KM 27(1988)
pag 27 - 46

ESTIMATING INPUT-OUTPUT FACTOR DEMAND EQUATIONS: SOME RESULTS ON HANOCH'S HCDES PRODUCTION FUNCTION*

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ABSTRACT

In this paper we apply the linearization procedure of Nakanishi and Cooper (1974) to Hanoch's (1975) HCDES production function in order to estimate input-output factor demand equations for the Netherlands using annual data for the years 1950-1968. A newly proposed structure of the covariance matrix, named HARBO after De Boer and Harkema ((1983), (1986)) is applied, which proves to be superior to the specification that is usually applied in empirical work.

1. Introduction

Hanoch's HCDES (homogeneous constant differences of elasticities of substitution) production function in input-output analysis received increasing attention in the Netherlands, c.f. Donkers and Kreyger ((1981), (1985)) and Van Zon ((1983), (1986)).

Hanoch (1975) defines an (implicit) cost function*** which - after application of Shephard's lemma - yields as functional form for (optimal) cost shares:

$$(1) \qquad w_i = \frac{e_i (p_i/c)^{\alpha_i}}{\Sigma_j e_j (p_j/c)^{\alpha_j}} \qquad i = 1,\ldots,n$$

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*** We do not deal with the (very) special case $\alpha_i = 0$.

with w_i: cost share of input i ($i = 1,\dots,n$),

p_i: price of input i, and

c: unit cost function which is a function of $p_i (i=1,\dots,n)$.

Contrary to Hanoch's assertion, it can be shown (Van Daal (1984)) that the production function that underlies (1) will be strictly quasi-concave if and only if:

$$(2) \qquad e_i > 0$$
$$\alpha_i \leq 1,$$

where the equality sign may apply for <u>at most</u> one i.

Donkers and Kreyger (1985) apply (1) to specify a model with 10 inputs and estimate the parameters using annual data that covers the period 1970 - 1979 (excluding 1971) for one sector, viz. Building materials. From (1) they derive:

$$\text{dlog } w_i = \alpha_i(\text{dlog } p_i - \sum_{j=1}^{n} w_j \text{dlog } p_j) - \sum_{j=1}^{n} \alpha_j w_j (\text{dlog } p_j - \sum_{k=1}^{n} w_k \text{dlog} p_k)$$

$$i = 1,..,n$$

Replacing the optimal budget shares w_i in the right-hand side by their realized values in the previous period and using the approximations:

$$(3) \qquad \text{dlog } z \approx \frac{z_t - z_{t-1}}{z_{t-1}} \qquad (\text{with } z = w_i, p_i,\ i = 1,..,n)$$

their discrete version reads:

$$w_{ti} - w_{t-1,i} = w_{t-1,i}(\alpha_i p^*_{ti} - \sum_{j=1}^{n} \alpha_j w_{t-1,j} p^*_{tj}),$$

$$\text{with } p^*_{ti} = \frac{p_{ti}}{p_{t-1,i}} - \sum_{j=1}^{n} w_{t-1,j} \frac{p_{tj}}{p_{t-1,j}}$$

Moreover, they use a specification of the covariance matrix which seems to be inspired by Theil's rational random behaviour (see Theil (1980)).

Van Zon (1983) applies (1) to specify a model with 9 inputs and uses annual data for the Netherlands that covers the period 1950 - 1968. He avoids the linearization (3) by transforming (1) according to:

$$(4) \qquad \log \frac{w_i}{w_n} = \log \left(\frac{e_i}{e_n}\right) + \alpha_i \log(p_i/c) - \alpha_n \log(p_n/c) \qquad i = 1,\dots,n-1$$

yielding (n-1) equations. This transformation has been introduced by Somermeyer

(1956) in the framework of the theory of consumer allocation where (1) is known as the indirect addilog model.

If no restrictions are imposed on the covariance matrix, Harkema (1984) has proved that the likelihood function associated with (4) is unbounded, when the number of observations is less than 2n. Since Van Zon distinguishes 9 inputs and disposes of 19 observations he is able to estimate this model by means of the unrestricted covariance matrix, but is so close to the lower bound of 2n that his estimates are likely to be quite unstable. A part of his disappointing results may be attributed to this fact. A second explanation for his disappointing results may be found in his treatment of "profits". In the derivation of the time-series for c "gross value of production" (including "profits") is used instead of "total cost of production" (excluding "profits") as advocated by De Boer and Donkers (1985).

In this paper we propose to apply (1) to specify a larger model than Donkers and Kreyger, and Van Zon, viz. a model with 13 inputs. Like Van Zon we use annual data for the Netherlands that covers the period 1950 - 1968 (see section 4), but we use "total cost of production" in the derivation of the time series for the unit cost c. As our number of observations is too small to admit an unrestricted covariance matrix, we use two restricted specifications, denoted by "DESO" and "HARBO", respectively. These specifications are discussed in section 3. Since the latter specification requires the use of all n equations, we apply a transformation of (1) due to Nakanishi and Cooper (1974), which reads:

$$(5) \qquad w_i \rightarrow \log w_i - \frac{1}{n} \sum_{j=1}^{n} \log w_j; \qquad i=1,\ldots,n$$

We shall analyze this transformation in more detail in section 2.
The organization of this paper is as follows: in section 2 we derive the model based on transformation (5), as well as a production model nested within Hanoch's, viz. CES (constancy of mixed value and volume coefficients, see De Boer and Donkers (1985)) and two models nested within CES: Cobb-Douglas (input-output analysis with constancy of value shares, see De Boer (1976)) and Leontief (input-output analysis with constancy of volume shares); in section 3 we discuss estimation and testing of the various production models, section 4 contains information on the data, whereas section 5 reports upon results.
It appears that HARBO may be profitably used when the number of observations is relatively small as compared to the number of inputs: only in two cases DESO is accepted at a 5% level of significance, whereas in all other cases DESO is strongly rejected.

At the level of aggregation considered in the paper, the concavity of Hanoch's function is not rejected in 6 out of 10 cases which is quite encouraging in view of the small number of observations and the large number of parameters to be estimated. For the other 4 sectors it seems worthwhile to try another level of aggregation. It turns out that Hanoch's model is by far superior to the (more simple) CES model.

From an economic theoretical point of view the CES model behaves nicely: all sectors show positive signs for the elasticity of substitution. The Leontief model is strongly rejected against CES so that the hypothesis of no substitution between inputs cannot be upheld. For 2 sectors the Cobb-Douglas model is not rejected at a 5% level of significance whereas for 1 sector it is not rejected at a 1% level of significance.

2. Derivation of the models

2.1 Hanoch's HCDES

Applying transformation (5) to (1), attaching a time index t=1,...,T, with T the length of the observation period, and adding a disturbance term u_{ti} leads to the following model:

$$\text{(6)} \qquad \log w_{ti} - \frac{1}{n}\sum_j \log w_{tj} = b_i + \sum_j \alpha_j(\delta_{ij} - \frac{1}{n})\log(\frac{p_{tj}}{c_t}) + u_{ti}$$

$$i = 1,\ldots,n$$
$$t = 1,\ldots,T$$

with:

$$\text{(7)} \qquad b_i = \log e_i - \frac{1}{n}\sum_j \log e_j,$$

and δ_{ij} the Kronecker delta, i.e., $\delta_{ij}=1$ for $i=j$ and $\delta_{ij}=0$ for $i \neq j$.

From (7) it is clear that:

$$\text{(8)} \qquad \sum_{i=1}^{n} b_i = 0.$$

Summation of (6) over all i and taking account of (8) leads to:

$$\text{(9)} \qquad \sum_{i=1}^{n} u_{ti} = 0 \qquad t = 1,\ldots,T,$$

i.c. to the well-known adding-up of the demand relations.

Defining:

$$[\log w_i]' = [\log w_{1i} \ldots \log w_{Ti}]$$

$$[\log(\frac{p_i}{c})]' = [\log(\frac{p_{1i}}{c_1}) \ldots \log(\frac{p_{Ti}}{c_T})]$$

and $\iota'_T = [1 \ldots 1]$

we can write (6) in matrix notation as:

(10) $$y_i = X_i \beta + u_i \qquad i=1,\ldots,n$$

where $$y_i = \log w_i - \frac{1}{n} \sum_{j=1}^{n} \log w_j$$

$$X_i = [0 \ \ldots \ \iota_T \ \ldots \ 0 \ \vdots \ -\frac{1}{n} \log(\frac{p_1}{c}) \ \ldots \ (1 - \frac{1}{n})\log(\frac{p_i}{c}) \ \ldots \ -\frac{1}{n} \log(\frac{p_n}{c})]$$

with ι_T as the i^{th} column of X_i

$$\beta' = [b_1 \ \ldots \ b_n \ \alpha_1 \ \ldots \ \alpha_n]$$

$$u_i' = [u_{1i} \ \ldots \ u_{Ti}].$$

Moreover, (8) can be written as:

(11) $$R\beta = 0,$$

with $$R = [\iota_n' \ \vdots \ 0_n'],$$

and (9) as:

(12) $$\sum_{i=1}^{n} u_i = 0_T.$$

The econometric model consisting of equations (10) - (12) will be analyzed in section 3.

2.2 Some special models

If:

(13) $$\alpha_i = \alpha \qquad i = 1,\ldots,n,$$

Hanoch's HCDES reduces to a CES production model with:

(14) $$\alpha = 1 - \sigma,$$

where σ denotes the Allen partial elasticity of substitution of the CES production function.

Then (6) reduces to:

(15) $$\log w_{ti} - \frac{1}{n}\sum_j \log w_{tj} = b_i + \alpha \sum_j (\delta_{ij} - \frac{1}{n})\log(\frac{p_{tj}}{c_t}) + u_{ti}.$$

Defining the X_i-matrix for the CES model to be:

$$X_i = [0 \; \dots \; \iota_T \dots \; 0 \;\vdots\; \sum_j (\delta_{ij} - \frac{1}{n})\log(\frac{p_j}{c})],$$

the β-vector to be:

$$\beta' = [b_1 \; \dots \; b_n \;\vdots\; \alpha],$$

and $$R = [\iota_n' \;\vdots\; 0],$$

the CES model can be written in the very same econometric structure (10) - (12). If:

(16) $$\alpha = 0, \text{ or equivalently, } \sigma = 1,$$

(c.f. (13) and (14)), we have the Cobb-Douglas model which, after substitution of (16) into (15), reads:

(17) $$\log w_{ti} - \frac{1}{n}\sum_j \log w_{tj} = b_i + u_{ti}.$$

If:

(18) $$\alpha = 1, \text{ or equivalently, } \sigma = 0$$

we have the Leontief model

(19) $$\log w_{ti} - \frac{1}{n}\sum_j \log w_{tj} - \sum_j (\delta_{ij} - \frac{1}{n})\log(\frac{p_{tj}}{c_t}) = b_i + u_{ti},$$

after substitution of (18) into (15).

By an appropriate choice of y_i (c.f. the left-hand side of (17) and (19)) both models can be written as:

(20) $$y_i = b_i \iota_T + u_i \qquad i = 1,\dots,n,$$

with:

(21) $$\sum_{i=1}^{n} b_i = 0, \text{ and}$$

$$(22) \qquad \sum_{i=1}^{n} u_i = 0_T.$$

The econometric structure (20) - (22) will be discussed in section 3.

3. Estimation of the models and some tests

3.1 Introduction

As usual, the vectors of disturbances $[u_{t1},\ldots,u_{tn}]$ $(t=1,\ldots,T)$ will be assumed to be identically and independently* distributed according to a multivariate normal distribution with zero mean and covariance matrix Ω_n. Because of (12) or (22) the rank of Ω_n will be n-1. Barten (1969), however, has shown that this singularity can be handled by simply deleting one arbitrary equation, say the n-th one. The matrix obtained by deleting the n-th row and column of Ω_n will be denoted by Ω_{n-1}.

As mentioned before, the likelihood function becomes unbounded when the number of observations is too small. The reason for this phenomenon is that the estimated covariance matrix $\hat{\Omega}_{n-1}$, becomes singular when T is smaller than 2n. Since we dispose of 19 observations and there are two sectors in the empirical application with n = 10 inputs and one with 11 inputs, it is impossible to estimate the model for these sectors without imposing restrictions on the covariance matrix; for the other sectors we either use n = 7 (1 sector), 8 (4 sectors) or 9 (2 sectors) and for these sectors T is so close to the underbound of 2n that the parameter estimates will be quite unstable. Therefore, we have decided to impose restrictions on the covariance matrix in order to reduce the minimum number of observations required to obtain a non-singular estimated covariance matrix.

3.2 The specification DESO

The first restricted covariance matrix we use is the one that is mostly used in applied research, see for instance Deaton (1975) and Solari (1971). This specification,denoted by DESO, reads:

* In the present paper we do not deal with any form of dynamic specification. The estimation of dynamic models is subject to current research. Some preliminary results have been presented in van Heeswijk, de Boer and Harkema (1986).

$$\Omega_n = \sigma^2(I_n - \frac{1}{n}\iota_n\iota_n') \tag{23}$$

Deleting an arbitrary equation from system (10) through (12) (the Hanoch and the CES model), it can be shown that maximization of the loglikelihood function subject to (11) with respect to σ^2 and β implies:

$$\hat{\beta} = (\sum_{i=1}^{n} X_i'X_i)^{-1}(\sum_{i=1}^{n} X_i'y_i), \tag{24}$$

i.c. application of ordinary least squares to the system* as a whole, and

$$\hat{\sigma}^2 = \frac{1}{T(n-1)} \sum_{i=1}^{n} \hat{u}_i'\hat{u}_i, \tag{25}$$

with

$$\hat{u}_i = y_i - X_i\hat{\beta}. \tag{26}$$

The covariance matrix of $\hat{\beta}$ can be shown to be equal to

$$\hat{\sigma}^2\{(\sum_{i=1}^{n} X_i'X_i)^{-1} - \frac{1}{Tn}\begin{bmatrix}\iota_n\iota_n' & & 0\\ & & \\ 0 & & 0\end{bmatrix}\}, \tag{27}$$

and the value of the loglikelihood function evaluated at the optimum is:

$$\log L(DESO) = -\tfrac{1}{2}T(n-1)(1 + \log 2\pi) + \frac{T}{2}\log n - \tfrac{1}{2}T(n-1)\log(\hat{\sigma}^2). \tag{28}$$

Deleting an arbitrary equation from system (20) - (22) (the Cobb-Douglas and the Leontief model)** leads to:

$$\hat{b}_i = (\iota_T'\iota_T)^{-1}\iota_T'y_i = \frac{1}{T}\sum_{t=1}^{T} y_{ti} \qquad i = 1,\ldots,n. \tag{29}$$

The estimate of σ^2 is obtained from (25) with

$$\hat{u}_i = y_i - \hat{b}_i\iota_T, \tag{30}$$

and the value of the loglikelihood function from (28).

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*) The unrestricted ordinary least squares estimator (24) can be shown to satisfy automatically constraint (11).

**) We have to estimate these models in order to evaluate the loglikelihood function for purposes of testing.

3.3 The specification HARBO

De Boer and Harkema (1983) specify the covariance matrix Ω_n as follows:

$$(31) \qquad \Omega_n = D_n - d^{-1}\delta_n\delta_n',$$

where

$$D_n = \begin{bmatrix} d_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & d_n \end{bmatrix}, \quad \delta_n' = [d_1 \; \ldots \; d_n], \quad d = \sum_{i=1}^{n} d_i$$

Obviously, for

$$(32) \qquad d_i = \sigma^2 \qquad i = 1,\ldots,n,$$

the HARBO specification (31) reduces to DESO (i.c. to (23)).

They show that the maximum likelihood estimates of d_i follow from the following system of equations:

$$(33) \qquad \hat{d}_i - \frac{\hat{d}_i^2}{\hat{d}} = \frac{1}{T}\hat{u}_i'\hat{u}_i \qquad i = 1,\ldots,n,$$

where $\hat{u}_i$ is specified in (26) for system (10) - (12) and in (30) for system (20) - (22). Apart from one special case that occurs with probability zero there is a unique solution to (33) that can be found by means of a one-dimensional search procedure that works very quickly. The algorithm is described in De Boer and Harkema (1986).

For model (10) - (12) it can be proved that

$$(34) \qquad \hat{\beta} = \left(\sum_{i=1}^{n} \hat{d}_i^{-1}X_i'X_i\right)^{-1}\left(\sum_{i=1}^{n} \hat{d}_i^{-1}X_i'y_i\right),$$

satisfying automatically constraint (11). The covariance matrix of $\hat{\beta}$ can be shown to be equal to:

$$(35) \qquad \left(\sum_{i=1}^{n} \hat{d}_i^{-1}X_i'X_i\right)^{-1} - \frac{1}{\hat{d}T}\begin{bmatrix} \hat{\delta}_n\hat{\delta}_n' & 0 \\ & \\ 0 & 0 \end{bmatrix},$$

and the value of the loglikelihood function evaluated at the optimum is:

$$(36) \qquad \log L(\text{HARBO}) = -\tfrac{1}{2}T(n-1)(1 + \log 2\pi) - \frac{T}{2}\log\left(\hat{d}^{-1}\prod_{i=1}^{n}\hat{d}_i\right)$$

For the system (20) - (22), we also have (29) in case the covariance matrix is specified to be HARBO. The estimates $\hat{d}_i$ follow from (33) with $\hat{u}_i$ as specified

in (30), the covariance matrix of the $\hat{b}_i$ follows from (35) with X_i a zero matrix apart from its i-th column, which equals ι_T, and the value of the loglikelihood function follows from (36).

3.4 Some specification tests

In the empirical part of this paper we test two kinds of hypotheses:

(i) we test different specifications of the covariance matrix against each other,

(ii) we test the different specifications of the model against each other.

ad (i) Since DESO is nested into HARBO, see (32), we can use the likelihoodratio test statistic

$$-2\log \lambda = -2(\log L(\text{DESO}) - \log L(\text{HARBO}))$$

which is approximately $\chi^2(n-1)$ distributed.

ad (ii) Since the CES model (15) is nested into Hanoch's model (6) see (13), the likelihoodratio test statistic

$$-2\log \lambda = -2(\log (\text{CES}) - \log (\text{HANOCH}))$$

is once again approximately $\chi^2(n-1)$ distributed.

Finally, Cobb-Douglas and Leontief are nested into CES, see (16) and (18), respectively. Consequently:

$$-2\log \lambda = -2(\log L(\text{Cobb-Douglas}) - \log L(\text{CES})), \text{ and}$$

$$-2\log \lambda = -2(\log L(\text{Leontief}) - \log L(\text{CES}))$$

are approximately $\chi^2(1)$ distributed.

As is well-known from empirical studies in the field of consumer demand (see Laitinen (1978) and Meisner (1979)) the likelihoodratio test statistic is biased towards rejection of the null-hypothesis when the number of observations is small as compared to the number of budget categories distinguished. In the empirical part of this paper we apply - where needed - a small sample correction factor that has recently been proposed by Italianer (1985).

Italianer decomposes the correction factor that Anderson (1958) derived for a specific testing problem into two factors and proposes to use this decomposition for more general problems such as the tests we described above.

The correction factor can be written as:

$$\frac{\frac{1}{2}(df_0 + df_1)}{\text{total number of observations}}$$

with df_0 : the number of degrees of freedom under the null hypothesis, and
df_1: idem, under the alternative hypothesis,

where the number of degrees of freedom is defined as the total number of observations minus the total number of parameters to be estimated (which is the sum of the number of model parameters and the number of covariance parameters). An example may clarify the procedure. Suppose we want to test CES against HANOCH with HARBO as maintained hypothesis. The total number of observations is $(n-1)T$ (recall that we deleted one equation), the number of model parameters under the CES specification is n, i.c. b_i $(i=1,\dots,n-1)$ and α, and the number of covariance parameters is also n, i.c. d_i $(i=1,\dots,n)$.
Hence:

$$df_0 = (n-1)T - n - n = (n-1)T - 2n$$

Similarly:

$$df_1 = (n-1)T - (2n-1) - n = (n-1)T - 3n + 1$$

4. Description of the data

Empirical application of the theory presented in the previous sections requires data on inputs and outputs expressed in constant prices and series of price index numbers.

In the Netherlands the input-output tables compiled by the Central Bureau of Statistics (C.B.S.) are expressed in current prices so that we need price index numbers in order to deflate the tables.

We got the appreciated cooperation of the Central Planning Bureau* (C.P.B.) that allowed us to use its data pertaining to the period 1950 - 1968.

For a description of the deflation of the matrices of intermediate and of final deliveries we refer to De Boer ((1982), ch. 6.2)**. Summation of all deflated deliveries yielded (rowwise) deflated gross value of production. As regards the primary inputs, the CPB supplied us with the price index for the imports of all sectors.

Consequently, we were able to derive deflated gross value added by (columnwise) subtracting from deflated gross value of production all deflated intermediate inputs as well as deflated imports. Gross value added can be split up into three components:

(i) labor services,
(ii) capital services,
(iii) profits

As argued by De Boer and Donkers (1985) "profits" just as "savings" in the theory of consumer allocation have to be excluded in the theory of costs. However, a part of "profits" can be attributed to renumeration for "labor services" of "self-employed".
Van Zon ((1983), (1986)) solved this in the following way: he calculated for each sector the wage-sum per wage-earner by dividing the total wage-sum (including social security payments by employers) by the number of wage-earners in that sector. This "average wage" was imputed to the number of "self-employed" in that particular sector and added to the total wage-sum obtaining in this way nominal and deflated amounts for "labor services".

Van Zon considered "capital services" to be measured by "depreciation charges". The price index was obtained from data on total nominal investment supply and total nominal investment demand for each sector. Using the RAS method he

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*) In this respect the authors sincerely wish to thank Mr. Van Nieuwenhoven for his kind allotment of precious time.

**) In that study we only disposed of the tables for 1958 - 1967. Since then, we could extend the period of observation to 1950 - 1968.

constructed a matrix linking nominal supply to nominal demand. This matrix was first deflated by means of investment supply price data and then aggregated to obtain a sector-specific price index for the sector demand for investment goods. This price index was used to deflate "depreciation charges".

The CBS distinguished 35 productive sectors for the period 1950-1968. As this is too detailed for our purposes, we decided to aggregate these 35 sectors into 10 new sectors. In table 1 we present the aggregated sectors in terms of the original CBS sectors. For the latter classification we refer to De Boer (1982), p. 74.

Table 1

Definition of aggregated sectors

	Aggregated sector	CBS sector No.
1.	Agriculture, forestry and fishing	1
2.	Extracting industry	2, 3
3.	Food, beverages and tobacco	4, 5, 6
4.	Textiles, footwear and other wearing apparel	7,8
5.	Chemicals, oil refineries	13
6.	Metal industry	15, 16, 17, 18, 19
7.	Construction	20
8.	Other manufacturing industries	9, 10, 11, 12, 14
9.	Transport, storage and communication	27, 28, 29
10.	Services	21, 22, 23, 24, 25, 26, 30, 31, 32,33, 34, 35

So we arrived at 19 nominal and deflated input-output tables with 10 intermediate inputs and 3 primary inputs: imports, labor services and capital services.

In applied research the optimal cost shares w_{ti} for a particular sector as well as the optimal (minimum) unit cost price are usually unknown and are replaced by observations on cost shares:

$$w_{ti} \approx \frac{\text{costs of input i in period t}}{\text{total costs in period t}},$$

and by observations on the unit cost prices:

$$c_t \approx \frac{\text{nominal total costs in period t}}{\text{deflated total costs in period t}}.$$

deflated costs of input j.

As a final remark, we left out items which have a cost share smaller than 1% of total costs (mainly zero entries), as the data for these inputs is quite unreliable, because of rounding errors.

5. Results

5.1 Estimates

In table 2 we present the results for Hanoch's model, i.c. (6). For economy of space we do not present the estimates of the constant terms b_i as they are not relevant from an economic-theoretical point of view*. Moreover, we only present the results for the covariance specification HARBO, since DESO is strongly rejected in a large majority of cases (see section 5.2).
Between brackets we present the standard errors of the estimates.

Only in two cases, sectors 7 and 9, the theoretical constraint of concavity, i.c.

$$(37) \qquad \alpha_i \leq 1 \qquad i = 1,\dots,n,$$

with the equality sign applying for at most one i, is met with.
For the other sectors, we also estimated the constrained version of the Hanoch model, i.c. with (37) imposed.**
In table 3 we present the results for the CES model, i.c. (15). All estimates of the substitution parameter σ are positive, as it should be. From an economic-theoretical point of view the CES model behaves nicely.

5.2 Testing specifications

Testing DESO against HARBO

In Table 4 we present the values of the likelihood ratio test statistic $-2\log\lambda$ as well as the critical point at a level of significance of 0,5%. We only applied Italianer's correction (see section 3.4) when the null-hypothesis DESO

*) For the same reason we do not present the estimates of the Cobb-Douglas and of the Leontief model.
**) The results may be obtained from the authors upon request.

was accepted at a level of significance of 0.5% or slightly exceeded the critical point. In the following two tables an asteriks indicates application of that correction factor.

In a large majority of cases DESO is strongly rejected against HARBO. At a 5% level of significance DESO is not rejected against HARBO only in the constrained Hanoch model for sectors 2 (with a corrected value of 13.867 and critical point 14.1) and 6 (corrected value 12.431 and critical point 12.6).
It seems safe to conclude that HARBO performs (much) better than DESO.

Testing model specifications against each other

Since DESO is strongly rejected against HARBO (except for 2 cases at a 5% level of significance), we only present the results for the covariance specification HARBO.

Hanoch constrained vs. Hanoch unconstrained

From the first column of table 5, it appears that in addition to sectors 7 and 9, concavity is also accepted for sectors 4, 5, 6 and 10, so that at this level of aggregation the concavity of Hanoch's function is accepted in 6 out of 10 cases which is quite encouraging in view of the small number of observations and the large number of parameters to be estimated. For the other 4 sectors it seems worthwile to try another level of aggregation.

CES vs. Hanoch

In the second column of table 5 we present the values of the likelihood ratio test statistic of CES against the unconstrained* Hanoch model. In view of the very large values of the test statistic we do not present critical points in the table. Obviously, the CES model to strongly rejected against the unconstrained Hanoch model in all cases.

* Note that the CES model is not rested within the constrained Hanoch model.

Table 2* Estimates of the parameter α_i of the Hanoch model

sector α_i	1	2	3	4	5	6	7	8	9	10
1	1.761	--	.982	--	--	--	--	2.367	--	--
	(.134)		(.370)					(.204)		
2	--	--	--	--	2.544	--	--	--	--	.419
					(.630)					(.269)
3	-1.336	--	1.127	--	.474	--	--	--	--	.792
	(.093)		(.232)		(.488)					(.119)
4	--	--	--	.404	--	--	--	-.737	--	--
					(.170)			(.191)		
5	.021	.375	-1.378	-1.219	-1.551	-.420	.816	-.321	.403	-.481
	(.110)	(.149)	(.203)	(.167)	(.500)	(.158)	(.127)	(.121)	(.165)	(.103)
6	-.132	.921	1.424	-.178	2.597	-.230	.235	.492	-.279	.527
	(.352)	(.422)	(.206)	(.296)	(.715)	(.192)	(.235)	(.360)	(.468)	(.178)
7	.852	--	--	--	--	--	.165	--	--	-2.819
	(.364)						(.525)			(.735)
8	.650	-.029	-1.325	.589	1.828	.422	.080	.091	.713	.666
	(.101)	(.247)	(.408)	(.274)	(.459)	(.121)	(.220)	(.091)	(.114)	(.107)
9	--	2.856	--	--	--	--	--	.694	-2.566	-.563
		(.484)						(.291)	(.666)	(.171)
10	.571	-2.050	3.938	1.930	1.294	2.897	.485	-.580	.313	1.009
	(.691)	(.478)	(.435)	(.273)	(.223)	(2.003)	(.299)	(.577)	(.458)	(.455)
11	.547	-.218	.173	-.832	-.326	-.039	.297	.470	.779	-.179
	(.072)	(.196)	(.064)	(.216)	(.144)	(.116)	(.156)	(.092)	(.103)	(.093)
12	-.370	-.233	.410	.659	.580	.428	-.144	.288	.278	.645
	(.053)	(.254)	(.019)	(.040)	(.038)	(.060)	(.102)	(.045)	(.063)	(.093)
13	-.019	.777	.151	1.480	.972	-.029	-2.330	.988	.708	.325
	(.137)	(.498)	(.181)	(.261)	(.220)	(.769)	(.501)	(.745)	(.216)	(.125)

* "--" denotes that the corresponding cost share is smaller than to 1% of total costs.

Table 3* Estimates of the substitution parameter of the CES model model

sector	1	2	3	4	5	6	7	8	9	10
α_1										
α	-.121	.048	.282	.409	.393	.282	.008	.294	.340	.416
	(.024)	(.085)	(.016)	(.013)	(.020)	(.016)	(.040)	(.032)	(.021)	(.021)
σ	1.121	.952	.718	.591	.607	.718	.992	.706	.660	.584

* The standard errors of σ are the same as those of α, of course.

Table 4 Values of test statistic: DESO vs. HARBO

sector	critical point at α = 0.005	Hanoch	Hanoch constrained	CES	Cobb-Douglas	Leontief
1	23.6	55.278	85.758	133.502	130.036	168.794
2	20.3	14.977*	13.867*	24.312*	24.433*	29.852
3	22.0	91.698	85.412	102.548	39.260	137.314
4	20.3	56.596	52.116	96.950	19.279*	97.310
5	22.0	83.116	107.200	133.714	90.742	57.546
6	18.5	12.772*	12.431*	13.498*	13.046*	62.290
7	20.3	35.284		111.420	114.916	118.524
8	23.6	88.314	152.698	226.358	222.666	138.802
9	20.3	57.900		84.176	58.210	69.610
10	25.2	251.410	251.418	161.936	122.726	134.892

In table 5 we present the values of the likelihood ratio test statistic with the critical point at a 5% level of significance between brackets.

Table 5 Values of the test statistic for alternative specifications of the model

	Hanoch constrained		CES vs	Cobb-Douglas	Leontief
sectors	vs.Hanoch		Hanoch vs.	CES (3.84)	vs CES (3.84)
1	16.112	(3.84)	167.788	6.248*	14.936
2	10.816	(3.84)	63.214	.260	51.544
3	13.168	(9.49)	92.166	73.244	99.802
4	5.870	(5.99)	78.578	81.558	93.654
5	8.287*	(9.49)	28.354	62.386	108.748
6	.810	(3.84)	28.974	24.762	85.896
7	concavity already accepted		37.546	.036	61.966
8	26.142	(5.99)	78.932	25.136	120.662
9	concacity already accepted		41.790	74.590	56.850
10	0.0003	(3.84)	159.730	44.342	95.812

Cobb-Douglas against CES and Leontief against CES

In the third and fourth columns of table 5, we present the results of testing Cobb-Douglas against CES and Leontief against CES, respectively. It appears that at a 5% level of significance Cobb-Douglas is only accepted for sectors 2 and 7. At a 1% level of significance Cobb-Douglas is also accepted for sector 1 (the critical point being 6.63). In all other cases Cobb-Douglas is strongly rejected. The Leontief model is strongly rejected against CES in all cases. Consequently, the hypothesis of no substutition between inputs cannot be upheld.

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