KM 41 (1992)
pg.109-125

THE DERIVATION OF A LONG TERM MILK SUPPLY MODEL FROM AN OPTIMIZATION MODEL

J.H.J. Roemen*

Abstract

In this paper we formulate an alpha-numerically specified model for the determination of the level of (des)investments in live stock by a dairy farm. From this model we derive reaction equations for the optimal level of the in- and outflow of dairy cows. Rearranging these equations provides a specification of the determinants of the milk supply in the long run. This model was used as a point of departure for the estimation of the long run milk supply elasticity in the Netherlands during the period 1969-1984. By starting from a model of the optimizing behaviour of the farmer, an underpinning of the milk supply model, to be used in estimation, is obtained. Having a micro-economic foundation, the derived specification gains cogency in comparison to a supply model specified on the basis of plausibility considerations and/or considerations related to the convenience of estimating.

* Department of Econometrics, Tilburg University, Postbox 90153, 5000 LE Tilburg, tel. 013-662054.

1. Introduction

In this paper, based on [10], we present a simplified model of the determinants of the milk supply of a dairy farm (or a country) in the long run. This supply is equal to the product of the average yield per cow and the number of lactating cows, so a change in the volume of this production can be realized by means of this average yield, via the number of lactating cows or via a combination of these two possibilities. In this analysis we take the yield development as autonomous, so the size of the dairy cow stock determines the development of the level of the long run milk production, other factors left aside. Changes in the size of this stock can be effected by putting in calf (inseminating) more (less) heifers this year than will be needed as replacements in the following year, by the level of culling or finally by a combination of these possibilities.

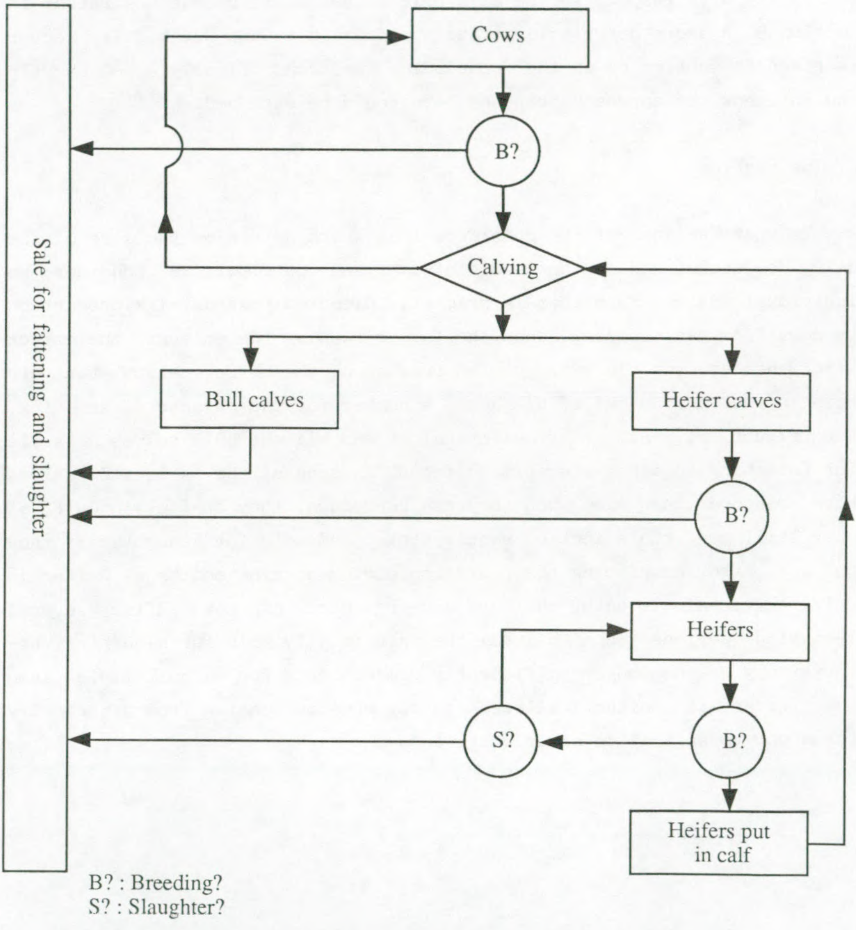
Because the individual farmer decides upon changes in this stock, we consider the underlying decision process at the farm as an obvious point of departure for modelling the milk supply. Every year such a farm has to take, in reaction to changing circumstances, decisions as to how the farm will be managed in that year. Furthermore, every year decisions must be taken on the direction and the volume of investments in live and dead stock and on whether these investments should be financed by own of borrowed funds. For simplicity's sake, however, we restrict us to the decisions with respect to the live stock. Although these decisions must always fit in the possibilities qua labour, dead stock and capital that the farm can dispose of, these factors will not be taken into consideration here. As a result of this assumption the derivation of the supply model does not come up for discussion in its full generality. However, by this simplification the exposition of the method followed gains clearness.

In section 2 we briefly describe the investment problem of the dairy farmer: the determination of the optimal size of the dairy cow stock. As a criterion in determining this size the farmer uses the maximization of the value of the (discounted) cash flows. The model of this problem is formulated in section 3. The objective function is alpha-numerically, that is in letters and numbers, specified. From this model we derive in section 4 decision rules for the optimal level of the (des)investments in the dairy cow stock. A rearrangement of these rules identifies the variables that determine the long run milk supply. This specification provides a well founded point of departure for the estimation of the long run milk supply elasticity with respect to the milk price, as will be demonstrated in section 5. A short description of an application to the Dutch dairy sector is given in section 6. In the concluding section, finally, we shortly indicate how the approach proposed here could be extended.

2. The problem

To get an idea of the considerations which determine the size of the dairy cow stock, and so the levels of in- and outflow, we consider an individual dairy farm that is primarily directed towards milk production by cows from own breeding. When the farmer wants to enlarge the stock size, he chooses in every period from among the heifer calves, that are born in that period out of his herd, a number for the purpose of breeding, see figure 2.1. All other heifer calves and all the bull calves he sells for fattening to other specialized farms. As soon as the selected calves have reached the age when they can reproduce, they are put in calf, if they still meet the selection requirements, and sold for slaughter if they do not. After completing the gestation period of nine months as heifer in calf, these animals enter the farm's dairy herd as cow. After several lactation periods (and calves) they are finally sold for slaughter, because they are no longer sufficiently productive. For simplicity's sake the farmer is neither allowed to buy breeding-cattle from other dairy farms nor to sell it to other dairy farms.

Figure 2.1. The development of the farm's live stock



Now, every year again the farmer faces the same problem. How many of the heifer calves born should be retained at the farm for breeding, how many heifers should be sold for slaughter or put in calf and finally how many cows should be culled. As soon as he has reached this decision, the development of the live stock in that period is known, given the opening stock and ignoring loss by death. We assume that the dairy farmer must take such decisions for T consecutive years. At the end of year T he sells his live stock to a new owner. Because revenues and expenses caused by the animals are distributed over their life time, this decision problem is really an investment problem.

In deciding upon these questions it holds that the possibilities in a particular period are partly dependent on decisions taken in the past, just as this period's decisions (co)determine the farm's future herd development. It also holds that the farmer in determining the size and age composition of the stock must take into account the capacities of labor, dead stock and funds he has at his disposal. His decisions must always fit within the framework given by these factors. In this paper however we will neither pay attention to the coherence and interaction between these factors and the live stock nor to the possibility and consequences of extending the capacities of these factors.

One can, on good grounds, hold the view that a farmer, in choosing from a set of alternatives, is satisfied, as soon as he reaches his aspiration level. Here, however, we will not proceed from a satisfying, but from a maximizing concept. The objective used here is maximization of the value of (discounted) cash-flows, generated by the farmer's decisions. This criterion, though one-sided, without doubt forms an important element in comparing alternatives, directly related as it is to the consumption possibilities of the production/consumption households considered here.

3. The model

We assume that the lactation and dry period together make up a year, so every cow in calf gives birth to one calf a year, with equal probability for a heifer or a bull calf. During the year following the birth a heifer calf enters the heifer (or yearling) category. Heifers can be put in calf or sold for slaughter, either in the year of entering the heifer category or later on. We suppose that between the moment of a heifer's insemination and its calving also lies a period of a year.

Let vk_t, p_t, v_t, c_t, $t = 0,1,\ldots,T$ denote the number of heifer calves, heifers, heifers in calf and lactating cows respectively at the farm at the beginning of year t, vvk_t, vp_t, vc_t the number of heifer calves, heifers and cows, sold for slaughter in year t, and d_t the number of heifers put in calf in year t. The development of the farm's herd can now be represented by the following equations, cf. figure 2.1:

$$\begin{aligned} vk_t &= \tfrac{1}{2}\,(v_{t-1} + c_{t-1}) - vvk_t \\ p_t &= vk_{t-1} + p_{t-1} - vp_t - d_t \\ v_t &= d_t \\ c_t &= v_{t-1} + c_{t-1} - vc_t \end{aligned} \qquad t = 1,\dots,T \qquad (3.1)$$

In matrix notation this reads

$$Y_t = C_1 Y_{t-1} + C_2 X_t, \qquad (3.2)$$

where

$$Y_t' = [vk_t,\ p_t,\ v_t,\ c_t],\quad X_t' = [vvk_t,\ vp_t,\ d_t,\ vc_t],$$

$$C_1 = \begin{bmatrix} 0 & 0 & \tfrac{1}{2} & \tfrac{1}{2} \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad C_2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Above we supposed that transactions in breeding-cattle between dairy farms don't take place. That means that the vector of decision variables, X_t, satisfies the condition of non-negativity. Further, of course, it holds that never more heifer calves, heifers or cows can be sold for slaughter (or inseminated) than available. However, in this paper we restrict us to the situation where X_t varies over its feasible region without its extreme values: we assume that X_t is always positive and never reaches its maximum. This assumption is founded on the consideration that in reality X_t will be positive most of the time and only seldom will take on its minimum or maximum value. Hence, the related restrictions can be left out in modeling the (des)investment problem. When the opening stock,

Y_0, is also positive, the vectors Y_t will be positive too. As a consequence the decision problem to be formulated at the end of this section is sizably simplified.

In deciding upon the size and age structure of the live stock the farmer's objective is to maximize the value of discounted cash flows evoked by his decisions. We suppose that the revenues can be represented by a linear function of the distinct cattle categories and the expenditures by a quadratic function. We achieve in such a manner, that the farm operates under at most constant returns, or, stated otherwise, we suppose that an optimal size of the live stock exists. The coefficients in these functions represent prices or the product of prices and quantities and the state of technology. Generally speaking, these coefficients will be time-dependent, as prices are not constant during the planning period and because substitution possibilities between inputs exist. In this analysis, however, we assume that the coefficients in the quadratic part of the objective function, apart from inflation, remain constant, whereas those in the linear term vary in time. This assumption is based on the consideration that the coefficients in the quadratic term rest on a number of different expenditures categories, in contrast to the revenues categories. Hence, opposite developments within these categories can possibly compensate each other, whereas such a compensation possibility doesn't exist within the homogeneous revenues categories.

The revenue sources of the farm are the delivery of milk to the dairy industry, the sale of heifer and bull calves, and the sale of heifers and culled cows for slaughter.

The level of milk production by the dairy herd depends on many factors. Important in the long term analysis here are breed, age composition and genetic potential of the average cow. Keeping breed constant we suppose that the revenues from milk in year t are

$$pm_t(1+g)^t\{a_1c_{t-1} + a_3v_{t-1} - a_5vc_t\}, \tag{3.3}$$

where pm_t denotes the price of milk in year t, g the genetic improvement in percent a year and a_1, a_3, a_5 the milk yield per dairy cattle category.

Revenues from the sale of cattle amount to

$$pk_t\{\frac{1}{2}(c_{t-1} + v_{t-1}) + vvk_t\} + pp_t.vp_t + pc_t.vc_t, \tag{3.4}$$

where pk_t, pp_t and pc_t denote the price of a calf, a heifer and a culled cow respectively. We suppose that these prices are independent of the numbers sold.

Expenditures are done for the acquisition of dead stock, the payment of interest and redemption of debt and for buying concentrates, fertiliser, fuel etc. In this paper, however, we confine us to those expenses which can without allocation be attributed to the live stock, e.g. concentrates. As stated, we assume, that all of the coefficients of the expenditures term change conform inflation during the planning period.

Within the expenses evoked by the live stock we discern on the one hand expenditures determined by the size of a cattle category and on the other hand expenditures determined by the age composition of a category. Leaving inflation aside a moment, the size dependent expenditures comprise the following four components, one for each cattle category,

$$\begin{aligned}
\frac{1}{2} b_1 vk_t^2 &= \frac{1}{2} b_1(\frac{1}{2} v_{t-1} + \frac{1}{2} c_{t-1} - vvk_t)^2 \\
\frac{1}{2} b_2 p_t^2 &= \frac{1}{2} b_2(p_{t-1} + vk_{t-1} - vp_t - d_t)^2 \\
\frac{1}{2} b_3 v_t^2 &= \frac{1}{2} b_3 d_t^2 \\
\frac{1}{2} b_4 c_t^2 &= \frac{1}{2} b_4(v_{t-1} + c_{t-1} - vc_t)^2
\end{aligned} \tag{3.5}$$

On top of these come the age dependent expenditures which arise when the animals within a category on average become older or younger,

$$\begin{aligned}
&\frac{1}{2} b_5(p_{t-1} - vp_t - d_t)^2 \\
&\frac{1}{2} b_6(c_{t-1} - vc_t)^2
\end{aligned} \tag{3.6}$$

If p_{t-1} is equal to $vp_t + d_t$, the breeding expenses for heifers in that period amount to $\frac{1}{2} b_2 vk_{t-1}^2$. If, however vp_t and d_t are both equal to 0, then these expenses total $\frac{1}{2} b_2(p_{t-1} + vk_{t-1})^2 + \frac{1}{2} b_5 p_{t-1}^2$.

The sum of the expenditures components (3.5) and (3.6) will be denoted by the symbol TE_t.

The net returns to the farmer in guilders of constant purchasing power can now be summarized by the following expression:

$$NR_t = \left[\prod_{j=1}^{t} (1+i_j)\right]^{-1} \{P'_{y,t}Y_{t-1} + P'_{x,t}X_t\} - \frac{1}{2}[Y'_{t-1}X'_t]\begin{bmatrix} A_1 & A_2 \\ A'_2 & A_4 \end{bmatrix}\begin{bmatrix} Y_{t-1} \\ X_t \end{bmatrix}, \quad t = 1,\ldots,T \tag{3.7}$$

where i_j denotes the inflation percentage in year j,

$$P'_{y,t} = [0,\ 0,\ \tfrac{1}{2}\,pk_t + pm_t(1+g)^t a_3,\ \tfrac{1}{2}\,pk_t + pm_t(1+g)^t a_1]$$

$$P'_{x,t} = [pk_t,\ pp_t,\ 0,\ pc_t - pm_t(1+g)^t a_5]$$

and

$$A_1 = \frac{\partial^2 TE_t}{\partial Y^2_{t-1}},\ A_2 = \frac{\partial^2 TE_t}{\partial Y_{t-1}\partial X_t},\ A_4 = \frac{\partial^2 TE_t}{\partial X^2_t}$$

For year T, the sale of the stock comes on top of the revenues in that year.

Now that a specification of net returns is available, the decision problem the farmer faces in the first year within the planning horizon can be represented by the following model, compare also [2] and [5], and [9],

$$\max F = \sum_{t=1}^{T} \beta^t \left[\prod_{j=1}^{t} (1+ {}_1i^E_j)\right]^{-1} \{{}_1P'^E_{y,t}Y_{t-1} + {}_1P'^E_{x,t}X_t\} +$$

$$- \frac{1}{2}[Y'_{t-1}X'_t]\begin{bmatrix} A_1 & A_2 \\ A'_2 & A_4 \end{bmatrix}\begin{bmatrix} Y_{t-1} \\ X_t \end{bmatrix} + \beta^T \left[\prod_{j=1}^{T} (1 + {}_1i^E_j)\right]^{-1} \{{}_1P'^E_{y,T+1}Y_T\} \tag{3.8}$$

subject to

$$Y_t = C_1Y_{t-1} + C_2X_t$$

$$Y_0 = \bar{Y}_0$$

Here β denotes the discount factor the farmer uses and ${}_1i_j^E$ the inflation percentage that he expects in year 1 to be valid for year j. The vectors ${}_1P_{y,t}^E$ and ${}_1P_{x,t}^E$ specify his expectations in year 1 with respect to the returns from milk delivery and cattle sales in year t,

$$ {}_1P'^{E}_{y,t} = [0,\ 0,\ \tfrac{1}{2}\cdot{}_1pk_t^E + {}_1pm_t^E(1+g)^t a_3,\ \tfrac{1}{2}\cdot{}_1pk_t^E + {}_1pm_t^E(1+g)^t a_1] $$

$$ {}_1P'^{E}_{x,t} = [{}_1pk_t^E,\ {}_1pp_t^E,\ 0,\ {}_1pc_t^E - {}_1pm_t^E(1+g)^t a_5] $$

Finally, the vector ${}_1P'^{E}_{y,T+1} = [{}_1pk_{T+1}^E,\ {}_1pp_{T+1}^E,\ \frac{1}{2}({}_1pp_{T+1}^E + {}_1pc_{T+1}^E),\ {}_1pc_{T+1}^E]$ denotes the prices the farmer expects to receive from selling his live stock to a new owner at the end of the planning horizon. The expected prices for the first year are, of course, equal to the actual prices in that year, i.e. ${}_1i_1 = i_1$, ${}_1P_{y,1}^E = P_{y,1}$ and ${}_1P_{x,1}^E = P_{x,1}$.

4. The solution

The decision problem (3.8) (and those for the following years which possess the same structure) can be solved in several ways. For the way by which this solution was obtained here, we refer to [10]. In this paper we only present the expressions for the decision variables in which we are specially interested: the investments, d_t, and the desinvestments, vc_t.

For the model considered here the optimal level of the inflow of heifers in calf in the dairy stock is given by the following decision rule, compare also [5];

$$ \begin{aligned} d_t = & -\frac{b_4+b_6}{n_1}\,pp_t + \frac{\beta(b_4+b_6)}{(1+{}_ti_{t+1}^E)n_1}\,{}_tpk_{t+1}^E + \frac{\beta(1+g)^{t+1}\{b_4(a_3-a_5)+b_6a_3\}}{(1+{}_ti_{t+1}^E)n_1}\,{}_tpm_{t+1}^E + \\ & + \frac{\beta b_4}{(1+{}_ti_{t+1}^E)n_1}\,{}_tpc_{t+1}^E + \frac{\beta^2 b_6}{(1+{}_ti_{t+1}^E)(1+{}_ti_{t+2}^E)n_1}\,{}_tpk_{t+2}^E + \\ & + \frac{\beta^2(1+g)^{t+2}b_6(a_1-a_5)}{(1+{}_ti_{t+1}^E)(1+{}_ti_{t+2}^E)n_1}\,{}_tpm_{t+2}^E + \frac{\beta^2 b_6}{(1+{}_ti_{t+1}^E)(1+{}_ti_{t+2}^E)n_1}\,{}_tpc_{t+2}^E, \end{aligned} \tag{4.1} $$

where $n_1 = \prod_{j=1}^{t} (1+i_j)(2\beta b_4 b_6 + b_4 b_3 + b_6 b_3)$.

For the optimal culling level we find

$$vc_t = \frac{b_4}{n_2} v_{t-1} + c_{t-1} - \frac{(1+g)^t a_5}{\prod_{j=1}^{t} (1+i_j) n_2} pm_t + \frac{1}{\prod_{j=1}^{t} (1+i_j) n_2} pc_t +$$

$$- \frac{\beta}{\prod_{j=1}^{t} (1+i_j)(1+{}_t i^E_{t+1}) n_2} {}_t pk^E_{t+1} - \frac{\beta(1+g)^{t+1}(a_1 - a_5)}{\prod_{j=1}^{t} (1+i_j)(1+{}_t i^E_{t+1}) n_2} {}_t pm^E_{t+1} +$$

$$- \frac{\beta}{\prod_{j=1}^{t} (1+i_j)(1+{}_t i^E_{t+1}) n_2} {}_t pc^E_{t+1}, \tag{4.2}$$

where $n_2 = b_4 + b_6$.

By substituting $\prod_{j=1}^{t} (1+i_j)$ by Π_t and the constellations of the other coefficients from the objective function in (4.1) by $co_{1,k}$, $k = 1,\dots,7$ and those in (4.2) by $co_{2,k}$, $k = 1,\dots,6$ we obtain the following expressions which are simpler to read:

$$d_t = -co_{1,1} \frac{pp_t}{\Pi_t} + co_{1,2} \frac{{}_t pk^E_{t+1}}{\Pi_t (1+{}_t i^E_{t+1})} + co_{1,3} \frac{{}_t pm^E_{t+1}}{\Pi_t (1+{}_t i^E_{t+1})} +$$

$$+ co_{1,4} \frac{{}_t pc^E_{t+1}}{\Pi_t (1+{}_t i^E_{t+1})} + co_{1,5} \frac{{}_t pc^E_{t+2}}{\Pi_t (1+{}_t i^E_{t+1})(1+{}_t i^E_{t+2})} +$$

$$+ co_{1,6} \frac{{}_t pm^E_{t+2}}{\Pi_t (1+{}_t i^E_{t+1})(1+{}_t i^E_{t+2})} + co_{1,7} \frac{{}_t pc^E_{t+2}}{\Pi_t (1+{}_t i^E_{t+1})(1+{}_t i^E_{t+2})} \tag{4.3}$$

$$vc_t = co_{2,1} v_{t-1} + c_{t-1} - co_{2,2} \frac{(1+g)^t pm_t}{\Pi_t} + co_{2,3} \frac{pc_t}{\Pi_t} - co_{2,4}.$$

$$\frac{{}_tpk^E_{t+1}}{\Pi_t(1+{}_ti^E_{t+1})} - co_{2,5}\frac{(1+g)^{t+1}\,{}_tpm^E_{t+1}}{\Pi_t(1+{}_ti^E_{t+1})} - co_{2,6}\frac{{}_tpc^E_{t+1}}{\Pi_t(1+{}_ti^E_{t+1})} \tag{4.4}$$

The investments in year t are according to (4.3) determined by the (deflated) actual and expected prices. The desinvestments in year t, (4.4), also depend on the size of the dairy herd at time t-1 and the inflow in period t-1. The weight of each variable depends on coefficients from the revenues and expenditures functions. Curious about (4.3) and (4.4) is that its variables only refer to the current and the future two years. One would rather expect the prices of all future periods to play a role. It must be admitted that these rules are hardly transparent. Though it is known on the basis of the derivation followed, that these rules rest on the equality of marginal revenues and costs, it turns out to be difficult to state this in economic terms.

By bringing c_{t-1} in (4.4) from the right to the left hand side and by substituting $c_{t-1}-vc_t$ by c_t-v_{t-1}, we obtain a specification of the determinants of the size of the dairy cow stock:

$$v_t = -co_{1,1}\frac{pp_t}{\Pi_t} + co_{1,2}\frac{{}_tpk^E_{t+1}}{\Pi_t(1+{}_ti^E_{t+1})} + co_{1,3}\frac{{}_tpm^E_{t+1}}{\Pi_t(1+{}_ti^E_{t+1})} +$$

$$+ co_{1,4}\frac{{}_tpc^E_{t+1}}{\Pi_t(1+{}_ti^E_{t+1})} + co_{1,5}\frac{{}_tpc^E_{t+2}}{\Pi_t(1+{}_ti^E_{t+1})(1+{}_ti^E_{t+2})} +$$

$$+ co_{1,6}\frac{{}_tpm^E_{t+2}}{\Pi_t(1+{}_ti^E_{t+1})(1+{}_ti^E_{t+2})} + co_{1,7}\frac{{}_tpc^E_{t+2}}{\Pi_t(1+{}_ti^E_{t+1})(1+{}_ti^E_{t+2})} \tag{4.5}$$

$$c_t = \overline{co}_{2,1}v_{t-1} + co_{2,2}\frac{(1+g)^t pm_t}{\Pi_t} + co_{2,3}\frac{pc_t}{\Pi_t} + co_{2,4}\frac{{}_tpk^E_{t+1}}{\Pi_t(1+{}_ti^E_{t+1})} +$$

$$+ co_{2,5}\frac{(1+g)^{t+1}\,{}_tpm^E_{t+1}}{\Pi_t(1+{}_ti^E_{t+1})} + co_{2,6}\frac{{}_tpc^E_{t+1}}{\Pi_t(1+{}_ti^E_{t+1})}, \tag{4.6}$$

where $\overline{co}_{2,1} = 1-co_{2,1}$.

In the rearrangement (4.5) and (4.6) the factors are identified which determine the optimal size of the dairy cow stock. Hence, these relations provide a well founded point of departure for investigations into the behaviour of the stock size and so cet.par. the level of long run milk supply.

As, finally, (4.6) specifies the size of the dairy cow stock as a linear function of, among others, expected milk prices, it can be seen as a variant of the supply model proposed by Nerlove [4]. This model is extensively used in agricultural supply studies, compare for instance [1], [6], [7], [8].

5. The long term elasticity of the milk supply with respect to the milk price

As holds for every milk supply specification, the relations (4.5) and (4.6) can for instance be used to assess to what extent the stock size reacts on changes in the producer's price of milk. This can be measured by means of the supply elasticity with respect to the milk price. This elasticity is defined as the ratio of the (percentage) change in the milk supply, mp, and the (percentage) change in the milk price, pm:

$$\frac{\frac{\Delta mp}{mp}}{\frac{\Delta pm}{pm}} = \frac{\Delta mp}{\Delta pm} \cdot \frac{pm}{mp} \qquad (5.1)$$

With $\overline{mgk}$ the average milk yields per cow and $\bar{c}$ the average number of dairy cows the split-up of milk supply can be represented as

$$mp = \overline{mgk} . \bar{c}, \qquad (5.2)$$

so that the elasticity is given by

$$\frac{\Delta \overline{mgk}}{\Delta pm} \cdot \frac{pm}{\overline{mgk}} + \frac{\Delta \bar{c}}{\Delta pm} \frac{pm}{\bar{c}} \qquad (5.3)$$

The first term can be seen as the short term elasticity and the second as the long term elasticity, the one in which we are interested here.

In period t the average size of the dairy cow stock at the farm amounts to

$$\bar{c}_t = \frac{c_{t-1}+c_t}{2} \tag{5.4}$$

The insertion of (4.6) in (5.4) gives this average as a function of, among others the prices and price expectations in that period. In view of the dependence of c_{t+1} on v_t, these prices also influence $\bar{c}_{t+1}$ and $\bar{c}_{t+2}$. Under the assumption that the milk price expectations in (4.5) and (4.6) only depend on pm_t, as far as it concerns milk prices, the effect of a change in the milk price in period t on the average size of the dairy cow stock is expressed by

$$\frac{\partial c_t}{\partial pm_t} + \frac{\partial c_t}{\partial_t pm^E_{t+1}} \frac{\partial_t pm^E_{t+1}}{\partial pm_t} + \frac{\partial c_{t+1}}{\partial v_t} \left[\frac{\partial v_t}{\partial_t pm^E_{t+1}} \frac{\partial_t pm^E_{t+1}}{\partial pm_t} + \frac{\partial v_t}{\partial_t pm^E_{t+2}} \frac{\partial_t pm^E_{t+2}}{\partial pm_t} \right], \tag{5.5}$$

or, shortly, by

$$\frac{\partial c_t}{\partial pm_t} + \frac{\partial c_{t+1}}{\partial v_t} \frac{\partial v_t}{\partial pm_t} \tag{5.6}$$

where $\frac{\partial c_t}{\partial pm_t}$ stands for the effect in the present period and $\frac{\partial c_{t+1}}{\partial v_t} \frac{\partial v_t}{\partial pm_t}$ for the effect in the two years to come.

Under the assumption just mentioned the long run elasticity, $\frac{\Delta \bar{c}}{\Delta pm} \frac{pm}{\bar{c}}$, can be determined by for instance the average of the elasticities in the several years.

$$\frac{\Delta \bar{c}}{\Delta pm} \frac{pm}{\bar{c}} = \frac{1}{T} \sum_{t=1}^{T} \left\{ \left[\frac{\partial c_t}{\partial pm_t} + \frac{\partial c_{t+1}}{\partial v_t} . \frac{\partial v_t}{\partial pm_t} \right] \frac{pm_t}{\bar{c}_t} \right\} \tag{5.7}$$

The elements $\frac{\partial c_t}{\partial pm_t}$, $\frac{\partial c_{t+1}}{\partial v_t}$ and $\frac{\partial v_t}{\partial pm_t}$ can be obtained by means of the estimates for the corresponding regression coefficients in the equations (4.5) and (4.6). Should the milk price expectations in (4.5) and (4.6) also depend on other milk prices than the one of period t, then (5.5) has to be adjusted accordingly.

The reaction equations (4.5) and (4.6) are derived at micro level, so the elasticity (5.7) can be estimated using data concerning individual farms given that the assumption of positivity of the decision variables X_t is fullfilled. However, if we assume that the same type of model as the one developed holds for all firms in the sector, the conditions for consistent aggregation are satisfied and estimation of (5.7) using data with respect to the sector as a whole is also allowed [3].

6. The application to the Dutch dairy sector

A variant of the model (4.5) and (4.6) was used for the estimation of the long term milk supply elasticity in the Netherlands during the period from 1969, say the start of the common agricultural market, untill the introduction of the super levy in 1984, on the basis of sector data. In comparison to (4.5) and (4.6) this variant was more comprehensive, as it comprised not only revenues and expenses caused by the live stock, but also receipts and expenditures evoked by the dead stock and the finance activity of the farm. However, the structure of this variant is identical to the one presented above: it has the same explanatory variables with the same lag structure.

In estimating the coefficients of this model it turned out to be impossible to maintain the full richness of the derived specification. Due to the small number of observations concerning the inflow of heifers in calf it was inevitable to reduce the number of explanatory variables. Fortunately, there were opportunities to do so. Firstly, one may safely assume that the price expectations for the two years following the decision period are highly correlated, because they rest on the same information set. So, to avoid collinearity one of them can be left out of consideration. Further it turned out that the present milk price, adjusted for inflation and increase in milk yield per cow, provides a very satisfactory forecast of the future milk price. Therefore, taking this forecast for the expected milk price, the present milk price suffices as explanatory variable. Finally, the prices for the different kinds of beef meat turned out to be strongly correlated, as was to be expected. So, instead of maintaining the three of them we used a linear combination of these prices. Of course, as a consequence of these reductions, part of the explanatory power, present in the original specification, is lost.

After these preparations the two reaction equations were estimated using the GLS method, as the equations share the person of the decision maker. The long term elasticity was estimated at 0,76. So during the period 1969-1984 a 1 percent milk price change resulted in a 0,76 percent change of the size of the dairy stock (and hence, ceteris paribus, the level of milk supply). For the period 1959-1979 Oskam and Osinga [7] also found an inelastic reaction using different models. For a more extended exposé of the approach followed here and the results reference is made to [10] and [11].

7. Conclusion

In this paper we have demonstrated, how, starting from an optimization model for the live stock (des)investment decision of a dairy farm, a specification of the determinants of the long run milk supply can be obtained. In comparison to a supply model, being (re)specified on the basis of plausibility considerations and/or considerations related to the convenience of estimating, such a derived model provides a well founded point of departure for investigations into long term milk supply.

For simplicity's sake, the production factors labour, dead stock and capital were left out of consideration in developing the supply model. However, the incorporation of these factors doesn't yield results essentially different from the ones obtained above, cf. [10].

As yet it has not been examined what form the decision rules (4.1) and (4.2) take on, when restrictions concerning the decision variables (or comparable restrictions with respect to labour, dead stock and capital) are active. Such an extension could bring within reach an underpinned investment model for the situation of for instance production rationing.

ontvangen 16-4-1992
geaccepteerd 16-6-1992

Acknowledgement

The author wishes to thank F. van der Duyn Schouten for a critical reading of the first version of this paper and A. Markink for programming support.

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