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A NONPARAMETRIC PROBABILISTIC MODEL FOR PARALLELOGRAM ANALYSIS

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Abstract

A nonparametric probabilistic model for parallelogram analysis can be regarded as a latent trait model with unimodal tracelines without specific assumptions about the functional form of the tracelines. Certain desirable empirically testable consequences can be derived, if two additional assumptions are postulated: total positivity of orders 2 and 3 for the traceline family. These rather technical assumptions are equivalent to two measurement related properties. The empirically testable consequences are formulated as properties of the correlation matrix and the conditional adjacency matrix. The correlation matrix is appropriate for distinguishing cumulative latent trait models from models for parallelogram analysis. The rows of the conditional adjacency matrix can be regarded as an estimate of smoothed-out tracelines of the row items. A test for the unimodality of these rows is also proposed, and can be regarded as a test for the unimodality of the tracelines.

Key words: Parallelogram analysis, latent trait, item response theory, unimodal tracelines, total positivity, unidimensional scaling, unfolding.

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1. Introduction

Unidimensional probabilistic parallelogram models for pick any/n data can be regarded as latent trait models, where persons and items are represented on the same scale, and where the probability of a positive response of a person to a given item tends to be higher when the distance between the position of the person and the position of the item on the latent trait is smaller. When the distance between the positions of the person and the item completely determines the person's response, we get the deterministic parallelogram model of Coombs (1950, 1964). Coombs formulated probabilistic versions of this deterministic model in terms of Single-Peaked Preference Functions: the person's preference is assumed to be a unimodal function of the positions of the items on the latent dimension, with its maximum on the person's position.

In the following, a mathematical model formulated in terms of item response theory will be presented as a probabilistic parallelogram (or unfolding) model for pick any/n data. This approach along the lines of item response theory deviates from Coombs' approach. The probabilistic model proposed here is not formulated in terms of Single-Peaked Preference Functions, but in terms of unimodal tracelines, i.e., the probability of a positive response to a given item is modeled as a unimodal function of the position of the person on the latent trait. However, if the preference is a monotone function of the distance between the position of the person and the position of the item, and if this function is the same for all persons, the model in terms of preference functions can be translated into a model formulated in terms of tracelines.

The set of items that together constitute the scale is supposed to be given. The items should have been selected in such a way that they are good indicators of the latent dimension. Persons are assumed to agree about the position of the items on the latent trait, although they themselves may have different positions on the latent trait. Each person is confronted with the same data set of items, and the response to each item can be positive ("agree") or negative ("disagree").

The aim of this nonparametric model for dichotomous responses on a set of items is

(1) to maintain the idea of unfolding and parallelogram analysis, viz. of modeling the associations between the responses by the existence of a latent trait, where closer proximity between persons' and items' positions leads to a higher probability of positive responses, and

(2) embody this idea in a minimal set of assumptions without specifying assumptions concerning the functional form of the tracelines or of the probability distribution of the person's positions on the latent trait,

(3) in such a way that consequences can be derived that reflect the basic idea expressed in (1), and that allow to test the model assumptions on the basis of the observed data.

In the following section our unidimensional model is formulated in terms of item response theory. In Section 3, properties of two diagnostic matrices are discussed, namely properties of the correlation matrix and of the conditional adjacency matrix. In Section 4, a statistical test for the properties of the conditional adjacency matrix is discussed. Section 5 presents the analysis of two empirical datasets. The last section contains a discussion of the proposed method.

2. Formulation of the Nonparametric Model for Parallelogram Analysis

As to the data structure, it is assumed that there are n items, indicated by i=1, ..., n; they are supposed to be located along the latent trait in the order given. The response of a person on item i is considered to be a random variable X_i; $X_i = 1$ if the person responds positively to item i, and 0 otherwise. A positive response to an item will also be expressed by stating that the person chooses this item. The vector of responses of a person is the random vector $X = (X_1, \ldots, X_n)$. We make the usual assumptions of the existence of a latent trait and local stochastic independence, reflecting the "pick any" data collection design and the unidimensionality of the set of items:

A1. There exists a unidimensional latent trait, β, such that every person has a position on this latent trait; the probability that a person with trait value β responds positively to item i is denoted $p_i(\beta)$. This probability, regarded as a function of β, is the traceline of item i.

A2. The responses of a person on the various items, given the latent trait value, are independent:

$$P(X=x|\beta=\beta_0) = \prod_{i=1}^{n} p_i(\beta_0)^{x_i}(1-p_i(\beta_0))^{1-x_i} .$$

The further assumptions, A3 to A5, are assumptions regarding the tracelines $p_1(\beta)$ to $p_n(\beta)$. For cumulative models, the tracelines would be required to be non-decreasing in β, see Mokken and Lewis (1982) and Sijtsma (1988). For parallelogram models, in contrast, the tracelines are required to be unimodal.

A3. For every item i, the traceline $p_i(\beta)$ is a unimodal function of β. Let δ_i be a value where $p_i(\beta)$ is maximal. It is possible that there is an interval of values where $p_i(\beta)$ is maximal, in which case δ_i is not unique. The δ_i can be chosen in such a way that they are non-decreasing as a function of i: $\delta_1 \le \delta_2 \le \ldots \le \delta_n$.

The value δ_i is regarded as the position of item i.

Assumption A3 is a statement about the conditional distributions of the response vector X, given values of β. To have properties of the parallelogram model that can be tested on the basis of a sample of persons, however, we need statements about the distribution of X without conditioning on the non-observable value of β, but under the assumption that β has been drawn at random from a population of β-values. It turns out that assumptions A1 through A3 do not suffice to derive testable consequences of the model. Therefore, we need two somewhat technical assumptions for the tracelines, which are expressed below in assumptions A4 and A5.

A4. The family of tracelines $\{p_i(\beta)|i=1,\ldots,n\}$ is Totally Positive of order two (TP_2), i.e. $p_j(\beta)/p_i(\beta)$ is a non-decreasing function of β, for all $\delta_j>\delta_i$.

This technical assumption is equivalent to the following measurement property concerning the relation between the order on the latent trait of the items and the order of the persons: When items i and j are ordered according to $\delta_j > \delta_i$, then knowing only that $X_j = 1$ should lead to a higher estimate of the person's latent trait value than knowing only that $X_i = 1$. For a proof, see Post & Snijders (1992) or Post (1992). These references also indicate that Assumption A4 is applicable to cumulative (monotone) models as well as parallelogram (unimodal) models.

We shall also need a somewhat more complicated analogue of A4, which is particularly appropriate for parallelogram models and not for cumulative models.

A5. The family of tracelines $\{p_i(\beta) \mid i=1,\ldots,n\}$ is Totally Positive of order three (TP_3), i.e. for all $\delta_k > \delta_j > \delta_i$, and $\beta_3 > \beta_2 > \beta_1$, the determinant

$$\begin{vmatrix} p_i(\beta_1) & p_i(\beta_2) & p_i(\beta_3) \\ p_j(\beta_1) & p_j(\beta_2) & p_j(\beta_3) \\ p_k(\beta_1) & p_k(\beta_2) & p_k(\beta_3) \end{vmatrix} \geq 0 .$$

This technical assumption is equivalent to the following measurement property concerning the relation between the conditional probability that a person has a central position on the latent trait, given that item i is chosen for i = 1, ..., n. For an item with a very small value of δ_i this probability will be low; for an item with a central position on the latent trait this probability will be larger, and for an item with a very high value of δ_i this probability will be low again. A further discussion is given in Post & Snijders (1992) and Post (1992).

Postulating A4 and A5 as additional assumptions makes it possible to derive testable consequences of the model defined by A1 to A3, as explained in the following section.

3. Two Diagnostic Matrices

How should one check in empirical data whether items form a scale according to parallelogram analysis as embodied in assumptions A1 to A5? In this section we discuss two diagnostic matrices, namely the correlation and conditional adjacency matrix. Certain properties for these matrices can be derived from assumptions A1-A5. A scale of items can be evaluated by checking these required properties. The conditional adjacency matrix is a variant of the adjacency matrix used as a diagnostic in MUDFOLD (Van Schuur, 1984, 1988; see also Van Schuur's contribution in this volume). For the latter matrix no properties could be derived under our model. For details and proofs, the interested reader is referred to Post and Snijders (1992) and Post (1992).

The Correlation Matrix

The correlation matrix is frequently used as a measure for association, but it also makes sense for item response models. In the cumulative Mokken model, for example, all correlations are necessarily nonnegative (Mokken, 1971). For the parallelogram model also negative correlations are possible. The correlation matrix is a square symmetric matrix with as its (i,j) element

$$R(i,j) = \frac{p(i,j) - p(i)p(j)}{\sqrt{[p(i)\{1-p(i)\}p(j)\{1-p(j)\}]}} \tag{1}$$

where p(i) is the proportion of persons in the sample choosing item i, and p(i,j) is the proportion of persons in the sample choosing both item i and item j.

If two items are close together on the latent trait, persons tend to either choose them both or not choose them both, resulting into a positive correlation. If two items are widely separated in the sense that where the traceline of the one item assumes large values, the traceline of the other assumes low values, then persons will tend to differ in response towards these two items, resulting in a negative correlation. Davison (1977) derived for a metric unidimensional unfolding model that the correlation matrix exhibits a simplex pattern (Guttman, 1954), i.e. the correlations are non-decreasing from the first column towards the diagonal and non-increasing from the diagonal towards the last column. It can be shown that for our model a weaker property is satisfied. If A1 to A5 hold then the population version of the correlation matrix exhibits the following sign pattern

1. Each row (column) has zero, one or two sign changes
2. If there are exactly two sign changes in a row (column), the first sign must be negative.

The Conditional Adjacency Matrix

The conditional adjacency matrix is a square asymmetric matrix with as its (i,j) element the relative frequency of persons responding positively on item i among persons responding positively on item j, i.e.

$$p(i|j) = p(i,j)/p(j) \tag{2}$$

where p(j) and p(i,j) are defined above (it is assumed that p(j)>0 for all j). Since in general no designs are considered in which persons are asked to respond twice independently on the same item, the diagonal elements of this matrix are not defined.

Consider the response of a person, drawn at random form a population of persons with a distribution of latent trait values with probability density function $g(\beta)$. Then the expected value of p(i|j) can be expressed as

$$P(X_i=1|X_j=1) = \int p_i(\beta)g_j(\beta)d\beta \ ,$$

where $g_j(\beta)$ is the conditional density function of β given that $X_j=1$,

$$g_j(\beta) = p_j(\beta)g(\beta)/ \int p_j(\beta)g(\beta)d\beta \ .$$

This shows that the i'th row of the conditional adjacency matrix can be regarded as an estimate of smoothed tracelines of item i, with smoothing kernel $g_j(\beta)$. It can be proven that the expected value of the conditional adjacency matrix has two properties under the nonparametric model, namely:

1. Each row exhibits a unimodal pattern, i.e., the elements first increase up to a maximum, after which they decrease.
2. The maximum of the row is in the column to the left of or identical to the column that contains the maximum of the following row, except for possible inversions around the diagonal, i.e., where the maximum in row i is in element (i,i+1) and the maximum in row i+1 is in element (i+1,i).

The latter needs some explanation. The maximum of the row must be defined with caution, because the diagonal element is not defined. If $p(i|i+1)$ is the maximal element in row i, then the 'true' maximum will be either in cell (i,i) or in cell (i,i+1). After all nothing is known about the diagonal element. Similarly, if $p(i+1|i)$ is the maximal element of row i+1, the 'true' maximum will be either in cell (i+1,i) or in cell (i+1,i+1). Therefore the case where the maximum in row i is in cell (i,i+1), and where the maximum in row i+1 is in cell (i+1,i) does not indicate that the second property is disturbed. In the following, property 1 is called the unimodality property, and property 2 the moving maxima property.

4. Testing the Properties of the Conditional Adjacency Matrix

Both properties of the conditional adjacency matrix can be inspected visually. But if the unimodality patterns in the rows are disturbed, or if the maxima do not have the required pattern, it still is a question whether these deviations are due to sample fluctuations or not. In Post (1992), a statistical test for the properties of the conditional adjacency matrix is derived, directed especially at the unimodality property. In the following, first a test statistic for the unimodality property for each separate row is discussed. After that we consider the positions of the maxima within the rows.

Denote by the vector $p(k|\cdot)$ the k'th row of the conditional adjacency matrix, by the vector $P(k|\cdot)$ its population version, and by $N^{-1}S(k)$ its estimated covariance matrix, where N is the number of persons. Our null hypothesis is that the row vector $P(k|\cdot)$ is unimodal. Note that this hypothesis does not include a statement about the position of the maximum element. Therefore we have to consider all possible positions of the maxima. Let $M(i)$ be the set of all unimodal vectors for which element i is the maximum. The proposed test statistic for our hypothesis is

$$\min_i \left[\min_{P(k|\cdot)\in M(i)} \{N(p(k|\cdot) - P(k|\cdot))'S(k)^{-1}(p(k|\cdot) - P(k|\cdot))\} \right] , \qquad (3)$$

Perlman (1969) derived upper and lower bounds for the asymptotic distribution of this test statistic under the null hypothesis, and also provided an explicit formula for calculating the p-level of the test statistic for a given data set.

The test procedure is as follows. The term between braces has to be minimized under the restriction that $P(k|\cdot)$ is unimodal with the maximum in element i. This minimization problem is a quadratic programming problem under a set of linear restrictions, and can be solved by several computer algorithms. The minimum has to be computed for each unimodal pattern $M(i)$, $i=1, \ldots, n$. For each given i the resulting p-value corresponds to a test of the hypothesis that the row is unimodal with the maximum in i. The second minimum (over i) yields the unimodal pattern with the maximum position that provides the largest p-value. In the following this p-value is called the maximal p-value. The maximal p-value corresponds to the test for the unimodality property. For details, see Perlman (1969) and Post (1992).

The procedure outlined in the last paragraph yields a matrix of p-values: element (i,j) is the p-value for the hypothesis that row i is unimodal with maximum in column j, see for example Table 4; the diagonal is empty. This matrix can be used to investigate the moving maxima property. There is also another reason to consider this matrix of p-values. If a row of the conditional adjacency matrix is nearly horizontal, several (or even almost all) of the p-values will be relatively large: the data are compatible with unimodal patterns with the maximum in several possible positions. Such a nearly horizontal row in the conditional adjacency matrix with many relatively large entries in the matrix of p-values suggests that the traceline for that row item is quite flat, which is not a very interesting case within the context of parallelogram analysis. In fact, such items are often poor items which should be detected, and in some cases they should be removed from the item set. This implies that the variation of the p-values in a row indicates the peakedness of the unimodal traceline, provided that the maximum of the p-values is not too small. A small variation of p-values indicates a poor item.

To summarize, the maximal p-value for each row can be used for testing the unimodality pattern of each row. The matrix of p-values for unimodal rows with maxima in given columns can be used to investigate the moving maxima property. Last but not least, the variation of the p-values within rows can be used as an indication for the peakedness of the unimodal tracelines of the row items.

5. Analysis of Two Empirical Data Sets

Attitude towards Nuclear Energy

Formann (1988) constructed a questionnaire about attitude towards nuclear energy. Five statements about nuclear energy were submitted to six hundred persons. We will use the scale found by the search procedure of MUDFOLD (see for details, the contribution about MUDFOLD by Van Schuur to this volume). Table 1 presents the scale defined by the five statements. The scale can be interpreted as a dimension ranging from strongly in favor of, to strongly against nuclear power.

The sign pattern of the correlation matrix in Table 2 is in agreement with the requirements: each row has exactly one sign change. The conditional adjacency matrix in Table 3 shows a perfect pattern. Each row satisfies the first property of the conditional adjacency matrix, a unimodal pattern,

Table 1
The Scale for the Statements about Nuclear Energy
Obtained by the MUDFOLD Procedure

A: In the near future, alternate sources of energy will not be able to substitute nuclear energy.
B: It is difficult to decide between different types of power stations if one carefully considers all their pros and cons.
C: Nuclear power stations should not be put into operation before the problems of radioactive waste have been solved.
D: Nuclear power stations should not be put into operation before it is proven that the radiation caused by them is harmless.
E: The foreign power stations now in operation should be closed down.

Table 2
The Correlation Matrix

Item	A	B	C	D	E
A	1	.09	.12	-.15	-.33
B	.09	1	.12	-.01	-.21
C	.12	.12	1	.13	-.20
D	-.15	-.01	.13	1	.32
E	-.33	-.21	-.20	.32	1

Table 3
The Conditional Adjacency Matrix and the
Maximal P-value for Unimodality per Row

Item	A	B	C	D	E	maximal p-value
A	-	<u>.37</u>[1]	.35	.29	.17	1.00
B	<u>.53</u>	-	.49	.46	.36	1.00
C	<u>.89</u>	.87	-	.85	.75	1.00
D	.73	.81	.83	-	<u>.94</u>	1.00
E	.26	.39	.45	<u>.57</u>	-	1.00

[1] The underlined elements are the maxima.

which is confirmed by the maximal p-values all being 1.00. The moving maxima property is also satisfied: the positions of the maxima are from top left down to the right.

The matrix of p-values presented in Table 4 gives p-values for all possible unimodality patterns for each row. The underlined elements are the p-values larger than 0.05. We propose that these are considered as candidates for the position of the maximum of the row. For the first row two positions for the maxima are likely, namely positions B and C. The two other possible unimodality patterns with the maximum in columns D and E are rather

Table 4
The Matrix of P-values with as its (i,j) Element the P-value for the Unimodality Pattern for Row i with Maximum in Column j.

Item	A	B	C	D	E
A	-	<u>1.00</u>[1]	<u>.87</u>	.00	.00
B	<u>1.00</u>	-	<u>.72</u>	<u>.08</u>	.00
C	<u>1.00</u>	<u>.96</u>	-	<u>.28</u>	.00
D	.00	.00	.00	-	<u>1.00</u>
E	.00	.00	.00	<u>1.00</u>	-

[1] The underlined elements are p-values larger than .05

unlikely. The second and third row both have three likely positions for the maxima. This implies that these rows have a horizontal pattern on a large part of the scale. From a measurement point of view these items have a rather limited value, because they do not give much information about the latent trait. The fourth and fifth rows are perfect. For both rows, there is one unimodality pattern for which the p-value is 1.00, and for all other possible unimodality patterns the p-values are 0.00. These items might have monotone tracelines.

We conclude that this scale of five items satisfies the assumptions of the nonparametric model. But because of the fact that rows B and C of the conditional adjacency matrix both could have three possible unimodal patterns, one might consider dropping one of these two.

Attitude towards Car-Use and Environment

Hoijtink introduced, in his editorial to this volume, data about the attitude towards car-use and environment. We have information about responses of 600 persons to ten statements concerning car-use in relation to environment. For 300 persons, we have pre-information campaign measures; for the other 300, we have post-information campaign measures. We shall analyze the scale obtained by the MUDFOLD procedure for all 600 persons (for details, see Van Schuur's contribution). The scale can be interpreted as a dimension ranging from strongly in favor of, to strongly against car use. In Table 5, the scale is presented.

The correlation matrix (Table 6) exhibits a perfect pattern: each row (column) has one sign change. There is a block structure in the pattern: the set of items consists of two subsets, the items within each subset having positive correlations. It is therefore possible that this scale consists of two cumulative scales, namely a scale formed by items A to E (with decreasing tracelines), and a scale formed by items F to J (increasing tracelines).

Checking the properties of the conditional adjacency matrix (see Table 7), we see that there are only small disturbances in the unimodality pattern of the first five rows. The disturbances are mostly due to items H and I. The maximal p-values, however, are very large for these rows which indicates that we cannot reject the hypothesis of unimodality on account of these data. The other rows show a perfect pattern, and have therefore maximal p-values equal to 1.00. Still, reversing items I and J would improve the pattern of the matrix. The moving maxima property is perfectly satisfied.

From the correlation matrix we had the indication that this scale consists of two cumulative scales. This is confirmed by this matrix. Recall that the rows of the conditional adjacency matrix can be regarded as estimates of smoothed tracelines. If items I and J are reversed, then we see that the first five rows each have a non-increasing pattern which suggests a non-increasing traceline, and that the last five rows each have a non-decreasing row which suggests a non-decreasing traceline. Comparing items C and D with each other, it is seen that these rows and columns are very similar.

Table 5
The Scale for the Statements about Car-Use and Environment
Obtained by the MUDFOLD Procedure

A: Instead of environmental protection measures with respect to car use, the road system should be extended.
B: Technically adapted cars do not constitute an environmental threat.
C: It is better to deal with other forms of environmental pollution than car driving.
D: Considering the environmental problems, everybody should decide for themselves how often to use a car.
E: Car use cannot be abandoned. Some pressure on the environment has to be accepted.
F: A cleaner environment demands for sacrifices like a decreasing car usage.
G: Car users should have to pay taxes per mile driven.
H: Putting a somewhat higher tax burden on car driving is a step in the direction of a healthier environment.
I: The environmental problem justifies a tax burden on car driving so high that people quit using a car.
J: People who keep driving a car, are not concerned with the future of our environment.

Table 6
The Correlation Matrix

Item	A	B	C	D	E	F	G	H	I	J
A	1	.28	.29	.28	.14	-.35	-.28	-.22	-.20	-.17
B	.28	1	.32	.23	.12	-.25	-.31	-.27	-.25	-.13
C	.29	.32	1	.24	.11	-.25	-.29	-.24	-.24	-.19
D	.28	.23	.24	1	.13	-.18	-.27	-.25	-.23	-.12
E	.14	.12	.11	.13	1	-.10	-.06	-.08	-.22	-.13
F	-.35	-.25	-.25	-.18	-.10	1	.29	.29	.22	.20
G	-.28	-.31	-.29	-.27	-.06	.29	1	.33	.29	.12
H	-.22	-.27	-.24	-.25	-.08	.29	.33	1	.30	.21
I	-.20	-.25	-.24	-.23	-.22	.22	.29	.30	1	.29
J	-.17	-.13	-.19	-.12	-.13	.20	.12	.21	.29	1

Table 7
The Conditional Adjacency Matrix and the Maximal P-value for Unimodality per Row

Item	A	B	C	D	E	F	G	H	I	J	maximal p-value
A	-	.47[1]	.45	.45	.39	.26	.20	.23	.15	.19	.97
B	.78	-	.70	.67	.63	.52	.42	.44	.32	.46	.60
C	.91	.86	-	.81	.76	.67	.59	.61	.50	.56	.98
D	.89	.80	.78	-	.75	.67	.58	.58	.49	.61	.77
E	.90	.87	.86	.86	-	.81	.81	.80	.65	.73	.91
F	.49	.61	.63	.65	.68	-	.86	.87	.93	.90	1.00
G	.23	.27	.32	.32	.39	.50	-	.61	.73	.53	1.00
H	.25	.28	.32	.32	.38	.49	.59	-	.73	.62	1.00
I	.06	.09	.11	.11	.12	.21	.29	.30	-	.39	1.00
J	.09	.13	.13	.14	.15	.22	.22	.27	.42	-	1.00

[1] The underlined elements are the maxima.

This could indicate that the positions of the items C and D are close together on the scale. The same can be said about items G and H.

From the matrix of p-values (Table 8) we get information about the peakedness of the tracelines. If the p-values are large within a row for several unimodal patterns, we regard the traceline of the row item as being rather flat on a large part of the trait.

Table 8
The Matrix of P-values with as its (i,j) Element the P-value for the Unimodality Pattern for Row i with Maximum in Column j.

Item	A	B	C	D	E	F	G	H	I	J
A	-	.97[1]	.79	.77	.00	.00	.00	.00	.00	.00
B	.60	-	.05	.01	.00	.00	.00	.00	.00	.00
C	.98	.50	-	.01	.00	.00	.00	.00	.00	.00
D	.77	.05	.00	-	.00	.00	.00	.00	.00	.00
E	.91	.64	.44	.44	-	.05	.05	.05	.01	.01
F	.00	.00	.00	.00	.00	-	.65	.85	1.00	1.00
G	.00	.00	.00	.00	.00	.00	-	.51	1.00	.16
H	.00	.00	.00	.00	.00	.00	.31	-	1.00	.85
I	.00	.00	.00	.00	.00	.00	.67	.73	-	1.00
J	.00	.00	.00	.00	.00	.01	.03	.18	1.00	-

[1] The underlined elements are p-values larger than .05

From Table 8, we learn that item E has a rather flat traceline on a large part of the trait. Six unimodal patterns are possible. This item is therefore a candidate for removal from the scale. The other items seem to have more peaked tracelines and seem therefore to be more discriminating.

Summarizing these findings we conclude that the scale obtained from MUDFOLD is a good scale, although we would prefer on account of the conditional adjacency matrix a reverse order of I and J, and remove on account of the matrix of p-values item E from the scale. Further, we should be aware of the relative closeness of items C and D, and of items G and H.

6. Discussion

In this paper we introduced a nonparametric parallelogram model for pick any/n data. This model is formulated in terms of item response theory with unimodal tracelines. Our goal was to formulate a model by a minimum set of assumptions without specifying the forms of the tracelines of items and the probability distribution of the persons. This should lead to testable consequences that reflect the basic idea of parallelogram analysis.

In addition to the usual assumptions, such as existence of the latent trait and local independence, we assumed unimodal tracelines. This, however, turned out to be insufficient for deriving testable consequences. Two additional, rather technical assumptions have to be postulated, namely the assumptions that the family of tracelines is totally positive of orders two and three. These assumptions are equivalent to two measurement related properties. The mathematical assumptions pose certain restrictions on the family of tracelines. For example, these are not satisfied by the family of PARELLA tracelines (Hoijtink, 1990, 1991; and see Hoijtink's contribution to this volume). This raises the question whether these mathematical assumptions are too restrictive for empirical applications. In Post (1992, Ch.4) this question is studied.

The correlation matrix and the conditional adjacency matrix turned out to be appropriate diagnostic tools. The correlation matrix is appropriate for distinguishing cumulative scale structures from parallelogram scale structures. The conditional adjacency matrix provides more information. The rows of this matrix can be regarded as estimates of smoothed tracelines of the row items. The test for the unimodality of the rows is useful but rather conservative, and not very powerful. The variation of the p-values of the possible unimodality patterns within a row turns out to be an appropriate diagnostic for the peakedness of the tracelines.

Because of the fact that the correlation matrix and the conditional adjacency matrix have properties that can be derived under a probabilistic model, and because of the fact that the latter matrix contains information about the form of the tracelines, they are essential additions to the diagnostic matrices used in MUDFOLD (Van Schuur, 1984, 1988), and PARELLA (Hoijtink, 1990, 1991). The variation of p-values for all possible unimodality patterns within a row is a useful diagnostic tool for recognizing flat tracelines. For details, see Post (1992), and Tempel (1991).

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