The distribution problem

### The Distribution Network

The materials, produced by the company, are sold in roll form in 14 countries in Western-Europe. Production is situated in 3 plants in Europe; in this study we assume that each plant produces unique products. Associated with each plant is a PSC where the produced rolls are prepared for transportation. From the PSC the products are transported either directly to the customers or to one of the six DC's where they are held in stock until ordered by a customer. Out of the 10,000 different products only some 300 are suitable for selection in the DC-stockmix. In figure 1 the locations of the distribution centres and plants are given.

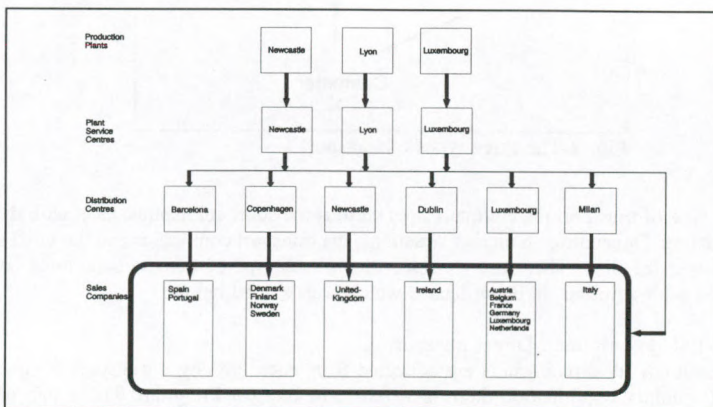


Fig. 1 The distribution network

If an ordered width is not the same as that of the produced roll, the roll must be slitted in the desired width. This slitting can be done in the PSC's as well as in the DC's. The slitting capacities are the only factors in the investigation which restrict the daily throughput volume in each location.

The objective of the company is *cost minimization* under certain *service requirements*. The costs to be involved in the minimization are the transport costs and the extra handling and packaging costs for orders shipped via a DC. Inventory costs are left out of consideration because their contribution was judged to be low because of the high inventory turnover and the limited number of products in the stockmix. The most important service requirements are *speed of delivery* and *reliability*. The speed of delivery is translated in a minimal percentage of orders shipped via a DC because delivery time via a DC is substantially shorter. This is due to the fact that in a DC the orders are shipped from stock while in a PSC the order still has to be produced (production to order). Reliability is translated in a maximal percentage of *orders shipped too late* as a result of insufficient slitting capacity in a location to slit all the orders the same day as picked from stock.

### Transport costs

In the distribution system we distinguish three types of transport (fig. 2):

- A. From PSC to customer: Direct transport
- B. From PSC to DC: Stock replenishments.
- C. From DC to customer: Domestic transport.

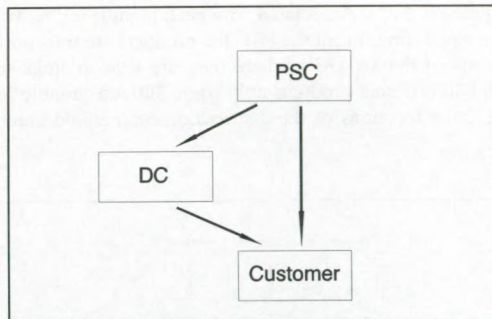


Fig. 2 The three types of transport

For each type of transport the company uses various transport companies, each with their own tariff structure. Depending on market situations, the transport companies and the tariff structures vary over the time. Therefore we selected for each type of transport the most common, sometimes a bit adjusted, tariff structure, which is described below.

- A. From PSC to customer: Direct transport.

Each day all direct orders are collected from each PSC by a transport company. For each PSC-country combination there is a basic cost rate per kilogram. The actual price per kilogram for a direct PSC-country order on a certain day is the basic rate divided by the average degree of load of all the direct vehicles loaded at that PSC that day. In formula,

$$DIRT_{p,d,t} = \frac{rate_{p,d}}{load_{p,t}/capa_{p,t}} = rate_{p,d} \frac{capa_{p,t}}{load_{p,t}}$$

with

- $DIRT_{p,d,t}$  = actual price per kg on day t for direct transport from PSC p to country d  
 $rate_{p,d}$  = basic cost rate per kg for direct transport from PSC p to country d  
 $load_{p,t}$  = total load in all the direct vehicles leaving PSC p on day t.  
 $capa_{p,t}$  = total capacity of all the direct vehicles leaving PSC p on day t.

To determine  $DIRT_{p,d,t}$  we have to know the total volume shipped directly from PSC p on day t. This volume depends on the stockmix and the break quantity of each DC; the factors which have to be determined in the investigation.

#### B. From PSC to DC: Stock replenishments.

For this type of transport time is not that crucial; a system of full truck loads is used. Replenishments for a certain DC leave as soon as the truck is fully loaded. So the price per kilogram of the PSC-DC trajet of an order delivered via a DC is fixed for each PSC-DC combination.

#### C. From DC to customer: Domestic transport.

Each day the orders packed at a DC are shipped to the customers. Besides the destination the cost rate per kilogram depends on the ordersize; the cost rate decreases as the ordersize increases. Several weight classes are distinguished. In each class there are basic rates per kilogram per DC-country combination. So the price per kilogram for domestic transport depends on the route and on the ordersize.

### 3. Analysis

Management preferred to have a solution procedure which works with real data, i.e. orders received over a past period, and use the outcomes for a coming period. This means that we consider a fixed set of orders, numbered 1,...,N for which the following data is available: quantity of order i in kilogram  $Q_i$ , order entry day  $t(i)$ , country of destination  $d(i)$ , the DC from which it can be delivered  $dc(i)$  and the PSC from which it can be delivered  $p(i)$ . The objective function of the distribution problem can be formulated as

$$\begin{aligned} \min C = & \sum_i Q_i (1 - O_i) DIRT_{p(i), d(i), t(i)} \\ & + \sum_i Q_i O_i SR_{p(i), dc(i)} \\ & + \sum_i Q_i O_i DT_{dc(i), d(i), q(i)} \end{aligned}$$

with

- $O_i$  = decision variable which is 1 when order  $i$  is delivered via a DC and else 0.  
 $DIRT_{p(i),d(i),t(i)}$  = direct transport cost per kg from PSC  $p(i)$  to country of destination  $d(i)$  on day  $t(i)$   
 $SR_{p(i),dc(i)}$  = stock replenishment cost per kg from PSC  $p(i)$  to DC  $dc(i)$   
 $DT_{dc(i),d(i),q(i)}$  = domestic transport cost per kg (incl. handling cost) of an order from DC  $dc(i)$  to country of destination  $d(i)$  and in weight class  $q(i)$

The decision variable  $O_i$ , which indicates whether order  $i$  is shipped directly to a customer or via a DC, is derived from the DC stockmix and the break quantity.

Before developing a solution method we first analysed the data to get a better understanding of the problem: what determines the total cost level? Which orders must be shipped directly and which orders via a DC?

First of all there is no cost difference between the various products; the origin-destination combination and the order pattern of a certain product determine the costs needed to ship the orders of a product. On forehand we can calculate the cost of a certain order when it is shipped via a DC; however we cannot calculate the cost when this order is shipped directly because this also depends on other directly shipped orders from the same PSC. So we cannot compare for an order the cost difference in shipping directly or via a DC. From the tariff tables it appears that direct shipping is in general cheaper than shipping via a DC. The reason is that with direct shipping there are no extra handling costs and only one transport type is used while via a DC the replenishment costs, the domestic costs and the extra handling costs must be added up.

### Decomposition of the problem into subproblems

In the previous section we indicated the problems we came across when determining the products that should be kept in stock. The discussion of the tariff structures of the three types of transport has shown the dependencies between the DC's. These properties call for an integrated model that can be simultaneously solved for the break quantities and the DC stockmixes. In that case the solution should invoke the lowest total costs while meeting the service requirements. The decision variables which represent the break quantities will be assumed to be continuous, although the use of ordersizes that are a multiple of 500 square meters ( $m^2$ ) makes them discrete. Continuity is justified by both the large number of orders and the wide range of possible order quantities. A major practical problem arises from the large number of products that can be stocked. Each product requires the use of a 0/1 stock mix variable to define whether it is stocked or not in a certain location. The range of about 300 products and the 6 DC's amounts to 1800 stock mix variables and 6 continuous BQ variables. Furthermore, the complex structure of the transportation costs makes the objective function nonlinear. Another disadvantage of an integral model is the acceptance by the management of the resulting stock lists and break quantities. It will merely be impossible to explain why this specific solution results in the lowest costs. Hence other methods will be tried out.

Decomposing the problem into subproblems has been the next step in the study. The most logical decomposition is a subproblem for each DC but, as argued before, this is impossible. We therefore apply a different decomposition: we solve the problem in two stages. In the first stage we determine the stockmix for each DC. In the second stage we determine an optimal set of break quantities, based on the selected stockmixes. In principle these

problems are connected, but taking that into account leads to a too complicated method.

### **Stage one: Determination of the stockmix**

In order to achieve service requirements and to keep storage costs at an acceptable level, each DC should only keep a limited number of products (management prefers 50) in stock. Because the products are physically identical, they can only be distinguished by the pattern in which they are ordered. We started therefore by analyzing the criteria on which the decision to stock a product should be based.

Firstly we note that the service objectives are formulated in terms of number of orders. This has lead us to base our analysis in number of orders as well. The service objectives and the maximum number of products that can be stocked in a DC suggest to stock the products which are ordered most often. These products can easily be determined by counting the number of times they were ordered during a period and selecting those 50 products that were ordered most often.

Examining the stocking decision more closely we find a paradoxical situation. Delivering orders via the DC's is more expensive than delivering directly, so the service objectives are best met with shipping small orders via the DC's. This also reduces the workload for the slitting machines in the DC's. For these reasons it would be recommendable to stock those products that are ordered in relatively small quantities. However, the cost structure for domestic transport, i.e. decreasing cost per kilogram as the ordersize increases, makes it worthwhile to stock those products that are ordered mostly in large quantities. To determine which effect has the largest impact on the total costs, we defined two more policies: stock those products that have the largest number of orders below and above the average ordersize, respectively. As the complexity of the problem makes it impossible to predict which policy will satisfy the restrictions at the lowest costs, we tested all three of them.

### **Stage two: Optimizing the break quantities**

For the second stage in our approach we considered three different methods to determine the breakquantities. To calculate total costs and service performance a simulation program has been developed. The program needs input in the form of orders that were entered during a period. For each order we need the following information: product, quantity, need for slitting, location of customer and the entry date. Furthermore, the program requires transport tariffs, slitting capacities and finally for each DC the list of stocked products and the break quantity. The output consists of the transport costs for replenishment, domestic and direct shipments, the number of orders that were shipped from each location and the number of orders that were shipped too late.

Delays in shipping are caused by a lack of slitting capacity in a location. In the simulation program slitting has been implemented with waiting lists according to the last in, first out (LIFO) policy. Daily practice in reality shows a first in, first out policy, which would lead to large numbers of orders being shipped too late in case there is not sufficient service capacity available on the slitting machine. To prevent service performance to drop below target, locations that suffer a lack of service capacity are in practice forced frequently to work overtime. Using the LIFO system approaches this overtime concept by keeping always the same orders for which there is no capacity available out of the system, rather than delaying all orders (the number of days an order is delivered too late does not play a role in the study).

### Three BQ optimization methods

Determination of the total costs for given stockmixes and break quantities takes a lot of time, because of the complex cost structure. To speed up optimization we therefore concentrated upon the development of simplified ways to compute the total costs. Three optimization methods (see fig. 3) were developed and tested:

- A. Enumeration
- B. Estimated cost function
- C. 80 % load for direct transport

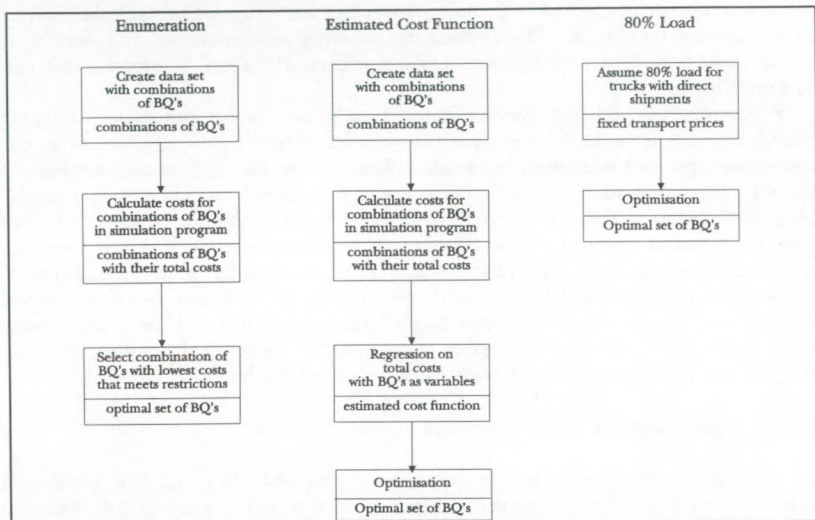


Fig. 3 Overview of the three BQ optimization methods

#### A. Enumeration

The first optimization method has its origin in the simulation program rather than being a simplified cost calculation method. The method compares as many sets of break quantities as can be calculated within a reasonable amount of time. Even if only break quantities are chosen that are a multiple of 1000 m<sup>2</sup> it is a practical impossibility to simulate all sets, so a selection has to be made. From the order pattern it is obvious that most ordered quantities fall somewhere in the range from 1000 m<sup>2</sup> to 10,000 m<sup>2</sup>. This would mean one million sets of BQ's. Furthermore, trials have shown that the most advantageous break quantities are either near zero or near the maximum ordered quantity (i.e. shipping everything via the DC). The most important DC's have another property: when their break quantity is below a certain value it becomes impossible to obtain the required percentage of total orders shipped via the DC's. These three observations make it possible to select sets of break quantities that provide best chances of finding feasible and low cost solutions. After simulating these sets of break quantities the final solution will be the set with the lowest total costs

satisfying all service constraints.

## B. Estimated cost function

The second method is based upon the idea that the total costs can be written as a simplified function of the break quantities. To obtain such a function we constructed a data set that contains many sets of break quantities, each with their costs calculated with the simulation program. The discussion concerning the selection of the sets of break quantities that was held for the enumeration method applies to this method as well.

Our first trial was to fit a linear function by ordinary least squares to this data set, but this yielded a poor fit ( $R^2 = 0.575$ ). We examined the data set more closely to obtain a global idea about the shape of the cost function. The function appeared to have its minima at both ends of the range of break quantities. This means that the costs for either shipping everything directly or via the DC's are below those where the break quantities invoke some of the orders to be shipped directly and some to be shipped via the DC's. This also means that there must be a point which has maximum costs somewhere in between, but its exact location (or the exact values for the break quantities) is unknown. We added additional variables to the linear function which express the (more or less) convex shape of the cost function:

$$C = \beta_0 + \sum_{dc} \beta_{dc} BQ_{dc} + \sum_{dc} \gamma_{dc} \sqrt{BQ_{dc} (BQ_{\max} - BQ_{dc})}$$

with

$BQ_{dc}$  = the break quantity in DC dc

$BQ_{\max}$  = the maximum value for the break quantity (100,000 m<sup>2</sup>).

The regression showed that this function fits the data rather well ( $R^2 = 0.93$ ).

## C. 80% Load for direct transport

The third method is based upon the assumption that the transport costs for each order should be a function of its weight and destination only, rather than depending on other orders as well. To achieve this property for both replenishment and direct shipments we have to assume that the trucks have a fixed load. Replenishment are transported in full truck loads. For direct transport the truck loads vary substantially, but they are on average 80% of the maximum capacity. Therefore we fixed the load for trucks containing direct shipments at 80%. The transportation costs can now be computed for each order separately as soon as it has been determined whether the order will be shipped from a PSC or from a DC. The cost function has been reduced to a sum over some simple multiplications:

$$\begin{aligned} \min C = & \sum_i Q_i (1 - O_i) DNEW_{p(i), d(i)} \\ & + \sum_i Q_i O_i SR_{p(i), dc(i)} \\ & + \sum_i Q_i O_i DT_{dc(i), d(i), q(i)} \end{aligned}$$

with

$DNEW_{p(i)d(i)}$  = direct transport cost per kg from PSC  $p(i)$  to country of destination  $d(i)$ , assuming 80 % load in the direct transport trucks.

### Implementation

The enumeration method is solely based on the simulation program and does not need further discussion. Implementing the other methods involves solving a model that can be formulated with the simplified cost functions as objective functions, and the service requirements and slitting capacities as the restrictions. The implementation of the slitting capacities is not as straightforward as it may look at first sight. The fact that 5% of all orders are allowed to be shipped too late, means that either a complicated system of waiting lists has to be formulated or simplified restrictions have to be defined. We have chosen to require for each location that the total quantity that has to be slit in the period is less than or equal to the total capacity that is available in that period. This results in a set of restrictions which is less restrictive than it should be. To determine the number of orders that have to be slit in a certain location we make use of tables that contain this information for a large number of BQ values. The information provided by these tables suffices for the estimated cost function to calculate both total costs and service performance. The 80% load for direct transport method needs some additional information to calculate the transport costs. It is not only necessary to determine the total shipped quantities per DC-country and plant-country combination, but also to have a distribution in weight classes for these quantities. This information is also contained in tables, but it makes the model used in this method considerably more complex than the model used in the estimated cost function. For both methods the resulting models make use of straightforward objective functions, but restrictions which are discontinuous in the break quantities. Consequentially, the nonlinear optimization was often trapped in local minima and many restarts had to be done.

## 4. Results

In this section computational results are presented. We first deal with the stockmix determination and then we evaluate the performance of the three BQ optimization methods. Finally, we compare the currently used stockmix and break quantities with the outcomes of the methods presented here.

The following basic figures apply to all three parts of the results. The minimum percentage of orders that should be shipped via the DC's was set at 74%, while the maximum percentage of orders that was allowed to be shipped too late was 5%. The break quantities could assume any value between 0 and 100,000 m<sup>2</sup>. Besides these basic cases we also did tests with other values for the service restrictions but the results are not given here.

### Which stockmix performs best?

The results in this and the next part were obtained by using both the estimated cost function and enumeration method. In table I we present the results for the three different stockmixes: stocking the products with most frequent small orders, all orders and large orders respectively.

**Table I** Results for the three different stockmixes

Stock most frequent			
	Small orders	All orders	Large orders
<i>DC Break quantity (in 1000 m<sup>3</sup>)</i>			
Bar	100	40	17
Cop	9	100	100
New	4	4	6
Dub	10	10	50
Lux	5	4	17
Mil	10	13	100
% Shipped via DC			
Total	74.6	74.7	74.6
% Shipped too late			
Total	4.0	4.7	3.2
Costs summary (in Dfl 1,000,000)			
DC activities			
Pack	0.45	0.48	0.69
Hand	0.35	0.38	0.54
Transport			
Repl	0.76	0.87	1.08
Dom	1.52	1.58	2.08
Direct	2.68	2.44	1.91
Total	5.76	5.75	6.30

The results clearly show which effect dominates: the diminishing costs per kilogram for delivering large orders via the DC's are less important than the lower costs for delivering orders direct from the plants. Stockmixes containing those products that are ordered most often in large quantities invoke higher overall costs. This means that extra costs for shipping more volume via the DC's dominate the reduced costs per kilogram for domestic transport. This amounts to 10% higher costs compared to the other stockmixes. The difference between the stockmixes based upon small and all orders is negligible (5.76 vs. 5.75).

### Which BQ optimization method to use?

Table II shows the results for the three methods. We used a stockmix that contains the most ordered (regardless of the quantity ordered) products. The results show that the solutions presented by the 80% load and the estimated cost function methods are equal. The results for the enumeration method are worse as far as total costs are concerned, but the slitting figures and service performance are slightly better.

**Table II** Results for the three BQ optimization methods.

	80% Load	Estimated cost function	Enumeration
<i>DC Break quantity (in 1000 m<sup>2</sup>)</i>			
Bar	45	45	50
Cop	9	9	17
New	4	4	3
Dub	10	10	50
Lux	5	5	6
Mil	10	10	6
<i>% Shipped via DC</i>			
Total	74.6	74.6	75.1
<i>% Shipped too late</i>			
Total	4.0	4.0	3.0
<i>Costs summary (in Dfl 1,000,000)</i>			
<i>DC activities</i>			
Pack	0.45	0.45	0.47
Hand	0.35	0.35	0.37
<i>Transportation</i>			
Repl	0.76	0.76	0.82
Dom	1.52	1.52	1.53
Dir	2.68	2.68	2.62
Total	5.76	5.76	5.81

Based on other values for the service restrictions, we chose to use a combination of the enumeration and estimated cost function method. The estimated cost function method performs in all cases equally well or better than the 80% load method. The enumeration method is less consistent. In some cases it provides the best solution, while in other cases results are worse. However, both the estimated cost function and the enumeration method are based upon a data set, which can be used for both methods. Most time consuming part of both methods is the construction of this data set, but once this set is available the time needed to obtain the final solution is relatively short.

A remark to be made is that the break quantities found may be quite sensitive, i.e. a slight change in the cost function may give considerable different break quantities. This can be observed in the comparison of the various methods. This effect seems more dramatic than it is, since the tail of the order size distribution is quite thin. Hence a change of a BQ from say 10,000 to 50,000 sqm may only effect some orders.

### Currently used stockmix and break quantities compared to our method

Table III shows the results for three cases. The first column represents the results given by the simulation program that were obtained when the actual stockmixes and break quantities are used. The second column shows a break quantity solution determined by our method, using actual stockmixes. The last column shows the results for the case in which we have used our method to determine both the stockmixes and the break quantities. For the latter two cases we used the restriction that the percentage of orders shipped via the DC's should at least be equal to the percentage obtained in the first case.

**Table III** Comparison of actual and optimized BQ's and stockmixes

	Actual stockmix, actual BQ's	Actual stockmix, optimized BQ's	Stock most frequent orders (all), optimized BQ's
<i>DC Break quantity (in 1000 m<sup>3</sup>)</i>			
Bar	24	100	10
Cop	21	100	11
New	10	9	0
Dub	10	15	10
Lux	20	100	5
Mil	21	14	5
% Shipped via DC			
Total	63.5	63.6	63.7
% Shipped too late			
Total	3.2	2.4	3.7
Costs summary (in Dfl 1,000,000)			
DC activities			
Pack	0.52	0.54	0.37
Hand	0.41	0.43	0.29
Transport			
Repl	0.94	1.02	0.65
Dom	1.63	1.65	1.22
Dir	2.35	2.19	3.03
Total	5.85	5.84	5.56

Comparing the first two columns shows that the currently used stockmixes do not leave much room for improvement. The actual stockmixes only allow minor adjustments to the break quantities, as otherwise the restriction of the minimal percentage of orders shipped via a DC is not met. The last column shows that a considerable reduction in the total costs can be obtained (5%) when the stockmixes are based upon the most ordered products. Alternatively, comparing the first column of table III (actual situation) with table II shows that an increase in delivery through a DC from 63% to 75% can be obtained against 2% lower costs!

## 5. Conclusions

In this paper we analysed an international distribution problem, with an interesting, but not yet published, structure. Operations research techniques were useful in determining stockmixes and break quantities for the distribution centres. Due to the complexity of the actual problem and the lack of structure in cost data, as well as the interaction between several quantities and time restrictions, we were only able to develop an heuristic solution procedure by combining simulation and nonlinear programming. Yet the company was fully satisfied by the combination of these techniques. More research is needed to investigate whether a further simplified model which can be solved analytically while still be of sufficient practical value.

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## References

- BLUMENFELD, D.E., BURNS, L.D., DAGANZO, C.F., FRICK, M.C. AND HALL, R.W. 1987. Reducing logistics costs at General Motors. *Interfaces*, **17** (I), 26-47.
- DAGANZO, C.F. 1991. *Logistics system analysis*. Lecture Notes in Economical & Mathematical Systems, no.361, Springer-Verlag, Berlin.
- KASTURI RANGAN, V. AND JAIKUMAR, R. 1991. Integrating distribution strategy and tactics: a model and an application. *Management Science*, **37**, 1377-1389.

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