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# MUDFOLD

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**Abstract**

This paper describes a nonparametric unfolding model that can find one or more unidimensional unfolding scales composed of maximal subsets of dichotomous variables. The model is intermediate between the perfect unidimensional unfolding model and the null model of statistical independence. In this and other respects it resembles Mokken's nonparametric cumulative scaling model (Mokken 1970; Mokken and Lewis 1982). The determination of parameter estimates and several goodness of fit criteria are presented. Some criteria -- notably the coefficients of homogeneity of items, triples of items, and of the scale as a whole -- are used in the search algorithm. Two applications to 'pick any/n' data are shown. The paper concludes with some extensions of the model beyond 'pick any/n' data.

Keywords: hierarchical clustering, nonparametric model, parallelogram analysis, pick any/n data, unfolding

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## 1. Introduction

MUDFOLD -- an acronym for Multiple UniDimensional unFOLDing -- is a nonparametric unfolding procedure. The term 'multiple' indicates that more than one unidimensional unfolding scale may be found in a pool of items. This paper describes the hierarchical (bottom-up) clustering procedure used by MUDFOLD to establish the order of a maximal subset of items along an initial unfolding scale, and then to determine whether further subsets of items also form unfolding scales. In this procedure MUDFOLD evaluates and selects among candidate unfolding orders of items by comparing the number of times the empirical dataset violates these orders under the deterministic model of perfect unfoldability with the number of violations expected under the null model of statistical independence. Additional goodness-of-fit diagnostics are also presented, as well as the procedure for identifying the scale value of the subjects (i.e., their order relative to each other). The paper concludes with two sample applications, along with brief remarks about how the model can be extended to other types of data.

## 2. The Deterministic Unfolding Model and the Null Model

### 2.1 The Deterministic Unidimensional Unfolding Model

In most unfolding models, items are represented as ordered points along a latent dimension. In MUDFOLD, however, they are represented as ordered intervals, as will be explained shortly. Subjects are represented in terms of their 'ideal points' on the dimension -- their location in the vicinity of the items to which they respond 'positively' (e.g., statements with which they agree). In the deterministic or 'perfect' unfolding model, subjects who respond positively to a given item should also respond positively only to items that are adjacent to it. Subjects who respond negatively to an adjacent item should also respond negatively to all items further to the left or right. Put differently, all the positive responses in a subject's response pattern should be adjacent to each other on the latent dimension. If positive responses are coded as 1 and negative responses as 0, subjects' response patterns to an ordered set of items should thus contain an uninterrupted set of 1's, possibly preceded and/or followed by a set of 0's.

In addition to parameters for item locations and subjejct locations, the deterministic unfolding model needs an additional parameter that indicates, for each item-subject combination, the point in the distance between the item and the subject at which the change occurs between a positive and a negative response. Such a distance parameter may be considered to be dependent on the subject; e.g., DeSarbo and Hoffman (1986) introduce a subject threshold parameter. In MUDFOLD, however, the distance parameter is regarded as dependent on the item. To incorporate information about the distance parameter, the item parameter is redefined in terms of <u>two</u> parameters which, taken together, define the position of item i as an interval instead of a point. Item i's 'left' parameter -- $\delta_{i01}$ -- gives the point on the dimension at which subjects represented to its right <u>begin</u> to give the positive response to the item, and item i's 'right' parameter -- $\delta_{i10}$ -- gives the point at which subjects represented still further to the right <u>stop</u> giving the positive response. These two points will be called 'item steps', following Molenaar's (1982) use of the term for analogous constructs in the nonparametric cumulative scale analysis of multicategory items.

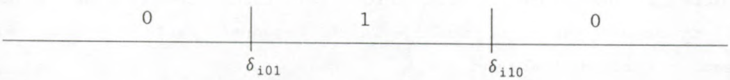


Figure 1
Latent Unfolding Dimension with the Two Item Steps for Dichotomous Item i: Only Subjects Located between $\delta_{i01}$ and $\delta_{i10}$ give the Positive (1) Response to Item i.

Although the location of item steps differs across items, we need to assume that the items can be ordered along the latent dimension in such a way that their left and right steps are sequenced in the same way. This assumption allows us to speak of the overall order of the items themselves, rather than only of their item steps.

## 2.2 The Null Model

In the deterministic unfolding model the probability that subject s will give the positive response to item i depends on the subject parameter and the two item parameters. The probability function - also known as 'trace line' or 'item characteristic curve' - is given in Figure 2 (left). The probability of a positive response changes as a step function from 0 to 1 and from 1 to

0 at the location of the item steps.

In evaluating the hypothesis that a dataset conforms to an unfolding scale, MUDFOLD tests this hypothesis against the null hypothesis that the responses to different items are independent. This null hypothesis is called 'the null model'. In any dataset to be evaluated for unfolding, each item has a certain 'popularity' -- i.e., a certain probability of receiving a positive response. This probability, denoted as p(i), is the same for each subject. The item characteristic curve is therefore a horizontal line according to the null model, rather than a step function (see Figure 2, right).

Under the assumption that the responses to different items are independent, the probability of each possible response pattern is equal to the product of the probability of each of the individual responses in the pattern. For instance, if p(A)=0.2, p(B)=0.4, and p(C)=0.7, the probability of the response pattern 101 to the triple ABC is equal to:

$p(ABC,101) = p(A).\{1-p(B)\}.p(C) = 0.2 \times 0.6 \times 0.7 = 0.084.$

To evaluate whether a dataset conforms to a unidimensional unfolding scale, MUDFOLD compares the number of response patterns that have a zero probability of occurrence under the deterministic unfolding model, but a nonzero probability under the null model. Which response patterns these are is discussed in the next section.


Figure 2
Trace Lines According to the Deterministic Unfolding Model (Left) and According to the Null Model (Right).

## 3. The Treatment of Model Violation in the MUDFOLD Model

### 3.1 Definition of Model Violation

A model violation will be defined as a response pattern that contains 0's between 1's. The smallest response pattern that can violate the deterministic unfolding model consists of three items: it is a triple with the response pattern 101. To understand this, consider what the responses to each of the three pairs of items in the response pattern (ijk,101) imply for the response to the third item. Let us assume that the items i, j, and k form an unfolding scale in this order. A subject who responds to item pair i,j with '10' (i.e., i=1 and j=0), must be represented to the right of item step i(01) (i's left item step, see Figure 1), but to the left of item step i(10) (i's right item step). If item i is represented to the left of item j, then item step i(10) is represented to the left of item step j(10). So if the subject is represented to the left of item step i(10), he must also be represented to the left of item step j(10). The 0 response to item j therefore implies that the subject must be represented to the left of item step j(01). If item j is represented to the left of item k, then item step j(01) is represented to the left of item step k(01). A subject who is represented to the left of item step j(01) is therefore also represented to the left of item step k(01), and so he should give the response 0 to item k; in other words, his response pattern to the triple i,j,k should be 100. The response 101 is therefore a model violation.

Similarly it can be shown that the response 01 to item pair j,k implies the response 0 to item i and the response 001 to triple i,j,k, and that the response 11 to item pair i,k implies the response 1 to item j, and the response 111 to the triple i,j,k. In each case the response 101 is a model violation.

The number of model violations (also called "errors") in a response pattern of more than three items is defined as the number of ordered triples with the 101 pattern. For instance, the response pattern 1010 to items A, B, C, and D, in this order, contains an error only in the triple ABC. The response pattern 1011, however, contains two errors: in triples ABC and ABD.

### 3.2 Evaluating the Amount of Error in a Dataset

In a dataset with n items, there are n(n-1)(n-2)/6 triples of items. For each of these triples it is possible to count the frequency with which two of the items are 'picked' (i.e., receive the positive response) but the third is not. If the items A, B, and C form a deterministic unfolding scale in this order, the response pattern ABC,101 has the frequency zero.

Since the order in which a set of items forms an unfolding scale is usually not known in advance, the MUDFOLD analysis considers all three orders of each triple -- e.g., ABC, BAC, and ACB (reflexions of these are equivalent for purposes of unfolding). If items A, B, and C conform to a deterministic unfolding scale, the observed absolute frequency of the error pattern for <u>one</u>

of the ordered triples -- i.e., O(ABC,101), O(ABC,011) or O(ABC,110) -- will be zero, and this will immediately indicate the unfoldable order of the items.

Unfortunately, in most empirical situations there are non-zero frequencies for the error patterns in all three permutations. We cannot simply accept the order with the smallest error frequency as indicating the unfoldable order of the items for two reasons. First, even the lowest number of errors may be too high in a substantive sense. Second, a low absolute number of errors may still imply a large *relative* number of errors, when the absolute error frequency is compared to the frequency that would be expected under the null model.

MUDFOLD proceeds by comparing the observed number of errors (O), observed in each permutation of each triple, with the number expected under the null model (E), e.g., O(ABC,101)/E(ABC,101), O(ABC,011)/E(ABC,011), and O(ABC,110)/E(ABC,110). The frequencies for response patterns 101, 011, and 110 that can be expected under the null model are calculated by simply multiplying the probability of these patterns according to the null model (see 2.2) by the sample size. The unfoldable order of a triple of stimuli is then taken to be the order with the smallest relative error frequency. Instead of using the relative error frequency, however, we use 1 minus this ratio as a coefficient of homogeneity, or H = 1 - O/E (c.f., Loevinger 1948). For each triple of items we therefore need to distinguish three different H-coefficients: H(BAC), H(ABC), and H(ACB).

A coefficient of homogeneity for the unfolding scale as a whole can also be defined as H = 1 - O/E. In this case O and E refer to the total amount of error observed and expected, respectively. These are the sums of the amount of error observed and expected, in all the n(n-1)(n-2)/6 triples that can be formed from the n ordered items in the unfolding scale. Similarly, a coefficient of homogeneity for each item i in a candidate unfolding scale H(i) can be determined by comparing the sums of the observed and expected number of errors in all those triples that contain the item.

On the basis of the null model we can postulate a distribution of the H-coefficient and establish the probability that a given H-coefficient in a sample might still have the value 0 in the population. Post (1988) has developed the T-statistic, interpretable as a z-score, for this purpose: if T > 2, then the probability that H is positive is smaller than 2.5% (one sided). But statistical significance does not always guarantee substantive relevance. In line with Mokken's (1970, 1982) experience with Loevinger's H-coefficient for cumulative scaling, we have adopted the rule of thumb that an H-coefficient larger than 0.3 is needed for a substantively acceptable unfolding scale. This rule has given acceptable results so far across different applications.

### 3.3 Additional Goodness-of-Fit Diagnostics

Three diagnostic matrices will be mentioned briefly here: the dominance matrix, the adjacency matrix, and the matrix of scale scores versus percentage of positive response to each item. Post gives a number of additional diagnostics elsewhere in this issue. The dominance matrix is a square matrix in which rows and columns identify the stimuli in their unfoldable order. Cell (i,j) of this matrix contains the percentage of subject who gave the positive response to item i but not to item j. The closer together the items are on the deterministic unfolding scale, the lower these percentages are. This means that rowwise the percentages should decrease from the extreme left to the diagonal, and increase from the diagonal to the extreme right. Cell values that violate this pattern of characteristic monotonicity indicate items that do not conform to the deterministic unfolding model.

The adjacency matrix is a square symmetric matrix in which rows and columns also identify the stimuli in their unfoldable order. Cell (i,j) of this matrix indicates the percentage of subjects who gave the positive response to both items i and j. The closer together the items are on the deterministic unfolding scale, the higher these percentages are. This means that rowwise the percentages should increase from the extreme left to the diagonal, and decrease from the diagonal to the extreme right (or: decrease from the diagonal to the bottom column, which is equivalent in a symmetric matrix). Cell values that violate this pattern of characteristic monotonicity indicate items that do not conform to the unfolding model.

The last diagnostic matrix is the table in which the item popularities of the items in their unfoldable order (columnwise) are given for each of the groups of subjects with the same scale score (rowwise). (The determination of subjects' scale scores is discussed below.) These percentages should show a single peaked pattern both rowwise and columwise. This table is adapted from an analogous table in the Mokken procedure, as devised by Molenaar (see Sijtsma, Debets, and Molenaar 1990).

### 4. Finding an Acceptable Unfoldable Order of the Items

Now that we have introduced the concept of model violation and the H-coefficient for a triple of items in each of its three permutations, we can define a search procedure that leads to an acceptable unfoldable order of the items. This search procedure consists of two steps: 1) find a startset; 2) add new stimuli to the startset as long as all requirements are fulfilled.

The startset can be a user-defined order of a (sub)set of items. If the user specifies a subset of items in a certain order, then the coefficients of homogeneity are calculated for the scale in this order. Ordered triples of items with a negative H(ijk)-coefficient are flagged, and their T-value is given. Starting with a user-defined ordering of items can be regarded as a confirmatory approach, in which the user asks whether or not these items form an unfolding scale in this order.

More often the startset is not predefined but rather is selected by the procedure as the 'best smallest unfolding scale' that can be found in the data. The best smallest unfolding scale consists of three items that satisfy a number of criteria based on the sign and the value of the H(ijk)-coefficients in their three permutations and on the absolute frequency of the most informative perfect patterns (i.e., 111, 011, and 110, if the error pattern is 101) (see Van Schuur, 1984, 1987 for a detailed explanation). The best smallest unfolding scale -- e.g., triple ABC -- can now be extended with a fourth item, e.g., D. In principle it is possible for the fourth item to be placed in any of the four positions in the existing unfolding scale: DABC, ADBC, ABDC, or as ABCD. The item that is added to the best triple, or to any existing k-item scale, must conform to a number of additional requirements, to do with the value of the H(i)-coefficients. This procedure continues as long as there are items left that conform to the requirements.

When an unfoldable order of a maximum subset of stimuli has been found there may be some items left over. The MUDFOLD procedure now tries to find an unfolding scale among these remaining items in the same way described above (hence the term 'multiple' unidimensional unfolding).

# 5. The Determination of Scale Values

## 5.1 Scale Values of Subjects in the Deterministic Model

Once the order of the items along the unfolding scale has been determined we can calculate scale values for the subjects. As defined in the deterministic model, each item brings with it two item steps. So k items lead to 2k item steps and 2k+1 diferent areas on the latent dimension. Only response patterns that contain at least one 1 can be used for determining scale values of subjects, and there are only 2k-1 of these, since the first and last area contain only 0's (see Figure 3).

We start with the extreme left area and give that the scale value 0. Subjects get one point for each item step they pass, moving along the latent continuum from left to right. A subject who gives the positive response to an item therefore has passed one item step (the left one) of that item, but not the second (right) one. A subject who gives the positive response to all the items in a three-item unfolding scale receives the scale value 1+1+1=3.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| item k: | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| item j: | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| item i: | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| area nr | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

Item steps (boundaries between areas, left to right): $\delta_{i01}$, $\delta_{j01}$, $\delta_{k01}$, $\delta_{i10}$, $\delta_{j10}$, $\delta_{k10}$

Figure 3
Scale Values for Subjects with Different
Admissible Response Patterns.

The negative response to an item, however, can be interpreted as either not having passed any of the two item steps, and so receiving no points, or both item steps, and so getting two points. The interpretation of a negative response as either zero or two points depends critically on the subject's responses to the other items, assuming at least one of them is positive. In the response pattern 100 for three items in their unfoldable order, the positive response is given to the leftmost item only. Since the subject has not yet passed the second item step of the first item, he cannot have done so for the last two items either, so the last two zeros are interpreted as worth

0 points. This response pattern is thus worth 1+0+0=1 points, so the scale value of subjects with this response pattern is 1. In the response pattern 001, in contrast, the positive response is given to the rightmost item, so the subject should be represented to the right of the threshold for the leftmost items. We assume therefore that both item steps of the first two items must have been passed, so the response pattern 001 gets 2+2+1=5 points. It turns out that if we define the scale values of the items as the odd-numbered rank orders (e.g., 1,3,5,7 ...), the definition of scale values for subjects is identical to the median value of the items picked by the subject (see Table 1).

A subject whose response pattern contains no positive responses at all cannot be given a scale value, since it is not clear which of the negative responses should be given 0 points and which two points. Such a person is regarded as a missing datum on the unfolding scale.

Table 1
Scale Values of Items and Subjects in a Deterministic Unfolding Scale

| A<br>1 | B<br>3 | C<br>5 | D<br>7 | E<br>9 | F<br>11 | G<br>13 | Scale values for<br><--- items | Scale value for<br>subjects ---> |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | median of 1,3,5 | or 1+1+1+0+0+0+0: 3 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | median of 5 and 7 | or 2+2+1+1+0+0+0: 6 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | median of 7,9,11,13 | or 2+2+2+1+1+1+1: 10 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | just 11 | or 2+2+2+2+2+1+0: 11 |

Subjects with imperfect response patterns (i.e., with 0's between the 1's) are given scale values as follows. A '1' means "one item step passed". A '0' means (a) zero item steps passed if the majority of 1's is to the right of this 0; (b) two item steps passed if the majority of 1's is to the left of this 0; or (c) one item step passed if there is an equal number of 1's to the left and right of this 0. Response pattern 1101 thus gets the scale value 3, response pattern 100111 gets the scale value 8, and response pattern 1110111 gets the scale value 7.

These rules can also be applied to more complicated response patterns. In the pattern 1010101, for instance, the first 0 is assigned 2 points, the second 1 point, and the last no points. The scale value of this response pattern is thus 7 (1+2+1+1+1+0+1, or as the median of 1,5,9,13).

## 6. Two Examples

### 6.1 The car-use data

MUDFOLD has been applied to the 'car-use' data introduced by Hoijtink in the first paper. It analyzed the data both as two separate datasets -- the pre- and the post-data -- and as a single joint dataset. The results differ only slightly, so I will focus on the analysis of the joint dataset. Table 2 shows the order of the items along the unidimensional unfolding scale according to the MUDFOLD procedure. The H-value of the scale is 0.39, and all H(i) values are over 0.30.

The interpretation of this scale is self-evident. Subjects with a low scale value (i.e., those with 1's on the left side of the scale) are in favor of car use and do not consider car use a threat to the environment. In contrast, subjects with high scale values (i.e., those with 1's on the right side of the scale) are strongly opposed to using a car and are very concerned about the future influence of car use on the environment.

Table 2
Results of the MUDFOLD Search Procedure for the Car Use Data.
Items are given in their unfoldable order. H=0.39.
p(i): proportion of subjects who agree with item i.
H(i): coefficient of homogeneity for item i.

| item | F | G | E | H | A | B | J | D | C | I |
|---|---|---|---|---|---|---|---|---|---|---|
| p(i) | 0.36 | 0.60 | 0.74 | 0.72 | 0.83 | 0.70 | 0.41 | 0.39 | 0.16 | 0.17 |
| H(i) | 0.53 | 0.40 | 0.37 | 0.34 | 0.38 | 0.41 | 0.35 | 0.39 | 0.47 | 0.33 |

Item F: Extend the road system; Item G: New cars are no threat; Item E: Other forms of pollution are worse; Item H: Everybody should decide for themselves; A: Pressure on environment must be accepted; B: Decrease car usage; J: Pay taxes per mile driven; D: Put a higher tax burden on car driving; C: Make taxes so high that people quit using a car; I: Car drivers are not concerned with the future of the environment. For a fuller formulation of the items, see Hoijtink, this issue.

The diagnostic matrices all show that the unfolding scale can be regarded as two cumulative scales of five items each. As can be seen from the column of p(i)'s (i.e., the proportion of subjects who give the positive response to item i), the column of p(i)'s is approximately single-peaked, going from 0.36 for item F to 0.83 for item A, and then back to 0.17 for item I. There are only two slight deviations: in the order of the items E and H and the items C and I.

The dominance matrix (not shown) displays a pattern of rowwise monotonicity that does not conform to the expected pattern in which the smallest percentages are found around the diagonal. Instead, the smallest percentages are consistently found around column A, the most popular item. With a few exceptions we find a pattern of rowwise monotonicity that accords better with the probabilistic cumulative scaling model, than with the unfolding model. The exceptions suggest that the order of the items E and H and perhaps J and D should be reversed.

Table 3, which presents the percentages of the positive response to each of the items by subjects in different scale score groups, shows the expected characteristic monotonicity pattern both rowwise and columnwise of a single peak in the order of the percentages. There are a few deviations in the expected order of 6% or less. The main rowwise disturbance concerns the order of items C and I, and the main columnwise disturbance concerns items A and H. These disturbances do not seem to be large enough to justify discarding any of the ten statements in the unfolding scale.

Table 3

Mean item scores for subjects in different scale score groups. The first two columns give the scale score(s) and the number of subjects in that scale score group. The other columns give the percentage of subjects in each scale score group that agrees with statement i.

| Scale score group | Items: N | F | G | E | H | A | B | J | D | C | I |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3- 5 | 94 | 85 | 96 | 97 | 78 | 78 | 9 | 1 | 1 | 0 | 1 |
| 6 | 78 | 62 | 94 | 100 | 100 | 83 | 45 | 9 | 14 | 0 | 4 |
| 7 | 103 | 49 | 78 | 93 | 87 | 92 | 74 | 25 | 17 | 3 | 8 |
| 8 | 62 | 31 | 56 | 82 | 98 | 98 | 84 | 37 | 32 | 5 | 11 |
| 9 | 71 | 18 | 58 | 73 | 70 | 87 | 87 | 44 | 49 | 15 | 24 |
| 10 | 56 | 4 | 39 | 55 | 64 | 100 | 100 | 68 | 64 | 14 | 16 |
| 11 | 51 | 8 | 25 | 49 | 43 | 73 | 94 | 75 | 71 | 31 | 22 |
| 12 | 41 | 2 | 12 | 34 | 41 | 88 | 100 | 100 | 90 | 39 | 49 |
| 13-16 | 44 | 2 | 7 | 16 | 14 | 32 | 95 | 86 | 93 | 89 | 59 |

As noted, scale values for the car-use data have been determined separately for the pre- and the post-datasets. The mean scale value for the pre-data is 8.2 and the mean scale value for the post-data is 8.4, but this difference is not significant (t=-0.92, p=0.36). Since the scale scores are in fact ordinal scores, a Kolmogorov - Smirnov 2-Sample Test was performed as well. This test also suggested that the two datasets were not significantly different (K-S Z=.90, 2-tailed p=.40).

**6.2 Formann's Nuclear Energy Data.**

In a second example, MUDFOLD is applied to Formann's nuclear energy data (see also Hoijtink, this issue). The results for this data set will be summarized briefly, since the principles have been illustrated with the previous example. The five items form an unfolding scale uniquely in the order established by Formann (see Table 4). The homogeneity coefficient for the scale as a whole is 0.44. The second item ("It is difficult to decide ...") is the worst-fitting item. In the search procedure it enters the scale as the last item, and causes the H-coefficient of a very good four-item scale (with H=0.73) to deteriorate.

Table 4
Summary of Search Procedure for Formann's Nuclear Energy Data.
p(i): percentage of respondents who agree with statement i.
H(i): coefficient of homogeneity of statement i.

| | P(I) | 3<br>H(I) | 4<br>H(I) | 5<br>H(I) |
|---|---|---|---|---|
| A) | 0.32 | 0.56 | 0.63 | 0.36 |
| B) | 0.47 | --- | --- | 0.33 |
| C) | 0.83 | 0.56 | 0.72 | 0.40 |
| D) | 0.81 | 0.56 | 0.76 | 0.54 |
| E) | 0.50 | --- | 0.79 | 0.65 |
| H-SCALE | | 0.56 | 0.73 | 0.44 |

Item A: Alternative sources are no substitute; Item B: It is difficult to decide; Item C: Solve problems of radio-active waste first; Item D: First show that radiation is harmless; Item E: Close foreign power stations. For a fuller formulation of the items, see Hoijtink, this issue.

The dominance matrix and the adjacency matrix (see Table 5) show only one deviation from the expected pattern of characteristic monotonicity: the percentage of subjects who respond positively to both of the first two items is too low, and the percentage who respond positively to the first but not the second is too high. This is an additional indication that the four-item scale without the second item is substantially better than the full five-item scale.

Table 5
Dominance and adjacency matrices for the five-item unfolding scale. In the dominance matrix, cell (i,j) contains the percentage of respondents who agreed with statement i but not j. In the adjacency matrix, cell (i,j) contains the percentage of respondents who agreed with both i and j.

| Dominance matrix | A | B | C | D | E |
|---|---|---|---|---|---|
| A) | - | 15 | 4 | 9 | 24 |
| B) | 30 | - | 6 | 9 | 29 |
| C) | 54 | 42 | - | 14 | 45 |
| D) | 58 | 44 | 12 | - | 35 |
| E) | 42 | 32 | 13 | 3 | - |

| Adjacency matrix | A | B | C | D | E |
|---|---|---|---|---|---|
| A) | - | | | | |
| B) | 17 | - | | | |
| C) | 29 | 41 | - | | |
| D) | 23 | 38 | 69 | - | |
| E) | 8 | 18 | 37 | 47 | |

Since the MUDFOLD procedure emphasizes model violations in triples of stimuli, it could be argued that it should not use response patterns that cannot contain such violations to evaluate the homogeneity of a set of items as an unfolding scale. In general, the response patterns that cannot falsify the unfolding model contain only '0's, only '1's, or a single '1': for a scale of five items these are thus the patterns 00000, 11111, 10000, 01000, 00100, 00010, and 00001. A large proportion of such patterns in a dataset may therefore overestimate the homogeneity of the data as an unfolding scale.

Three of the subjects of the nuclear energy dataset agreed with none of the five statements, forty agreed with only one, and twenty four agreed with all five. If we leave out these subjects from a MUDFOLD analysis, we find that the dataset of the remaining 533 subjects does indeed show less structure (i.e., lower H(i)-coefficients). MUDFOLD gives the same four item scale (second item left out) with H=0.61. The second item is not admitted for two reasons. First, two of the triples of stimuli now have a negative H(ijk)-value - H(ABC) and H(BCE). Second, forcing the second item in the scale decreases the homogeneity of the scale as a whole below the rule-of-thumb boundary of 0.30 (to H=0.29). In addition, it decreases the item homogeneity values of the first three items below that boundary (H(A)=0.23, H(B)=0.20, and H(C)=0.22)). The four remaining items ACDE conform to all the requirements of an unfolding scale, however.

## 7. Extensions

The MUDFOLD procedure has been described so far for dichotomous rating data, or 'pick any/n' data, as Coombs (1964) called them. Some additional applications are discussed in Van Schuur (1984, 1987, 1988, 1989). For example, MUDFOLD can also be used on 'pick k/n' data, in which each response pattern is required to contain the same number (k) of positive responses to the set of n stimuli. Pick k/n data often arise through the procedure of dichotomizing (full or partial) rank order data with the k highest rankings considered positive and the remaining n-k negative. The only difference with the procedure described above is the calculation of the expected number of errors. The dependencies of the responses in a pick k/n error pattern are taken into account.

Another extension is the application of MUDFOLD to ordered multicategory rating data (see Van Schuur, in press). For items with c categories, there are 2(c-1) different item steps. The order of the item steps for each pair of adjacent categories is the same, and this order determines the order of the items along the unfolding scale. Model violation is defined in terms of ordered triples of stimuli in which the middle item has a lower category value than either of the outer ones (assuming that 0 is the lowest response and a higher value indicates a more positive response). Scale values are determined as the number of item steps passed. The dominance matrix retains its meaning, as does as the table of scale scores versus item means.

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