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PARELLA: A PARAMETRIC APPROACH TO PARALLELOGRAM ANALYSIS

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Abstract

The PARELLA model is a parametric parallelogram model that can be used for the measurement of latent attitudes or latent preferences. The data analyzed are the dichotomous responses of persons to items, with a one/zero indicating agreement/disagreement with the content of the item. The model provides a unidimensional representation of persons and items. The response probabilities are a function of the distance between person and item: the smaller the distance, the larger the probability that a person will agree with the content of the item.

This paper will briefly discuss the procedure (based on EM and MML) by which the parameters of the PARELLA model are estimated, discuss two goodness-of-fit tests (one for differential item functioning, and one for the adequacy of the item characteristic curve) and provide examples concerning the measurement of the attitude with respect to nuclear power stations, and the measurement of the attitude in the car-environment controversy.

Key words: parallelogram analysis, unfolding, item response theory.

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1. Introduction

It is not unusual that social scientists need measurements of latent person traits such as preferences and attitudes. Item response theory (Hambleton and Swaminathan, 1985) provides the means by which these measurements can be obtained. The first step is the construction of a set of items indicative of the latent trait of interest (see Table 2 for items used for the measurement of attitude towards nuclear plants, and Table 5 for items used for the measurement of attitude in the car-environment issue). The second step is the collection of the the dichotomous responses X (1/0 indicating agree/disagree) of a sample of persons to these items (see Table 1 for the responses to the items concerning nuclear power stations). In the third step an item response model is used to relate the responses of persons to items to locations of persons and items on the trait of interest. The locations of the items constitute the grid of the measurement instrument, the locations of the persons constitute the measures of the attitude at hand.

In the PARELLA model (Hoijtink, 1990; 1991; 1992; Hoijtink and Molenaar, 1992) the probability of a positive response (1) (also called the item characteristic curve) is a function of the distance between person location β, indexed a=1,...,N, and item location δ, indexed i=1,...,n,

$$P(X_{ai}=1|\beta_a,\delta_i,\gamma) = P_{ai} = 1 / (1 + |\beta_a-\delta_i|^{2\gamma}). \qquad (1)$$

If the distance is large, the attitude of the person and the attitude expressed by the item are rather different, and the probability of a positive response is small. If the distance is small, the attitude of the person and the item agree to a large extent, and the probability of a positive response is large.

The PARELLA model is the probabilistic counterpart of Coombs' parallelogram model (Coombs, 1964). There the probability of a positive response equals 1 if a person is within a distance τ of the item and 0 otherwise. Data according to Coombs' model are hardly ever observed in practice. Many persons will respond negatively/positively to one or more items within/outside a distance τ. If the parameter γ is large (say 4 or more) (1) cannot be distinghuished from Coombs' model and responses are completely determined by the distance between person and item (τ=1). The smaller the value of γ the more random characteristics of either person or item influence the response.

2. Model Properties

The representation of items and persons resulting from an analysis with the PARELLA model is unidimensional i.e. items and persons are represented on the same dimension. This implies that the responses of persons to items are mainly determined by the (psychological) distance between the attitude of the person, and the attitude expressed by the item. If the interest is in the measurement of attitude with respect to nuclear plants (see Section 6), the responses of each person should mainly be determined by their attitude with respect to nuclear plants. Since empirical data are hardly ever so perfect that attitude is the only determinant of the responses (in which case Coombs' model would apply), the PARELLA model allows for random characteristics of either person or item to interfere.

When γ is larger than .4, usually columns and rows of the data matrix can be reordered such that the resulting pattern of zero's and one's resembles a parallelogram. When γ is smaller than .4 such a reordering does not exist. This indicates that whenever γ is larger than .4 the attitude to be measured is the main determinant of a person's response, otherwise the data are more or less random.

An important property of all parametric item response models and thus of the PARELLA model is the sample invariance of the item locations i.e. the grid of the measurement instrument constituted by the locations of the items is invariant across (relevant) samples of persons. It is not unusual to compare the measures obtained with a measurement instrument. For example, which of these persons have a positive attitude towards nuclear plants, or, did the attitude in the car-environment issue change after an information campaign. These comparisons are only valid if the measurement instrument used is the same for all persons, or, the sample of persons used before and after the information campaign, otherwise the inferences may be seriously biased. An example from the realm of manifest traits will clarify this matter. Suppose the length of women and men has to be compared. Using a yardstick with a grid in centimeters for the women and a grid in inches for the men, would lead to the surprising conclusion that on average women are larger than men. In Section 5 a test for sample invariance of the item locations will be discussed.

Another property of the PARELLA model is local stochastic independence. This property implies that for each person the responses to the items have to be mutually independent. Using this property the probability of response vector **X** can be written as

$$P(\mathbf{X}_a|\beta_a) = \prod_i P_{ai}^{X_{ai}}(1-P_{ai})^{1-X_{ai}}. \qquad (2)$$

Unless indicated otherwise, all sums and products run from a=1,...,N, i=1,...N, q=1,...,Q, and, g=1,...,G.

The item characteristic curve specified by the PARELLA model (1) is based on proximity relations between person and item. The smaller/larger the distance between person and item, the larger/smaller the probability of a positive response. Usually not all items constructed are in accordance with the PARELLA model. Some will have an item characteristic curve that is flat instead of single peaked i.e. the probability of a positive response does not depend on the attitude of the persons. Such an item affects the discriminative power of the measurement instrument and has to be removed from the item set. In Section 5 a test for agreement between empirical (i.e. reconstructed from the data) and PARELLA item characteristic curve will be discussed.

3. Parameter Estimation

The parameters of the PARELLA model are estimated using marginal maximum likelihood, an method proposed by Bock and Aitkin (1981) which is based on the EM-algorithm (Dempster, Laird and Rubin, 1977). Direct estimation of the parameter vector $(\boldsymbol{\delta},\boldsymbol{\beta},\gamma)$ would lead to inconsistent estimates of $\boldsymbol{\delta}$ and γ since the number of parameters to be estimated increases with sample size (the location of each person has to be estimated, i.e. $\boldsymbol{\beta}$ is random). This problem disappears if instead of $\boldsymbol{\beta}$ the marginal density function $g(\beta)$ is estimated. This can be done without making any assumptions with respect to the parametric shape of $g(\beta)$, if it is approximated by a stepfunction with nodes $\mathbf{B}$ and weights $\boldsymbol{\pi}$ both indexed $q=1,\ldots,Q$.

The likelihood of the parameters $(\boldsymbol{\delta},\gamma,\boldsymbol{\pi})$ given the data $\mathbf{X}$ is given by

$$\log L(\boldsymbol{\delta},\gamma,\boldsymbol{\pi}|\mathbf{X}) = \sum_a \log \left(\sum_q P(\mathbf{X}_a|B_q)\ \pi_q \right), \tag{3}$$

under the restrictions that

$$\sum_i \delta_i=0, \quad \text{and,} \quad \sum_q \pi_q=1. \tag{4}$$

The nodes B are not estimated, but chosen such that the weight of the first and the last node are in the interval <.005,.025> with the other nodes equally spaced in between (Hoijtink, 1990). In this way at least 95% of the density function is located between the first and the last node.

If the data contain more than one sample (i.e. men and women, or samples analyzed before and after an information campaign), the sub-sample structure (samples indexed $g=1,\ldots,G$) can be incorporated in the model. The product over sub-samples of (3) under the restriction of equal item and γ parameters across sub-samples (due to the property of sample invariance), but with sub-sample specific density functions of the person parameters, renders

$$\log L(\boldsymbol{\delta},\gamma,\boldsymbol{\pi}^1,\ldots,\boldsymbol{\pi}^G|\mathbf{X}^1,\ldots,\mathbf{X}^G) = \sum_g \left[\sum_{a\epsilon g} \log \left(\sum_q P(\mathbf{X}_a|B_q)\ \pi^g_q \right), \right]. \tag{5}$$

under the restrictions that

$$\sum_i \delta_i=0, \quad \text{and,} \quad \sum_q \pi^g_q=1, \quad \text{for } g=1,\ldots,G. \tag{6}$$

The estimation procedure is an iterative sequence across three stages in which subsequently the item locations, the density function of the person parameters (for one or more samples), and the parameter γ are updated using the EM-algorithm. Standard errors of the estimates are obtained by inversion of the Hessian matrices of the respective likelihood functions. The details will not be presented here, the interested reader is referred to Hoijtink (1990; 1991), and Hoijtink and Molenaar (1992).

4. Person Parameter Estimation

Once the item locations, the γ parameter, and the density function of the person parameters for one or more samples are estimated, an estimate of each person's location and its standard error can be obtained. The expected a posteriori estimator (EAP) (Bock and Aitkin, 1981) is the mode of the a posteriori density function of the person location conditional on the response vector **X**:

$$\mathrm{EAP} = \sum_q B_q \; P(B_q | \mathbf{X}_a), \tag{7}$$

The standard error of the EAP estimate is

$$\sigma_{\mathrm{EAP}} = \sqrt{\left[\sum_q (B_q - \mathrm{EAP})^2 \; P(B_q | \mathbf{X}_a)\right]}, \tag{8}$$

where, from Bayes theorem,

$$P(B_q | \mathbf{X}_a) = P(\mathbf{X}_a | B_q) \; \pi_q \; / \; \left(\sum_q P(\mathbf{X}_a | B_q) \; \pi_q\right). \tag{9}$$

5. Model Fit

As implied in Section 2, three conditions have to be fulfilled for data to accord with the PARELLA model: γ should be larger than .4 (otherwise the data are more or less random); the item locations and the γ parameter should be sample invariant; and, the item characteristic curve specified by the PARELLA model (1) should be adequate. This section will provide goodness of fit tests for sample invariance and adequacy of the item characteristic curve.

The hypothesis of sample invariant parameters is formulated as

$$H_0: \boldsymbol{\delta}^1 = \ldots = \boldsymbol{\delta}^G \text{ and } \gamma^1 = \ldots = \gamma^G.$$

It can be tested with a likelihood ratio test of L1 (the product of (3) across sub-samples i.e. no equality constraints on the item and γ parameters) against L2 ((5) i.e. item and γ parameters are constrained to be equal across sub-samples):

$$LR = -2 \log(L2/L1). \tag{10}$$

LR is asymptotically chi-square distributed with (G-1)n degrees of freedom (Hoijtink and Molenaar, 1992).

If LR is larger than the α-th percentile of a chi-square distribution with (G-1)n degrees of freedom, the hypothesis of sample invariant item and γ parameters has to be rejected. The next step is to determine which item (or the γ parameter) causes the trouble, and to remove that item from the item set. The hypothesis of sample invariance at the item and γ parameter level is formulated as

$$H_0 : \delta_i^1 = \ldots = \delta_i^G, \text{ for } i=1,\ldots,n,$$

and,

$$H_0 : \gamma^1 = \ldots = \gamma^G.$$

These hypotheses can be tested using Wald statistics:

$$CHI_i = \sum_g [(\hat{\delta}_i^g - \bar{\delta})^2 / (\hat{\sigma}_i^g)^2], \tag{11}$$

for $i=1,\ldots,n$, where, $\hat{\delta}_i^g$ and $\hat{\sigma}_i^g$ denote the item parameter estimate in sub-sample g and its standard error respectively, and

$$\bar{\delta} = \sum_g (\hat{\delta}_i^g / (\hat{\sigma}_i^g)^2) / \sum_g (1/(\hat{\sigma}_i^g)^2). \tag{12}$$

If in (11) and (12) $\hat{\delta}_i$ and its estimated standard error are replaced by $\hat{\gamma}$ and its estimated standard error the Wald statistic for the γ parameter is obtained.

Each Wald statistic is asymptotically chi square distributed with G-1 degrees of freedom (Hoijtink and Molenaar, 1992). If CHI is larger than the α-th percentile of a chi-square distribution with G-1 degrees of freedom, the hypothesis of sample invariance for the corresponding parameter has to be rejected. Since the Wald statistics are correlated, usually only the item with the largest CHI value is removed from the item set, after which the estimation/testing sequence is repeated for the reduced item set.

The adequacy of the PARELLA item characteristic curve (1) for the data at hand, is tested for each item via a comparison of the latter with the empirical item characteristic curve (which has to be reconstructed from the data). This is done with the SUM-statistic. Its theoretical distribution is unknown, but some relevant percentiles where determined via a simulation study (95-th=.65, 99-th=.78).

The SUM-statistic is defined as

$$SUM_i = [\sum_q |N_{qi}(\text{empirical}) - N_{qi}(\text{PARELLA})|]/\sqrt{N}, \tag{13}$$

for i=1,...,n. Where

$$N_{qi}(\text{empirical}) = \sum_a X_{ai} P(B_q|X_a), \text{ for } q=1,\ldots,Q, \tag{14}$$

denotes the expected number of persons at node q giving a positive response to item i as reconstructed from the data, and,

$$N_{qi}(\text{PARELLA}) = N\ \pi_q\ P_{qi}, \text{ for } q=1,\ldots,Q. \tag{15}$$

denotes the expected number of persons at node q giving a positive response to item i as predicted by the PARELLA model.

The basic idea behind the SUM-statistic is to compare the empirical proportion of persons giving a positive response to item i with (1) at each node q. If the differences are small, (1) provides an adequate description of the response process for the item at hand: the empirical choice proportions will be small at nodes located at a large distance from the item at hand, and large at nodes located at a small distance from the item. Since both the empirical choice proportions and (1) have to be multiplied with the same number (N_q) to arrive at N_{qi}(empirical) and N_{qi}(PARELLA), a comparison of the latter is equivalent to a comparison of the former.

6. Measurement of the Attitude with respect to Nuclear Plants

Formann (1988) provides an example of the measurement of the attitude towards nuclear plants. In Table 2 five items that were used as indicators for this latent trait are displayed. Looking at the phrasing of the items, it may be clear that they are ordered from pro to contra nuclear plants.

It is plausible that persons who are in favor of nuclear plants will agree with items 1 and 2, and disagree with the other items. Persons who are against nuclear plants will only agree with items 4 and 5, and persons with a neutral attitude will only agree with item 3. Stated otherwise, persons will probably agree with items that are an expression of their own attitude i.e. items that are located on a small distance of the person location.

Table 1
Response Patterns (Responses Ordered According to the Order of the Items in Table 2) and Observed Frequency

Resp.	Freq.	Resp.	Freq.
10000	6	11000	3
11100	18	01100	22
01110	65	00111	118
00011	39	00001	11
00000	3	01000	6
00100	14	00010	3
10100	22	10010	3
01010	4	01001	2
00110	41	00101	1
11010	2	10110	37
10101	2	10011	5
01101	1	01011	16
11110	52	11101	2
11011	2	10111	15
01111	61	11111	24

Since proximity relations appear to be the main determinant of the responses of persons to items, the PARELLA model may be suited to analyse the data that were collected by Formann. As can be seen in Table 1, most persons have a response vector that consists of a string of 1's bordered on one or both sides by a string of 0's (486 of 600 respondents). These persons could have been modelled with Coombs' parallelogram model. However, since not all persons have such 'perfect' response patterns (i.e. responding positively/negatively to all items within/outside a threshold τ) a probabilistic parallelogram model like the PARELLA model can be used to model these data.

Table 2
Item Phrasings

Item	Phrasing
1.	In the near future alternative sources of energy will not be able to substitute nuclear energy
2.	It is difficult to decide between the different types of power stations if one carefully considers all their pros and cons
3.	Nuclear power stations should not be put into operations before the problems of radio-active waste have been solved
4.	Nuclear power stations should not be put in operation before it is proven that the radiation caused by them is harmless
5.	The foreign power stations now in operation should be closed

In Table 3 the results of two analyses are displayed. The estimated order of the item locations resulting from analysis 1 corresponds with the expected order displayed in Table 2. The value of the γ parameter is 1.5, indicating that some noise interferes in the responses, but that distance between person and item on the latent trait is the main determinant of the response. Looking at the test for the agreement between the empirical and PARELLA item characteristic curve (column headed SUM), it can be seen that both item 2 and item 5 appear to have an item characteristic curve which is flat, i.e., the probability of a positive response does not depend on the location of a person on the latent trait.

Since flat items affect the discriminative power of the measurement instrument, they should be removed from the item set. However, since the SUM-statistics are mutually correlated, one is well advised to remove items one by one, the worst fitting first.

The columns presented under the heading Analysis 2 give the results of a PARELLA analysis omitting item 2. As can be seen the order of the remaining items is stable, the value of the γ parameter increases, and the fit is satisfactory. Consequently it may be concluded that the PARELLA model provides an adequate description of the responses to the remaining items.

Since the fit of item 5 is only slightly better than the fit of item 2 (.99 versus 1.0), the removal of item 2 instead of item 5 is rather arbitrary. Removal of item 5 also resulted in a set of four items with an adequate fit (the results will not be presented here). This leaves the choice to the researcher which of both four item sets (one without item 2, and one without item 5) is prefered.

Table 3
Item and γ Parameter Estimates

Item	Analysis 1			Analysis 2			Choice Proportion
	Item Location	SE	SUM	Item Location	SE	SUM	
1	-1.09	0.04	.48	-1.38	0.03	.42	0.32
2	-0.84	0.03	1.0				0.47
3	-0.02	0.04	.12	-0.14	0.03	.13	0.83
4	0.64	0.03	.16	0.33	0.03	.15	0.81
5	1.31	0.03	.99	1.20	0.03	.26	0.50
γ	1.50	0.10		1.75	0.10		

In Table 4 the estimated density function of the person locations is displayed. Most persons (93%) are located in the range <-.5,.5>. Looking at the phrasings of the items in this range (items 3 and 4), it may be concluded that most persons in this sample are moderately against nuclear plants i.e. nuclear plants should not be put in operation before they are proven to be save. Some persons (3%) are definitely against nuclear plants (located around -2.0), and some persons (5%) are in favor of nuclear plants (located around 2.5).

Table 4
Density Function of the Person Locations, Mean μ, and Variance σ^2 Resulting from Analysis 2 (N=600, μ=0.11, σ^2=0.50)

Node	Weight	SE
-2.50	0.01	0.08
-1.49	0.02	0.04
-0.47	0.43	0.05
0.54	0.50	0.04
1.55	0.03	0.05
2.57	0.01	0.14
3.58	0.01	0.11

7. Measurement of Change in Attitude with respect to the Car-Environment Issue

Doosje and Siero (1991) wanted to measure the change in attitude with respect to the car-environment issue, after a pro-environment information campaign. Some of the items they used to measure this attitude are displayed in Table 5. As can be seen the items are ordered from pro-environment to pro-car. Item 10 was omitted from the analyses to be presented because a prelimimary analysis showed that its item characteristic curve was too flat. This is not really surprising considering the content of the item, both persons with a pro-environment and pro-car attitude may agree with this item.

In Table 6 the results of three analyses are presented. The columns headed 'pre and post meas.' provide the results of an analysis in which the item and γ parameters are constrained to be equal for the sample of person in the pre, and the sample of persons in the post measure. The columns headed 'pre measure' and 'post measure', provide the results of the analyses in which the item and γ parameters are unconstrained.

Looking at the estimated order of the items on the latent trait (the second column in Table 6), it can be seen that it corresponds with the expected order (see Table 5). The γ parameter has a value of 1.5, indicating that the distance between person and item is the main determinant of the responses. Looking at the agreement between the empirical and the PARELLA item characteristic curve (the column headed SUM), it can be seen that the fit of item 7 is questionable. However since the fit of each item is tested at α=.05, it is not surprising that one of the items appears to have a bad fit. Consequently it was decided not to remove item 7 from the item set.

Table 5
Item Phrasings

Item	Phrasing
1.	People who keep driving a car, are not concerned with the future of our environment.
2.	The environmental problem justifies a tax burden on car driving so high that people quit using a car.
3.	Putting a somewhat higher tax burden on car driving is a step in the direction of a healthier environment.
4.	Car users should have to pay taxes per mile driven.
5.	A cleaner environment demands for sacrifices like a decreasing car usage.
6.	It is better to deal with other forms of environmental pollution then car driving.
7.	Considering the environmental problems, everybody should decide for themselves how often to use the car.
8.	Technically adapted cars do not constitute an environmental threat.
9.	Instead of environmental protection measures with respect to car use, the road system should be extended.
10.	Car use cannot be abandoned. Some pressure on the environment has to be accepted.

For a valid comparison of the measures obtained before and after the information campaign, the measurement instrument should be invariant, i.e. the item and γ parameters should be the same before and after the information campaign. The likelihood ratio test for sample invariance of the item and γ parameters rendered LR=16.65 with 9 degrees of freedom, which is not significant at the α=.05 level. Consequently the hypothesis of sample invariant parameters cannot be rejected. Looking at the tests for sample invariance at the item/γ level (column headed CHI), this result is confirmed.

Since the measurement instrument is invariant before and after the information campaign, the measures obtained before and after can be compared. In Table 7 the density functions of the person locations before and after are displayed (note that the samples before and after are independent). The mean of the person locations in the pre-measure (.46) is somewhat more to the pro-car side of the latent trait, than the mean in the post-measure (.35). To determine if the shift is significant the following Z-test was computed:

$$Z = [\mu(\text{pre}) - \mu(\text{post})] \,/\, \sqrt{[\sigma^2(\text{pre})/N(\text{pre}) + \sigma^2(\text{post})/N(\text{post})]}, \quad (16)$$

Table 6
Item/γ Parameter Estimates and Item/γ Level CHI Tests
(LR = 16.65, DF = 9)

Item	Pre and Post Meas. Item Location	SE	SUM	Pre Measure Item Location	SE	Post Measure Item Location	SE	CHI	Choice Prop.
1	-1.52	.08	.13	-1.52	.16	-1.53	.10	.00	.17
2	-1.51	.07	.66	-1.46	.13	-1.56	.10	.36	.16
3	-0.88	.03	.12	-0.92	.06	-0.85	.04	.77	.39
4	-0.85	.03	.19	-0.85	.06	-0.84	.04	.01	.41
5	-0.30	.06	.17	-0.36	.12	-0.24	.06	.70	.70
6	1.01	.03	.45	1.01	.05	1.01	.04	.02	.74
7	1.06	.03	1.05	1.04	.05	1.07	.05	.18	.72
8	1.28	.03	.52	1.34	.07	1.23	.04	1.41	.60
9	1.72	.07	.75	1.72	.12	1.72	.08	.00	.36
10									.81
γ	1.51	.11		1.40	.22	1.62	.14	.67	

which is to a test of the equality of two correlation coefficients as estimated from two independent samples (Sachs, 1974, pp. 333-336). The resulting Z-value was 2.52 which is significant at the α=.006 level. Consequently it may be concluded that the information campaign led to a significant shift of the attitudes in the car-environment issue towards the pro-environment side of the latent trait.

Table 7
Density Function of the Person Locations: Pre Measure (N=300, μ=0.46, σ^2=0.34); Post Measure (N=300, μ=0.35, σ^2=0.23)

Node	Weight	SE	Weight	SE
-1.05	0.014	0.013	0.011	0.017
-0.48	0.104	0.039	0.097	0.039
0.09	0.317	0.044	0.400	0.046
0.66	0.392	0.061	0.418	0.066
1.23	0.132	0.041	0.064	0.034
1.80	0.041	0.021	0.01	0.018

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