KM 41 (1992)
pg.127-145

THE UNDERSHOOT OF THE RE-ORDER LEVEL FOR ITEMS SOLD SINGLY, a case study

Abstract

For the design of a new automatic order system a study is made of some properties of a periodic review stock control system for items sold singly. When at the end of a day the stock level is lower than a minimum level m an order is made to increase the stock level up to a maximum level M . The undershoot is the difference between the re-order level m - 1 and the stock level m - d on the moment an order is made. The probabilities P(d) are calculated without approximations for several values of D = M - m . For the demand a Poisson distribution is assumed. First the probabilities of all stock levels below the minimum stock level are calculated for one specified day after the last order. This gives the probability distribution of orders in time. The undershoot distribution is found by adding the results for all days. Special cases are D = 0 and D = ∞ . The case D = ∞ corresponds with the approximation known for large order quantities. This approximation appears to apply for all the order quantities that can be expected in the near future. Expressions for the mean and the standard deviation of the undershoot were achieved by looking at the moment that the stock falls below the minimum stock. The results are compared with experimental data for car parts sold singly. Corrections needed for the calculation of m are discussed.

Keywords: Inventory, Undershoot, Distribution

H.E. Schut
Automatiseringscentrum PON BV
P.O. Box 339, 3830 AJ Leusden
The Netherlands

1. Introduction

About twenty years ago a car factory introduced a computerised order system to supply car dealers automatically with parts from regional centres. Every month orders were made to bring the stock levels again up to the maximum level M . Only parts were selected that had a regular demand. The mean demand did not need to be high, a mean monthly sales of one item is normal for car parts. The Dutch importer adopted the system but after some years this importer changed from monthly to weekly and finally to daily orders and supplies. A minimum stock level m was introduced to limit the number of orders. Now both levels m and M were calculated for the selected parts and the dealers could add minimum stock levels for slow moving parts.

After the reduction of the review time and the introduction of a minimum stock level considerable reductions in stock value were achieved but some problems remained unsolved. Both levels m and M were calculated as constant time supplies, the mean demand in one month multiplied by a factor. This approach was easy to implement in computer programs but it is known to be not optimal [14]. After many changes in the programs a new system with more theoretical background was develloped and recently implemented. For $D = M - m$ a modified EOQ formula is now being used. One aspect of the calculation of the minimum stock in this new system, the estimation of the undershoot u of the re-order level $m - 1$, is the main subject of this article.

When at the end of a working day the economic stock is $m - d$, $d \geq 1$, an order is made in a batch process in the evening. The most usual situation is $d = 1$ but $d > 1$ also occurs frequently. The delivery time, also called replenishment lead time [14], is two days. This delivery time is not much longer than the review time, one day, and so the undershoot $u = d - 1$ cannot be neglected in the calculation of the minimum stock m . In the literature on stock control the undershoot is often not even mentioned [2,3,4,5,6,16]. For all the expressions that can be found for the mean and variance of the undershoot [7,13,14] approximations were used and some results are only valid for special cases such as sporadic demand or continuous review.

Expressions for the mean and variance of u for continuous review and discrete demand are given by Silver and Peterson [14] which they say to derive from Karlin [9]. An approximation for $P(u)$ for periodic review with u as a continuous variable, is given by Tijms and Groenevelt [15]. These results were found by using renewal theory and assuming D is large. Tijms and Groenevelt say that $D+1 > 1\frac{1}{2} a$, a is the number of items sold in one review period, is enough for a good approximation.

Expressions for the mean and variance of u in case of periodic review are given by Hill [7], who uses methods similar to those of Hadley and Whitin [6]. Hill assumes large order quantities but does not say how large orders have to be. Hill distinguishes two components of the undershoot, closely related to the two components of the demand distribution [4,13]. One component of the undershoot is caused by the sales transaction that brings the stock under the minimum stock. The other component is caused by the possibility of more than one sales transaction in the review period. The first component is only important for parts not sold singly. Most car parts are sold singly so that case had to be understood first. This proved to be also a good start for the more complicated situation of parts not sold singly.

From literature study and from De Kok [11] it became clear that not much is known about the undershoot distribution in case of small order quantities. There is a general tendency to reduce order quantities to achieve flexible production and inventory systems [5]. A determination of the accuracy of the usual approximations by comparing approximated and unapproximated results may therefore also be useful for other situations.

The probabilities $P(d)$ are calculated for periodic review and Poisson demand without using approximations. Graphs are given of the probabilities $P(d,n)$ of a stock level $m - d$ on the moment an order is made, n days after the previous order. The calculation of $P(d,n)$ gives an indication of the dispersion in the time between orders. Expressions for the mean and the variance of the undershoot are derived for two special cases. The numerical results are illustrated with experimental data. Some corrections of the minimum stock are discussed.

2. General case

The probabilities $P(d)$ can be calculated by summation of $P(d,n)$ over all possible daynumbers n . So the first question to be answered is: What are the probabilities $P(d,n)$ of a stock level $m - d$ at the end of day n while on day $n - 1$ the stock level was not less than the minimum m ? To answer this question the distribution of the demand $A(k)$ in a period of k days must be known for any integer $k \geq 1$. If the sales transactions are independent and consist of one unit, a Poisson distribution can be used with mean ka and a number of units sold j :

$$P(A(k) = j) = \frac{e^{-ka} (ka)^{j}}{j!} \tag{1}$$

The stock level $V(k)$, k days after the last order, is:

$$V(k) = M - A(k) \tag{2}$$

The probability $P(d,1)$ that stock level $m - d$ is reached one day after the order generation is:

$$\begin{aligned} P(d,1) &= P(V(1) = m - d) \\ &= P(A(1) = D + d) \\ &= \frac{e^{-a} a^{D+d}}{(D + d)!} \end{aligned} \tag{3}$$

The calculation of $P(d,n)$ with $n > 1$ is more complicated because there are several stocks levels $m + i$ possible at day $n - 1$. A summation has to be made over all possible values of i , $0 \leq i \leq D$. Sales during the first $n - 1$ days and sales at day n are independent. Hence the probabilities of reaching level $m + i$ from M , and $m - d$ from $m + i$ can be multiplied to give $P(d,n,i)$, the probability of reaching stock level $m - d$ from level $m + i$ at day n :

$$P(d,n,i) = P(V(n-1) = m + i)\, P(A(1) = d + i)$$
$$= P(A(n-1) = D - i)\, P(A(1) = d + i)$$

$$= \frac{e^{-na}\,(n-1)^{D-i}\,a^{D+d}}{(D-i)!\,(d+i)!} \qquad (4)$$

The probabilities $P(d,n)$ can now be calculated from:

$$P(d,n) = \sum_{i=0}^{D} P(d,n,i) \qquad (5)$$

where $n > 1$. The expression for $P(d)$ is:

$$P(d) = P(d,1) + \sum_{n=2}^{\infty} \sum_{i=0}^{D} P(d,n,i) \qquad (6)$$

In (4), (5) and (6) there is no direct dependence on m or M. Only the values of a, d and D have to be varied to get an impression of the behaviour of $P(d,n)$ and $P(d)$. For numerical results see Table 1 and Figure 1a and 1b.

3. Special case: D = 0

For slow moving service parts $M = m$ and hence $D = 0$ is the most usual situation. If $D = 0$ is substituted in (3), (4) and (5), the expression for $P(d,n)$ reduces to an equation which also can easily be derived directly:

$$P(d,n) = P(A(n-1) = 0))\, (P(A(1) = d)$$

$$= \frac{e^{-na}\,a^{d}}{d!} \qquad (7)$$

For this special case the summation over n can be carried out as a simple summation of a geometric series, giving:

$$P(d) = a^d / ((e^a - 1)\, d!) \qquad (8)$$

With (8) the results in Table 1 for $D = 0$ were checked. The results (7) and (8) can be seen as an exponential decay process as described in atomic physics. If $D = 0$ then $P(d)$ given by (8) is the transition probability from state $V = M$ to state $V = M - d$. If $D > 0$ there are possible states in between M and $m - d$ and the decay process becomes more complicated.

4. Mean and variance if D = 0

From (8) expressions for the mean and variance of d can be derived. In these derivations d is a more natural choice for the stochastic variable than u . From (8) and the definition of the expectation value it can be shown that if $D = 0$:

$$E(d) = a / (1 - \exp(-a)) \qquad (9)$$

$$E(d^2) = E(d)\,(1 + a) \qquad (10)$$

The mean of the undershoot u is:

$$\mu = E(d) - 1 \qquad (11)$$

The variance is both for d and u given by:

$$\sigma^2 = E((d - E(d))^2) = E(d)\,(1 + a - E(d)) \qquad (12)$$

From (10) and (12) it follows that if $D = 0$ and $a \gg 1$ the mean and variance of d are both equal to a . From (8) it can also directly be seen that if $a \gg 1$ then $P(d)$ is a Poisson distribution with mean a .

Using a second order approximation for the exponent in (9) it can be seen that if $D = 0$ and $a \ll 1$ the mean and variance of u are equal to $\frac{1}{2}a$. For these first two moments a Poisson distribution with mean $\frac{1}{2}a$ can be fit to $P(u)$.

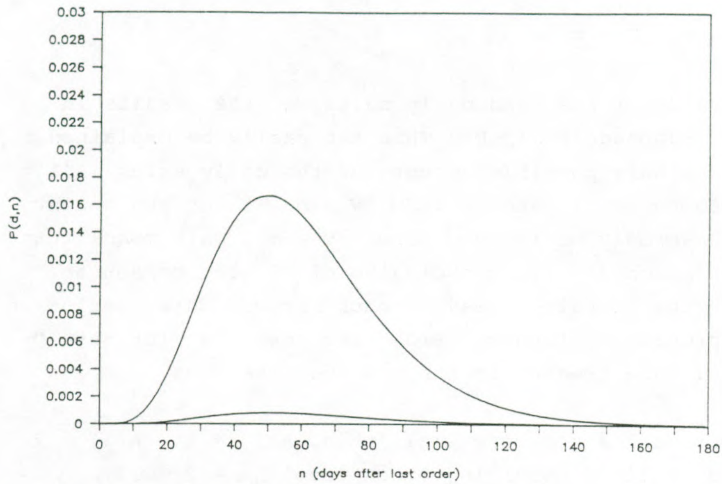

Figure 1a: Probabilities of a stock level m - d , when n days after the previous order the next order is made.
Upper curve: a = 0.1 , D = 5 , d = 1
Lower curve: a = 0.1 , D = 5 , d = 2

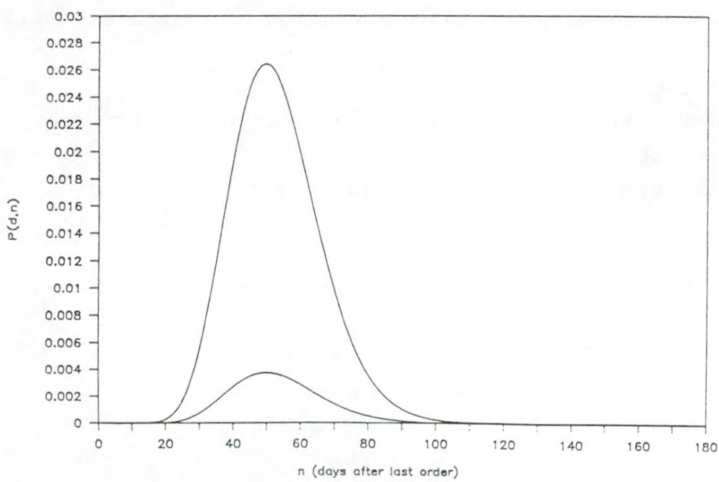

Figure 1b: Probabilities of an order point m - d , when n days after the previous order the next order is made.
Upper curve: a = 0.3 , D = 15 , d = 1
Lower curve: a = 0.3 , D = 15 , d = 2

5. Special case: $D = \infty$

For a small value of the mean daily sales a the results in Table 1 are independent of D. This can easily be explained. If $a \ll 1$ the only possible amounts of the daily sales $A(1)$ can be assumed to be 1 and 2 . So $V = m - 2$ at the moment of ordering, can only be reached from $V = m$. This means that for the calculation of $P(2)$ the value of D can be set to zero although the actual D may be much larger. This conclusion reduces the problem of finding $P(d)$ for small a for any D to the special case treated in the previous sections.

If $a < 1$ but a is not very small then $A(1) = 1$, $A(1) = 2$ and $A(1) = 3$ will be possible. Stock level $m - 2$ will usually be reached from m and $m + 1$. So $P(d)$ will hardly depend on D if $D > 1$. In Table 1 it can be seen that, in general, for any a there is a D' so that for $D > D'$ the values of $P(d)$ do not change anymore within the accuracy of the table. This is approximately the formal definition of the statement that the limit of $f(d,D) = P(d)$ exists for D to "infinity". An expression for the calculation of this limit will now be derived.

The value of $P(d,1)$ given by (3) can be seen to be very small if $D \gg a$, so this term in (6) can be neglected. The remaining expression for $P(d)$ can be written as:

$$P(d) = \sum_{i=0}^{D} \{P(A(1) = d + i) \sum_{n=2}^{\infty} P(A(n - 1) = D - i)\} \qquad (13)$$

The summation over n can be written as:

$$\sum_{n=2}^{\infty} P(A(n - 1) = D - i) = \frac{a^{D-i}}{(D - i)!} \sum_{n'=0}^{\infty} n'^{D-i} e^{-n'a} \qquad (14)$$

where $n' = n - 1$ is used. The term for $n' = 0$ could be added because its value is zero.

Figure 1a and 1b show that the distribution in time is spread over many days so the summand in (14) may be treated as a continuous variable. This can also be seen from the distribution of the demand in D / a days, this is a Poisson distribution with mean D and standard deviation $\sqrt{D}$. If $D >> a$, then also $D / \sqrt{D} >> a$, so the uncertainty in the moment of reorder must be much larger than one day. From these arguments we may conclude that the last summation in (14) can be replaced by an integral. The result is remarkably simple:

$$\frac{a^{D-i}}{(D-i)!} \sum_{n'=0}^{\infty} n'^{D-i} e^{-n'a} = \frac{a^{D-i}}{(D-i)!} \int_0^{\infty} x^{D-i} e^{-ax} dx$$

$$= 1 / a \qquad (15)$$

Substitution of (15) in (13) gives:

$$P(d) = (1 / a) \sum_{i=0}^{D} P(A(1) = d + i) \qquad (16)$$

Again using $D >> a$, the last summation can be extended to infinity and so:

$$P(d) = (1 / a) \; P(A(1) \geq d) \qquad (17)$$

The final result (17), derived for Poisson demand, is a special case of more general expressions for $D = \infty$ [14,15]. Numerical results are given in Table 1.

Expression (16) suggests that all stock levels before an order is made have the same probability. This is not true, low stock levels are more probable on the day before an order is made. A direct 'physical interpretation' of the expressions (16) or (17) is not possible. The derivation of the mean and variance from (16) or (17) is also not simple. For that purpose another approach of the undershoot is chosen.

6. Mean and variance if $D = \infty$

For the general result (4) and hence for (17) two periods were considered, before and after the beginning of the day that the order is made. If $D = \infty$ can be assumed it is useful to look more closely at the moment that the stock falls below the minimum stock level m .

For items sold singly, the undershoot is the demand after the moment at time t when the stock level $V = m - 1$ is reached. The time t is expressed as a fraction of the review time. The undershoot distribution is the composition of the distribution of t and the demand in the period between t and 1 . If $D >> a$ the distribution of t will be uniform, the dispersion in the moment of re-order is much larger than one day. In that case the mean value of t is $\frac{1}{2}$. The mean undershoot is the mean demand in a period with average duration $\frac{1}{2}$, so:

$$\mu = \tfrac{1}{2} a \qquad (18)$$

Also the variance of a composed distribution can be derived from the mean and variance of the composing distributions [2,16]. In case of a uniform distribution of t the variance of t is $1/12$ and the variance of the undershoot is given by:

$$\sigma^2 = \tfrac{1}{2} a + a^2 / 12 \qquad (19)$$

From (19) it can be seen that if $a << 1$ the dispersion in t can be neglected. In that case the dispersion in the demand is relatively large and (19) reduces to:

$$\sigma^2 = \tfrac{1}{2} a \qquad (20)$$

The expressions (18) and (20) illustrate that for $a << 1$ the undershoot distribution is close to a Poisson distribution with mean $\frac{1}{2} a$ as was found for $D = 0$ and $a << 1$. In contrast to (11) and (12) the expressions (18) and (19) can also be derived from results in literature [7,14]. For the application of the standard results for continuous review [14] the transaction size must be assumed to be the sales in one review period.

Table 1

The probability distribution P(d) , m - d is the stock when an order is made, for several values of D = M - m and the mean daily sales a . The undershoot is u = d - 1 .

a	d	D=0	D=1	D=3	D=5	D=∞
0.05	1	0.975	0.975	0.975	0.975	0.975
	2	0.024	0.024	0.024	0.024	0.024
0.1	1	0.951	0.952	0.952	0.952	0.952
	2	0.048	0.047	0.047	0.047	0.047
	3	0.002	0.002	0.002	0.002	0.002
0.3	1	0.858	0.864	0.864	0.864	0.864
	2	0.129	0.123	0.123	0.123	0.123
	3	0.013	0.012	0.012	0.012	0.012
0.5	1	0.771	0.787	0.787	0.787	0.787
	2	0.193	0.180	0.180	0.180	0.180
	3	0.032	0.029	0.029	0.029	0.029
1	1	0.582	0.630	0.632	0.632	0.632
	2	0.291	0.266	0.264	0.264	0.264
	3	0.097	0.081	0.080	0.080	0.080
	5	0.005	0.004	0.004	0.004	0.004
3	1	0.157	0.261	0.325	0.317	0.317
	2	0.236	0.273	0.269	0.266	0.267
	3	0.236	0.214	0.190	0.192	0.192
	5	0.106	0.070	0.059	0.062	0.062
5	1	0.034	0.086	0.194	0.213	0.199
	2	0.085	0.144	0.207	0.197	0.192
	3	0.141	0.182	0.189	0.172	0.175
	5	0.177	0.153	0.108	0.107	0.112
	7	0.105	0.069	0.041	0.046	0.048
	10	0.018	0.009	0.005	0.006	0.006

7. Applicability of the approximation $D = \infty$

The results in Table 1 can be used to see in which cases the usual approximation $D = \infty$ holds. For parts with $a < 0.5$ the approximation $D = \infty$ can be used even if $D = 0$, in that case the probabilities $P(d)$ do hardly depend on D .

In general the actual value of D must be larger than a value dependent on a , as was shown in the beginning of section 5. From Table 1 it can be seen that if $D + 1 > 1\frac{1}{2}\, a$ the $D = \infty$ approximation can be used, as was suggested by Tijms en Groeneveld [15] and De Kok [10]. In the automatic order system the standard economic order quantity will be used with some corrections, if $a > 0.5$ roughly $D + 1 = EOQ$ will hold. So the approximation $D = \infty$ can be used if:

$$\sqrt{(500\ a\ Cb\ /\ (Cv\ P))} > 1\tfrac{1}{2}\ a \qquad (21)$$

with Cb = order costs in Fl, Cv = stock costs in one year for Fl 1,- of stock and P = sales price in Fl. The number of working days in a year is set to 250. For parts send to car dealers approximately $Cb / Cv = 15$ and so:

$$a < 3333\ /\ P \qquad (22)$$

Parts with a price above Fl 1000,- will not be automatically delivered so (22) holds for $a < 3,3$. Expensive parts sold singly have at most $a = 3$ so the approximation $D = \infty$ can be applied in the automatic order system.

They approximation $D = \infty$ may not hold in the future. If the stock costs Cv are increased by a factor 4, as was advised recently [5] to avoid hidden problems, some care has to be taken with the approximation $D = \infty$. Also the tendency to reduce the number of stock points [12] can make the appoximation $D = \infty$ invalid. Daily delivering orders may become too expensive and the review time might become one week, which would increase the mean sales in one review period by a factor 5. In that case, to keep the required service level, the approximation $D = 0$ can be used for parts with $a > D + 1$.

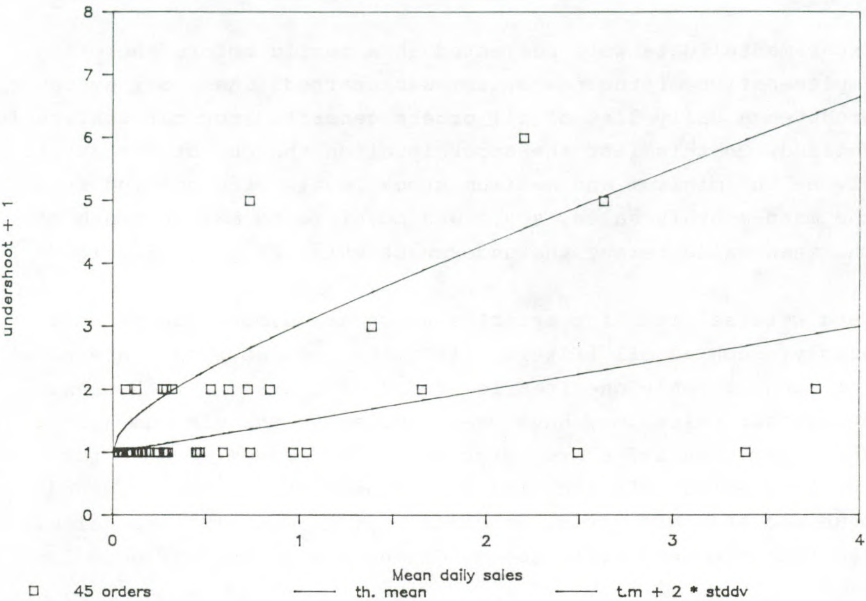

Figure 2: The undershoot of the minimum stock level for 45 orders of articles sold singly. The straight line gives the expectation value and the curved line the expectation value plus two times the expected standard deviation, both as calculated from the mean daily sales. The data points can also be compared with the probabilities in Table 1 for $D = \infty$, with d = underhoot + 1 and a = mean daily sales.

8. Experimental data

Experimental data were collected in a period before the implemenation of the new system was started. The order system produced a daily list of all orders generated for car dealers in Holland. On this list the stock level on the end of the day is given. The minimum and maximum stock levels were derived from the mean monthly sales, so it was possible to make a graph of the mean sales versus the undershoot which is given in Figure 2.

Data were selected for articles which are almost always sold singly, such as oil filters, air filter elements and antenna's. For each car only one item is needed. For small a the real undershoot values may have been limited by the minimum stock. The order quantities are large enough to assume $D = \infty$ for the theoretical distribution of the undershoot. The expected mean and standard deviation given by (18) and (19) are calculated from the mean daily sales. Graphs are given in Figure 2.

The mean of the daily sales a is calculated from a moving average of the monthly sales. If less than one item is sold in one month the error in a can be quite large [4]. A simulation showed that the standard deviation in a may range from 10% for $a > 0.5$ up to 30% for $a < 0.5$. In case of a trend in the sales data the error in a can even be much larger. All points were omitted where a trend or irregularity in the sales could be recognised. For $a < 0.5$ two moving averages with a different time constant could be compared. Some of the omitted points with a positive trend in the sales data showed a large undershoot with a probability lower than 0.5% in Table 1. This analysis showed that including a correction for the trend in the calculation of the minimum stock is quite useful.

In Figure 2 most of the data points scatter around the expected mean value of the undershoot but still some data points deviate more than two standard deviations from the mean. Deviations larger than 2σ are not directly in conflict with the probabilities given in Table 1 under $D = \infty$. Some data points with 4σ deviation from the mean still have a probability of about 3%. These deviations illustrate that the probability distibution

P(d) is quite asymmetric. This is mainly caused by using a demand distribution for a period of only half a day. For the new expression of m the demand in a period of about three days is considered and only a small correction for asymmetry is applied.

In Figure 2 there remains one point, with $a = 0.7$ and $d = 5$, which has a probability in Table 1 lower than 1%. This still can be a normal undershoot but there may be another explanation. In a more recent data set also one case of extreme large undershoot seemed to occur. A stock correction on the same day, triggerd by the order picking, appeared to be the reason of the low administrative stock level at the end of the day.

9. Time between orders

The time between two subsequent orders is equal to the time to sell at least the minimum order quantity $D + 1$. The graphs given in Figure 1a and 1b show that the mean time between two subsequent orders T is approximately $(D + 1) / a$. A more accurate expression for T can be obtained by adding the expected undershoot (18) to the minimum order quantity. In case of Poisson demand and $D \gg a$ the result is:

$$T = (D + 1) / a + \tfrac{1}{2} \qquad (23)$$

Expression (23) can also be derived by using a result from renewal theory [8] and the relation between mean and standard deviation for a Poisson distribution.

In the literature on stock control no expressions were found for standard deviation in the time between orders. The graphs in Figure 1a and 1b show that the dispersion in the time between two orders is large if $a \ll 1$. This implies that, in case of many dealers and a large assortment of slow moving articles, the stream of automatically generated orders to the central store will be almost constant. The number of orders in one day will not vary very much. In the old system the montly dealer orders were spread over the days in one month. This is not neccessary anymore, which simplifies the system.

10. Conclusions and discussion

Calculations of undershoot probabilities were used to test the applicability of the approximation $D = \infty$ and for an interpretation of experimental data. For the current situation the order quantities are large enough to assume $D = \infty$.

For the expression of m in the new automatic order system, using $D = \infty$, a uniformly distributed variable is added to the delivery time. Half a day is added to the delivery time of two days to obtain the mean lead time. The variance of the lead time is $1/12$, which cannot be neglected for fast moving parts. This approach is also used for parts not sold singly. The first two moments were determined for the composition of the distributions for the lead time, customer arrival and transaction size. In the expression for m a separate term is included for the event that brings the stock under the minimum stock.

When the stock costs or the review time increase, the assumption $D = \infty$ may not be valid anymore for expensive fast moving parts. For these parts the approximation $D = 0$ can be used. The approximation $D = 0$ can also be simplified, a worst case approximation can be applied. The first customer can be assumed to arrive directly at the beginning of the review period and the minimum stock can be calculated by including a full review period in the lead time, as was suggested by Brown [1]. This approach must not be used for all parts because this would increase the stock value of slow moving parts without a reason.

Looking at the moments when things happen made it possible to understand the system, which increased the flexibility in the design process. The dispersion in the time between orders was found to be large enough to result in a constant flow of orders. Poisson demand could be extended to compound Poisson demand. Fluctuations in the delivery time could be included by adding another uniformly distributed variable to the delivery time. The time oriented approach could also be explained to the people who support the dealers. This gave them more background to tell the dealers what they can expect from the new system: lower stock values and still a good service degree for all parts in stock.

Glossary

a = mean number of items sold in one review period
Cb = order costs in Fl
Cv = stock costs in one year for Fl 1,- of stock
d = difference between minimum stock and the stock on the moment an order is made
D = M - m
i = difference between stock on the day before an order is made and the minimum stock level
j = number of items sold in k days
k = number of days in the period for which a Poisson distribution is defined
m = minimum stock level
M = maximum stock level
P = sales price in Fl
P(d) = probability of stock m - d on the moment, at the end of the day, that an order is made
P(u) = probability of undershoot u
P(d,n) = probability of stock m - d at the end of day n
P(d,n,i) = probability of stock m - d at the end of day n while at the end of day n - 1 the stock is m + i
σ = standard deviation of the undershoot
t = time between the last stock review and the moment that the stock becomes smaller than the minimum stock m
T = mean time between two subsequent orders
u = undershoot, u = d - 1
μ = mean undershoot
V = stock level
= inventory position
= stock on hand - backorders + orders - committed
V(k) = stock level k days after the last order

(Remark: in the text all the variables and arithmetical expressions are surrounded by spaces.)

ontvangen 4-3-1992
geaccepteerd 26-5-1992

References

[1] Brown, R.G., "Statistical Forecasting for Inventory Control", McGraw-Hill, New York, 1959.

[2] Beek, P. van, "Modelling and analysis of a multi-echelon inventory system, a case study", European Journal of Operational Research 6 (1981) 380 - 385.

[3] Christopher, M., "The Strategy of Distribution Management", Quorum Books, Connecticut, 1985.

[4] Croston, J.D., "Forecasting and stock control for intermittent demands", Operational Research Quarterly 23 (1972), 289 - 303.

[5] Eekhout, L.H.M. van, "Rekenen aan voorraadbeheer", in E. Mollema (Conference Manager), 'Nieuwe inzichten in voorraadbeheer', Institute for International Research, World Trade Center, Amsterdam, Januari 29, 1991.

[6] Hadley, G. and Whitin, T.M., "Analyses of Inventory Systems", Prentice-Hall, Englewood Cliffs, N.J., 1963.

[7] Hill, R.M., "Stock control and the underhoot of the reorder level", Journal of the Operational Research Society 39 (1988) 173 - 181.

[8] Hill, R.M. and Odoni, R.W.K., "Time to sell a given quantity of stock", Journal of the Operational Research Society 39 (1988) 969 - 972.

[9] Karlin, S., "The application of renewal theory to the study of inventory policies", in: K.J. Arrow, S. Karlin and H. Scarf, 'Studies in the Mathematical Theory of Inventory and Production' Stanford University Press, Stanford, California, 1958, 270 - 297.

[10] Kok, A.G. de, "Approximations for operating characteristics in a produktion-inventory model with variable production rate", European Journal of Operational Research 29 (1987) 286 - 297.

[11] Kok, A.G. de, personal communication (1989).

[12] McCraith, A.J., "Case-study: inventory reduction on a grand scale by British Aerospace", in E. Mollema (Conference Manager), 'Nieuwe inzichten in voorraadbeheer', Institute for International Research, World Trade Center, Amsterdam, Januari 29, 1991.

[13] Schultz, C.R., "Forecasting and inventory control for sporadic demand under periodic review", Journal of the Operational Research Society 38 (1987), 453 - 458.

[14] Silver, E.A., and Peterson, R., "Decision Systems for Inventory Management and Production Planning", 2nd edn., Wiley, New York, 1985.

[15] Tijms, H.C. and Groeneveld, H., "Simple approximations for the reorder point in periodic and continuous review (s,S) inventory systems with service level constraints", European Journal of Operational Research 17 (1984) 175 - 190.

[16] Van der Veen, B., "Safety stocks - an example of theory and practice in O.R.", European Journal of Operational Research 6 (1981) 367 - 371.