KM 32(1989)
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A DYNAMIC SPECIFICATION OF AN AIDS IMPORT ALLOCATION MODEL[1]

by **B.J. van Heeswijk**[2], **P.M.C. de Boer**[3] and **R. Harkema**[3]

Abstract

In this paper we use the first-order autoregressive scheme in order to introduce dynamics into the AIDS model. We also consider the theoretical restrictions of additivity, homogeneity and symmetry, and use two different specifications of the covariance matrix. We estimate the models using import allocation data for the UK 1952-1979 of five EEC countries and test different specifications against each other.

1. INTRODUCTION

Winters (1984) applies the Almost Ideal Demand System (AIDS) in order to explain import allocation of the United Kingdom, 1952-1979. As usual, the theoretical homogeneity and symmetry restrictions from demand theory are rejected. One of the reasons that Winters advances to explain this phenomenon is **dynamic misspecification**. Deaton and Muellbauer (1980), who introduced this model, state: "We also find that imposition of homogeneity generates positive serial correlation in the errors of those equations which reject the restrictions most strongly; this suggests that the now standard rejection of homogeneity in demand analysis may be due to insufficient attention to the dynamic aspects of consumer behavior."

Another reason advanced in literature on demand models (see for instance Laitinen (1978) and Meisner (1979)) is that asymptotic tests of homogeneity and symmetry are biased towards rejection of the null hypothesis when the number of budget categories (or import sources in the framework of import allocation) is large as compared with the number of observations, so that the number of degrees of freedom, i.e. the number of observations minus the total

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1 The authors are very much indebted to Prof. L. Allen Winters for putting his data at our disposal.

2 BSO, Utrecht.

3 Erasmus University Rotterdam, P.O. Box 1738, NL 3000 DR Rotterdam, The Netherlands, phone-number 010-4081517/4081271.

number of parameters to be estimated, is relatively small. In order to increase that number it may be worthwhile to impose restrictions on the contemporaneous covariance matrix of the disturbances.

In this paper we pay attention to both problems: on the one hand we introduce dynamics by means of the first-order autoregressive scheme and on the other hand we consider not only the covariance matrix that - apart from additivity - is unrestricted, but also a restricted specification that appeared to perform well in the context of this model before (see De Boer, Harkema, and Van Heeswijk (1987)).

In section 2 of this paper we discuss the different versions of the AIDS model that we estimate in the empirical part, section 3 deals with specifications of the covariance matrix and with some tests, whereas in section 4 we apply the models to import allocation data of the United Kingdom 1952-1979 borrowed from Winters (1984). Presumably because of lack of degrees of freedom, we were unable to obtain convergence for the 10-country model of Winters in case of the most general dynamic structure dealt with in this paper. Therefore, we decided to restrict the analysis to a smaller number of countries, i.c. to imports from five EEC countries: Belgium, FRG, France, The Netherlands and Italy.

2. SPECIFICATION OF THE MODELS

For the explanation of the import allocation we follow Winters (1984) by adopting the simplified AIDS model including a dummy to measure the access of the United Kingdom to the EEC in 1972:

$$(2.1) \qquad w_{ti} = \alpha_i + \delta_i dd_t + \sum_{j=1}^{n} \gamma_{ij} \log(p_{tj}) + \beta_i \log(Y_t/P^*_t) + u_{ti},$$

with w_{ti} : share of imports from country i (= 1, ..., n) in total imports at time t (= 1, ..., T),

$dd_t = 0 \quad 1 \leq t < T_1$

$dd_t = 1 \quad T_1 \leq t \leq T$

T_1 being the year of access to the EEC,

p_{tj} : price of imports from country j,

Y_t : total imports,

$\log P^*_t = \sum_{i=1}^{n} w_{ti} \log p_{ti}$, the Stone-index, and

u_{ti} : a disturbance term.

Economic theory imposes four types of restrictions three of which can be met with by imposing restrictions on the parameters:

(i) **additivity:**

$$\Sigma_i \alpha_i = 1,\ \Sigma_i \delta_i = \Sigma_i \gamma_{ij} = \Sigma_i \beta_i = 0,$$

(ii) **homogeneity:**

(2.2) $$\Sigma_j \gamma_{ij} = 0,$$

(iii) **symmetry:**

(2.3) $$\gamma_{ij} = \gamma_{ji}, \text{ and}$$

(iv) **negativity:** letting δ_{ij} denote the Kronecker delta, the matrix K_t with typical element:

$$K_{tij} = \gamma_{ij} + \beta_i \beta_j \log(Y_t/P^*_t) - w_{ti}\delta_{ij} + w_{ti}w_{tj},$$

has to be negative semi-definite for all $t = 1, \ldots, T$. This restriction cannot be imposed a priori, but has to be verified for each datapoint separately.

We introduce the dynamic aspect by means of the first-order autoregressive scheme:

(2.4) $$u'_t = u'_{t-1} R + e'_t \qquad t = 2, \ldots, T$$

with $u'_t = [u_{t1} \ \ldots \ u_{tn}]$ and R the matrix of autocorrelation parameters with typical element r_{ki}. The vectors of disturbances $e'_t = [e_{t1} \ \ldots \ e_{tn}]$ are assumed to be independently distributed according to a normal distribution with zero mean and covariance matrix Ω_n:

(2.5) $$e' = [e'_2 \ \ldots \ e'_T] \sim N(0, I_{T-1} \otimes \Omega_n),$$

with "$\otimes$" denoting the Kronecker matrix product.

Assuming in addition to (2.5) that $E(u_t') = 0$ (t = 1, ..., T), it can be shown that additivity implies:

$$(2.6) \qquad R\iota_n = k\iota_n,$$

where k is an arbitrary constant and ι_n denotes the summation vector, i.e., the vector with all elements equal to one and

$$(2.7) \qquad e_t'\iota_n = 0$$

(c.f. Berndt and Savin (1975)).

As a consequence, the covariance matrix Ω_n is singular. This singularity can easily be handled by deleting an arbitrary equation from system (2.1), say the n^{th} one (c.f. Barten (1969) and Berndt and Savin (1975)). The covariance matrix that is obtained when the n^{th} row and column of Ω_n are deleted, will be denoted by Ω_{n-1}.

In matrix notation, where the definition of the matrices easily follows from equations (2.1) and (2.4) (see the appendix), the dynamic AIDS model can be written as:

$$(2.8) \qquad W_{(1)}^{(n)} = X_{(1)}B^{(n)} + U_{(1)}^{(n)}$$

$$(2.9) \qquad U_{(1)}^{(n)} = U_{(T)}R^{(n)} + E^{(n)},$$

where the lower index (1) (respectively (T)) denotes that the first **row** (respectively the T^{th}) has been deleted and the upper index (n) denotes the deletion of the n^{th} **column.**

Additivity implies:

$$(2.10) \qquad U_{(T)}\iota_n = 0$$

Hence, the matrix $U_{(T)}$ is of order $(T-1) \times n$ with rank $(n - 1)$; consequently the matrix $U_{(T)}'U_{(T)}$ is singular. In order to handle this singularity Berndt and Savin (1975) define:

$$(2.11) \qquad \bar{R} = \begin{bmatrix} (r_{11} - r_{n1}) \dots (r_{1n} - r_{nn}) \\ \vdots \qquad\qquad \vdots \\ (r_{n-1,1} - r_{n1}) \dots (r_{n-1,n} - r_{nn}) \end{bmatrix}.$$

It easily follows from (2.6) that:

$$(2.12) \qquad \bar{R}\iota_n = 0.$$

Using (2.10), (2.9) can be rewritten as:

$$(2.13) \qquad U^{(n)}_{(1)} = U^{(n)}_{(T)} \bar{R}^{(n)} + E^{(n)}, \text{ with:}$$

$$(2.14) \qquad U^{(n)}_{(T)} = W^{(n)}_{(T)} - X_{(T)} B^{(n)}.$$

This type of dynamics is called "R" in the sequel.
Apart from "R" we use in the empirical part of the paper two other specifications:

(i) the "**diag-R**" model,
where we assume the R-matrix to be diagonal. Because of (2.6) all autocorrelation parameters should be equal to each other:

$$(2.15) \qquad R = kI,$$

(ii) the "**static**" model, where[4]:

$$(2.16) \qquad R = 0.$$

As a summary, we have three different specifications of the AIDS model: "additivity", "homogeneity" and "symmetry", and three different types of dynamics: "R", "diag-R" and "static". Consequently, we have 9 different specifications. In the appendix we present an iterative procedure to obtain the maximum likelihood estimates for the specification with symmetry and homogeneity constraints and with dynamical specification "R"; the maximum

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4 In this case there is no need to delete the first observation. However, for reasons of comparability, we nevertheless decided to do so.

likelihood estimates for all other specifications can be obtained in a similar way.

3. SPECIFICATIONS OF THE COVARIANCE MATRIX AND SOME TESTS

3.1. Specifications of the covariance matrix

In the empirical part of this paper we consider two specifications of the covariance matrix Ω_n in (2.5):

(i) a specification which (apart from "additivity") is unrestricted, denoted by UNREST

(ii) a restricted version proposed by de Boer and Harkema (1983), denoted by HARBO.

UNREST: as is wellknown, the maximum likelihood estimator of Ω_{n-1} is given by

$$\hat{\Omega}_{n-1} = \frac{1}{(T-1)} \hat{E}^{(n)\prime}\hat{E}^{(n)}$$

with

$$\hat{E}^{(n)} = \begin{bmatrix} \hat{e}_{21} & \cdots & \hat{e}_{2,n-1} \\ \vdots & & \vdots \\ \hat{e}_{T1} & \cdots & \hat{e}_{T,n-1} \end{bmatrix}$$

The loglikelihood function evaluated at the optimum is:

$$\log L(\text{"UNREST"}) = -\frac{(T-1)}{2}(n-1)(1+\log[2\pi]) - \frac{(T-1)}{2}\log[|\hat{\Omega}_{n-1}|]$$

HARBO: this specification reads:

$$\Omega_n = D_n - \frac{1}{d}\delta_n\delta_n'$$

with

$$D_n = \begin{bmatrix} d_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & d_n \end{bmatrix},$$

$$\delta'_n = [d_1 \dots d_n] \text{ and } d = \sum_{i=1}^{n} d_i.$$

De Boer and Harkema (1983) show that the maximum likelihood estimates of the parameters d_i ($i = 1, \dots, n$) follow from the following system of equations:

$$\text{(3.1)} \qquad \hat{d}_i - \frac{\hat{d}_i^2}{\hat{d}} = \frac{1}{(T - 1)} (\hat{E}'\hat{E})_{ii} \qquad\qquad i = 1, \dots, n$$

where $(\hat{E}'\hat{E})_{ii}$ denotes the i^{th} diagonal element of the $(n \times n)$-matrix $\hat{E}'\hat{E}$.
Apart from one special case that occurs with probability zero, there is a unique solution to (3.1) that can be found by means of a **one-dimensional search procedure** that works very quickly.
The algorithm is described in de Boer and Harkema (1986).

The loglikelihood function evaluated at the optimum is shown to be:

$$\log L(\text{"HARBO"}) = - \frac{(T - 1)}{2}(n - 1)(1 + \log[2\pi]) - \frac{(T - 1)}{2}\log\left[\hat{d}^{-1}\prod_{i=1}^{n} \hat{d}_i\right].$$

3.2. Testing some specifications

In this paper we consider three different types of specifications of the model, viz., specifications according to

(i) **theoretical restrictions:** additivity, homogeneity and symmetry,

(ii) **dynamics:** "R", "diag-R" and "static",

(iii) **covariance matrix:** UNREST and HARBO

Within each type, the specifications are nested into each other, so that we may apply the likelihood ratio test. In table 1 we specify the (asymptotic) distributions of the test statistics.

Table 1. Asymptotic distributions of the test statistics.

additivity	$\underline{\chi^2(n-1)}$	homogeneity	$\underline{\chi^2(\frac{1}{2}(n-1)(n-2))}$	symmetry
"R"	$\underline{\chi^2(n(n-2))}$	"diag-R"	$\underline{\chi^2(1)}$	static
UNREST	$\underline{\chi^2(\frac{1}{2}n(n-3))}$	HARBO		

As is wellknown from consumer demand theory (see Laitinen (1978) and Meisner (1979)) the likelihood ratio test statistic is biased towards rejection of the null hypothesis when the number of observations is small as compared to the number of budget categories distinguished. In the empirical part of this study we apply a small sample correction factor that has been proposed by Italianer (1985). He decomposes the correction factor that Anderson (1958) derived for a specific testing problem into two factors and proposes to use this decomposition for more general problems such as the tests we described above.

Italianer's correction factor can be written as:

$$\frac{\frac{1}{2}(df_0 + df_1)}{\text{total number of observations}}$$

with df_0: the number of degrees of freedom under the null hypothesis, and
df_1: idem, under the alternative hypothesis,

where the number of degrees of freedom is defined as the total number of observations minus the total number of parameters to be estimated.

4. RESULTS

4.1. Introduction

As mentioned before, we applied the models described above to explain import allocation of the United Kingdom for the period 1952-1979. We confined ourselves to imports from the following EEC countries: Belgium, France, FRG, the Netherlands and Italy. For a description of the data we refer to Winters (1984).

In table 2 we summarize the loglikelihood values of the 18 models that we have estimated.

Table 2. Loglikelihood values of the models.

		"symmetry"	"homogeneity"	"additivity"
"UNREST"	"R"	369.3086	372.1832	379.8158
	"Diag-R"	347.9753	357.4660	361.8565
	"Static"	331.9888	345.9188	353.3503
"HARBO"	"R"	363.9592	366.8945	373.7769
	"Diag-R	340.4059	349.0699	352.0520
	"Static"	316.1061	333.7372	341.8468

4.2. Testing theoretical restrictions

From table 2 the values of the L.R. test statistics for testing homogeneity against additivity and symmetry against homogeneity can be calculated right away. The values that are presented in table 3 are those that result from the application of Italianer's correction in order to countervail the bias of the L.R. test statistic.

Table 3. Corrected values of the L.R. test statistics for testing theoretical restrictions.

	Homogeneity vs. Additivity (critical value at 5% : 9.49)		Symmetry vs. Homogeneity (critical value at 5% : 12.59)	
	HARBO	UNREST	HARBO	UNREST
Static	10.96*	9.36	25.47*	18.83*
Diag-R	3.98	5.45	12.35	12.65*
R	7.27	7.35	3.37	3.03

* Rejected at a 5% level of significance.

We observe that in the static model all theoretical restrictions are rejected except for homogeneity in case of UNREST. In the dynamic models, however, all theoretical restrictions are accepted except for symmetry in case of UNREST and a diagonal R matrix. Obviously, we find support for Winters' assertion that the common rejection of the theoretical restrictions is, among others, due to dynamic misspecification.

In the context of the static Rotterdam model of consumer expenditure, Laitinen (1978) has derived the exact distribution of the Wald test statistic for testing homogeneity against additivity in case of UNREST so that we are able to compare the conclusion that results from the corrected L.R. test with that from an exact test. Adapting Laitinen's proof to cope with the constant term and the dummy in (2.1), it can be shown that the distribution of the Wald test statistic is Hotelling's T^2 which is itself distributed as a multiple $(n - 1)(T - n - 3)/(T - 2n - 1)$ of a F-distributed random variable with $(n - 1)$ and $(T - 2n - 1)$ degrees of freedom, i.e. $4.75 \times F(4,16)$. At a size of 5% the critical value is 14.28, whereas the value of the test statistic turns out to be 13.95, so that the null hypothesis homogeneity is only marginally accepted. This is exactly the same conclusion as drawn from the corrected L.R. test, where the realization was 9.36 as compared with the critical value at 5% of 9.49. This result fully agrees with recent simulation results (see de Boer and Harkema (1988)), which also point out that Italianer's correction factor performs remarkably well. As a final remark, we mentioned in section 2 that the theoretical restriction of "negativity" can only be verified for each datapoint separately. It turned out that only for the year 1973 the symmetric R-model in case of HARBO yielded a negative semi-definite K-matrix.

4.3. Testing dynamic specifications

In table 4 we present the corrected values of the L.R. test statistics for testing the dynamic specifications.

Table 4. Corrected values of the L.R. test statistics for testing dynamic specifications.

	Static vs. Diag-R (critical value at 5% : 3.84)		Diag-R vs. R (critical value at 5% : 25.00)	
	HARBO	UNREST	HARBO	UNREST
Additivity	13.32*	10.32*	25.15*	19.12
Homogeneity	21.15*	14.86*	21.95	16.76
Symmetry	36.22*	22.35*	31.62*	26.67*

* Rejected at a 5% level of significance.

We find that the static model is always strongly rejected against the most simple dynamic specification, i.e. the "diag-R" specification. For HARBO the "diag-R" model is marginally rejected against the more general "R" model in case of additivity, accepted for homogeneity and strongly rejected against the "R"-model when symmetry is imposed (at a 0.5% level of significance the critical value is 32.8). For UNREST, finally, the "diag-R" model is strongly accepted for additivity and homogeneity, whereas it is just rejected for symmetry. Only for HARBO under the symmetry restriction, the "R"-model clearly performs better than the "diag-R" model.

4.4. Testing specifications of the covariance matrix

Finally, we derive from table 2 the corrected values of the L.R. test statistics for testing the different specifications of the covariance matrix. The values involved are presented in table 5.

Table 5. Corrected values of the L.R. test statistics for testing HARBO against UNREST (critical value at 5% : 11.07).

	Additivity	Homogeneity	Symmetry
Static	14.59*	16.36*	23.09*
Diag-R	12.26*	11.12*	10.86
R	5.87	5.53	6.19

* Rejected at a 5% level of significance.

We observe that for the simplest models, i.e. the static ones, HARBO is strongly rejected against UNREST. For the intermediate models, i.e. the diag-R ones, there is no clear preference at a 5% level of significance, but for the most general models, i.e. the R-models, HARBO is strongly accepted against UNREST. We conclude that, as could be expected, HARBO performs better as compared with UNREST according as the dynamic specification of the model is less rigid.

5. CONCLUSION

Summarizing our main findings, we observe that under the most general dynamic specification considered in this paper, i.e. the R-model, all theoretical restrictions as well as the HARBO specification of the covariance matrix are accepted. But even under an inflexible dynamic specification like the diag-R model, it appears that all theoretical restrictions are accepted except for symmetry in case of UNREST, which is only marginally rejected. Moreover, it appears that under this specification no clear preference exists for either the HARBO or the UNREST specification of the covariance matrix. Evidently, the results of our analysis lend support to the hypothesis that the common rejection of the theoretical restrictions in demand models is due to dynamic misspecification as well as the bias towards rejection of the null hypothesis in asymptotic tests. In addition, we conclude that restricting the contemporaneous covariance matrix of the disturbances provides an acceptable way of enlarging the number of degrees of freedom when the number of demand categories is large as compared with the number of observations.

Appendix

In order to write the model in matrix notation, we define:

$$W = \begin{bmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{T1} & \cdots & w_{Tn} \end{bmatrix}$$

(A.1)
$$X = \begin{bmatrix} & 0_{T_1-1} & \log[Y_1/P_1^*] & \log[p_{11}] & \cdots & \log[p_{1n}] \\ \iota_T & & \vdots & \vdots & & \vdots \\ & \iota_{T-T_1+1} & \log[Y_T/P_T^*] & \log[p_{T1}] & \cdots & \log[p_{Tn}] \end{bmatrix}$$

with 0_{T_1-1} a (T_1-1)-vector with all elements equal to zero, and ι_T, ι_{T-T_1+1} vectors with all elements equal to one of orders T and $(T-T_1+1)$, respectively;

(A.2)
$$B = \begin{bmatrix} \alpha_1 & \cdots & \alpha_n \\ \delta_1 & \cdots & \delta_n \\ \beta_1 & \cdots & \beta_n \\ \gamma_{11} & \cdots & \gamma_{n1} \\ \vdots & \ddots & \vdots \\ \gamma_{1n} & \cdots & \gamma_{nn} \end{bmatrix}$$

$$U = \begin{bmatrix} u_{11} & \dots & u_{1n} \\ \vdots & \ddots & \vdots \\ u_{T1} & \dots & u_{Tn} \end{bmatrix}, \text{ and } E = \begin{bmatrix} e_{21} & \dots & e_{2n} \\ \vdots & \ddots & \vdots \\ e_{T1} & \dots & e_{Tn} \end{bmatrix}.$$

The model to be estimated (compare (2.8), (2.13) and (2.14)) can then be written in vector notation as:

$$\text{(A.3)} \qquad \text{vec } W_{(1)}^{(n)} = [I_{n-1} \otimes X_{(1)}] \text{ vec } B^{(n)} + \text{vec } U_{(1)}^{(n)}$$

$$\text{(A.4)} \qquad \text{vec } U_{(1)}^{(n)} = \left[(\bar{R}^{(n)})' \otimes I_{T-1}\right] \text{ vec } U_{(T)}^{(n)} + \text{vec } E^{(n)}$$

$$\text{(A.5)} \qquad \text{vec } U_{(T)}^{(n)} = \text{vec } W_{(T)}^{(n)} - (I_{n-1} \otimes X_{(T)}) \text{ vec } B^{(n)},$$

with

$$\text{(A.6)} \qquad \text{vec } E^{(n)} \sim N(0, \Omega_{n-1} \otimes I_{T-1}).$$

From (A.3) - (A.6), it follows that the likelihood function may be written as:

$$\text{(A.7)} \qquad L\left[W_{(1)}^{(n)}\right] = (2\pi)^{-\frac{1}{2}(n-1)(T-1)} \left|\Omega_{n-1}\right|^{-\frac{1}{2}(T-1)} .$$

$$\exp\left\{-\tfrac{1}{2} z' \left[\Omega_{n-1}^{-1} \otimes I_{T-1}\right] z\right\}$$

with

$$\text{(A.8)} \qquad z = \text{vec } W_{(1)}^{(n)} - \left[I_{n-1} \otimes X_{(1)}\right] \text{vec } B^{(n)} +$$

$$-\left\{\bar{R}^{(n)'} \otimes I_{T-1}\right\} \left\{\text{vec } W_{(T)}^{(n)} - \left[I_{n-1} \otimes X_{(T)}\right] \text{vec } B^{(n)}\right\}$$

The homogeneity and symmetry constraints can be written as a system of linear constraints on vec $B^{(n)}$:

$$\text{(A.9)} \qquad A \text{ vec } B^{(n)} = 0$$

where A is a matrix of order $\left[\frac{1}{2}n(n-1)\right] \times \left[(n-1)(n+3)\right]$.

The maximum likelihood estimates are obtained from the following iterative procedure:

(1) specify initial estimates for $\bar{R}^{(n)}$ and Ω_{n-1}, for example, $\hat{\bar{R}}^{(n)} = 0$ and $\hat{\Omega}_{n-1} = I_{n-1}$;

(2) estimate $\hat{B}^{(n)}$, given $\hat{\bar{R}}^{(n)}$ and $\hat{\Omega}_{n-1}$;

(3) calculate a new estimate for $\hat{\bar{R}}^{(n)}$, given $\hat{B}^{(n)}$ and $\hat{\Omega}_{n-1}$;

(4) calculate a new estimate for $\hat{\Omega}_{n-1}$, given $\hat{B}^{(n)}$ and $\hat{\bar{R}}^{(n)}$;

(5) repeate steps (2), (3), and (4) until convergence.

In order to derive the maximum likelihood estimator $\hat{B}^{(n)}$ in step 2, we rewrite z as defined in (A.8) as

$$\text{(A.10)} \qquad z = \text{vec}\left[W^{(n)}_{(1)} - W^{(n)}_{(T)}\bar{R}^{(n)}\right] - D \text{ vec } B^{(n)}$$

with

$$D = \left[I_{n-1} \otimes X_{(1)}\right] - \left[\bar{R}^{(n)'} \otimes X_{(T)}\right]$$

From (A.10), it is easily seen that the **unconstrained** maximum likelihood estimator, vec $\hat{B}^{(n)}_u$, is given by

$$\text{(A.11)} \quad \text{vec } \hat{B}^{(n)}_u = \left[\hat{D}'\left(\hat{\Omega}^{-1}_{n-1} \otimes I_{T-1}\right)\hat{D}\right]^{-1}\left[\hat{D}'\left(\hat{\Omega}^{-1}_{n-1} \otimes I_{T-1}\right)\text{vec}(W^{(n)}_{(1)} - W^{(n)}_{(T)}\hat{\bar{R}}^{(n)})\right].$$

The **constrained** maximum likelihood estimator, vec $\hat{B}^{(n)}_c$, is equal to:

$$\text{(A.12)} \qquad \text{vec } \hat{B}^{(n)}_c = \text{vec } \hat{B}^{(n)}_u - \hat{C}A'(A\hat{C}A')^{-1}A \text{ vec } \hat{B}^{(n)}_u$$

with

$$\text{(A.13)} \qquad \hat{C} = \left[\hat{D}'\left(\hat{\Omega}^{-1}_{n-1} \otimes I_{T-1}\right)\hat{D}\right]^{-1}$$

The maximum likelihood estimator $\hat{\bar{R}}^{(n)}$ in step 3 is obtained by rewriting z as:

$$(A.14)\qquad z = \mathrm{vec}\left[W^{(n)}_{(1)} - X_{(1)}B^{(n)}\right] - \left\{I_{n-1} \otimes \left[W^{(n)}_{(T)} - X_{(T)}B^{(n)}\right]\right\}\mathrm{vec}\ \bar{R}^{(n)}$$

Then, it easily follows that $\hat{R}^{(n)}$ is given by

$$(A.15)\qquad \hat{R}^{(n)} = \left[\hat{U}^{(n)\prime}_{(T)}\ \hat{U}^{(n)}_{(T)}\right]^{-1}\left[\hat{U}^{(n)\prime}_{(T)}\ \hat{U}^{(n)}_{(1)}\right]$$

where $\hat{U}^{(n)}_{(T)}$ and $\hat{U}^{(n)}_{(1)}$ are obtained by substituting $\hat{B}^{(n)}$ into (2.14) and (2.8), respectively. Finally, the derivation of the maximum likelihood estimator $\hat{\Omega}_{n-1}$ in step 4, has already been dealt with in section 3.

Special cases

Additivity: use the unconstrained maximum likelihood estimator vec $\hat{B}^{(n)}_{u}$ instead of vec $\hat{B}^{(n)}_{c}$.

Homogeneity: redefine the matrix A such that only the homogeneity constraints are taken along. Note that the order of A becomes $[n - 1] \times [(n - 1)(n + 3)]$.

Static model: replace $\hat{R}^{(n)}$ by the zero matrix.

Diag-R model: replace $\hat{R}^{(n)}$ by $\hat{k}I_{n-1}$, the maximum likelihood estimator $\hat{k}$ being given by

$$(A.16)\qquad \hat{k} = \frac{\mathrm{tr}\left\{\hat{U}^{(n)\prime}_{(T)}\hat{U}^{(n)}_{(1)}\ \hat{\Omega}^{-1}_{n-1}\right\}}{\mathrm{tr}\left\{\hat{U}^{(n)\prime}_{(T)}\hat{U}^{(n)}_{(T)}\ \hat{\Omega}^{-1}_{n-1}\right\}}$$

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Ontvangen: 30-09-1987
Geaccepteerd: 30-05-1989