KM 24(1987)
pag 93 - 113

Construction and analysis of Social Accounting Matrices, an application to Colombia

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Abstract

The paper reports on a social accounting matrix of Colombia for 1970, which formed the base year data for an economy-wide model emphasising growth and income distribution among several population groups. After a brief description of the estimation of the SAM, and a specification of exogenous and endogenous accounts in the SAM, two types of analysis are performed.

First, the SAM is tested for its predictive ability. Even though the outcome is mixed, the analyses illuminates major changes which occurred in the Colombian economy during the seventies. Secondly, overall accounting multipliers are examined and decomposed in terms of transfer, open-loop and closed-loop effects.

1. Introduction

In the past decade there has been considerable interest and momentum in the design, construction and use of social accounting matrices (SAM) in developing countries [1)] The case for the construction of a SAM is based on several arguments: it is a convenient way of systematizing related data from diversified sources on production, income and expenditure, and therefore, is an efficient tool for directing attention to data gaps, ignored interactions, significant linkages, etc.; besides, the transactions in the SAM can be conceived to represent the functioning of the economy in the short run, and therefore, the SAM can be utilized for predictions and for an analysis of multiplier properties of the system, even though these uses are limited by the static nature of the matrix.

In general economy-wide models which treat the interactions between categories of production, income and expenditure make use, by implication, of an explicit or implicit SAM for at least an initial year for which the underlying model is validated. The generation of a SAM can be seen as the by-product of the specification and testing of such models. By the same

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1) See, among others, Pyatt and Round (1985), Hayden en Round (1980) and Pyatt and Roe (1977).

reasoning, any SAM has an underlying model and it pays to make both the specified model and the accounting matrix explicit and in harmony with each other for purposes of calibration.

The present paper, in affiliation with the above, reports on a SAM which is the by-product of such a modelling application to Colombia. The model has been estimated on the basis of data during the sixties and seventies, it includes dynamic elements and allows for price variations. The specification of the matrix will reflect the focus of the model which gives special emphasis on: (a) the duality of the economy as incorporated via disaggregations into rural/urban, modern/traditional modes of production, and physical/human capital resources, and (b) the basic needs situation for various population groups; i.e., primarily the expenditure on food, housing, health and education.

The above specifications of the underlying model will be reflected in the SAM which is constructed and estimated in sections 2 and 3. In the remainder of the paper we look at the SAM as if it is a set of equations which model the Colombian economy in a consistent but static way. The matrix is put to two main uses: as an aid, though limited, in predicting the course of the economy, section 4; and via an analysis of multipliers more insight is gained into the structural properties of the economy, section 5.

2. Characteristics

Social Accounting Matrices are compiled according to the same accounting principles as input-output tables, each transaction is recorded twice so that any ingoing in one account must be balanced by an outgoing of another account. In contrast, SAM contains a complete list of accounts describing income, expenditure and production flows. These accounts are grouped into different sets of accounts as indicated below.

1) the wants account

2) The factors of production account;

3.) The institutions account which can be further disaggregated by type of institution as far as current transactions are concerned:

3.1 Households account,

3.2) Firms account,

3.3) Government account; and in an aggregate form :

3.4) The institutions capital account,

4) The activities account,

5) The rest of the world account.

These accounts are disaggregated to give a SAM of 33x33 in Table 1. The focus of attention of the disaggregations lies with the current household account.The disaggregation of households emphasises dualities in the location of population (rural, urban), mode of earning (modern employment, self employment, inactive), and occupational characteristics of worker (within modern employment a distinction is made between employer, non-manual and manual worker).

The first set of accounts in Table 1 is the wants accounts, rows and columms 1 to 6. This is a new set of accounts that focuses on the consumer expenditure by type of good. It is more informative to know, for instance, how much is being spent on food products as well as knowing the amount of consumer goods produced by agriculture, than only being aware of the total consumer demand for agricultural produce only. This interest in consumer demand by type of good stems from the policy questions adressed by basic needs approaches - or as sometimes called - social planning. [2)]

Rows 1 to 6 contain one large block of entries on the intersects with columns 10 to 19 giving the breakdown of final consumer demand over the six wants categories and over the ten household groups. The outgoings of the wants account, columns 1 to 6, are entered as incomings to the activities account, rows 23 to 30. This block of entries converts the broad categories of consumer demand such as food, etc. into the more well-known sectoral classification. For example, column 1 contains the sectoral breakdown of food consumption: 30 percent of food is produced directly by agriculture, 67 percent of food is supplied by the food-processing industry.

The next set of accounts is the factor account, showing for instance, that the largest part of urban labour income originates in the services sectors, while the largest source of rural labour income is agriculture.

Next are the institutional current accounts. In this SAM, household groups receive a mixture of labour and capital income. It can be seen that the groups of urban employed and urban self-employed dominate urban labour incomes and that rural workers and rural self-employed dominate rural labour remuneration. The distribution of capital income over institutions also involves firms and government. Additional sources of income to households are transfers from government and transfers from the rest of the world.

The expenditures by the different institutions are directly readable from their respective rows.

As for the institutions capital account, all savings of the institutions

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2) An early attempt to model basic needs approaches can be found in Cohen (1975).

Table 1. The disaggregated SAM, Colombia 1970.

	WANTS ACCOUNTS						FACTOR ACCOUNTS			INSTITUTIONS CURRENT ACCOUNTS						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1 FOOD	-	-	-	-	-	-	-	-	-	11280.4	3461.9	1140.6	10680.1	4822.2	1036.6	3830.6
2 NONFOOD	-	-	-	-	-	-	-	-	-	6433.6	1623.9	779.1	5443.9	2240.2	472.8	1172.1
3 HOUSING	-	-	-	-	-	-	-	-	-	4837.8	953.8	567.6	3865.8	2456.4	170.8	353.8
4 HEALTH	-	-	-	-	-	-	-	-	-	745.9	304.6	177.5	682.5	426.5	59.6	171.5
5 EDUCATION	-	-	-	-	-	-	-	-	-	1387.8	237.9	223.2	1121.8	366.6	64.1	89.5
6 OTHER SOC. SERVICES	-	-	-	-	-	-	-	-	-	2542.6	298.5	711.2	2169.5	751.4	169.0	225.9
7 URBAN LABOUR REM.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8 RURAL LABOUR REM.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9 GROSS PROFITS	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10 URBAN EMPLOYED	-	-	-	-	-	-	13259.4	-	16625.7	-	-	-	-	-	-	-
11 URBAN WORKER	-	-	-	-	-	-	6866.2	-	-	-	-	-	-	-	-	-
12 URBAN CAPITALIST	-	-	-	-	-	-	1392.4	-	2694.9	-	-	-	-	-	-	-
13 URBAN SELF EMPLOYED	-	-	-	-	-	-	9304.8	-	16419.1	-	-	-	-	-	-	-
14 URBAN INACTIVE	-	-	-	-	-	-	4528.7	-	6502.3	-	-	-	-	-	-	-
15 RURAL EMPLOYED	-	-	-	-	-	-	-	1208.7	958.1	-	-	-	-	-	-	-
16 RURAL WORKER	-	-	-	-	-	-	-	5526.7	-	-	-	-	-	-	-	-
17 RURAL CAPITALIST	-	-	-	-	-	-	-	481.0	1178.5	-	-	-	-	-	-	-
18 RURAL SELF EMPLOYED	-	-	-	-	-	-	-	5486.4	4304.2	-	-	-	-	-	-	-
19 RURAL INACTIVE	-	-	-	-	-	-	-	1271.1	822.3	-	-	-	-	-	-	-
20 FIRMS	-	-	-	-	-	-	-	-	15492.4	-	-	-	-	-	-	-
21 GOVERNMENT	-	-	-	-	-	-	-	-	1882.2	1767.9	148.3	490.8	1317.2	642.4	64.0	63.9
22 AGGR CAPITAL ACCOUNT	-	-	-	-	-	-	-	-	-	1130.2	-109.3	20.1	583.3	856.7	136.8	-328.1
23 AGRICULTURE	13661.6	23.5	425.3	-	-	148.5	-	-	-	-	-	-	-	-	-	-
24 MINING	-	3.7	67.9	-	-	54.4	-	-	-	-	-	-	-	-	-	-
25 ELABORATED COFFEE	853.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
26 INDUSTRY	29885.2	12034.2	43.0	1064.2	-	1628.1	-	-	-	-	-	-	-	-	-	-
27 ELECT, GAS & WATER	-	-	-	-	-	934.1	-	-	-	-	-	-	-	-	-	-
28 MODERN SERVICES	-	751.6	13626.8	-	-	3378.5	-	-	-	-	-	-	-	-	-	-
29 PERSONAL SERVICES	-	7739.5	-	1944.3	3632.0	1207.6	-	-	-	-	-	-	-	-	-	-
30 GOVT. SERVICES	33.8	394.5	68.4	69.0	99.3	90.7	-	-	-	-	-	-	-	-	-	-
31 INDIRECT TAXES	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
32 IMPORT DUTIES	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
33 REST OF THE WORLD	-	-	-	-	-	-	-	-	-	173.7	-	3.1	89.5	131.6	20.9	-
34 TOTALS	44434.3	20947.0	14231.4	3077.5	3731.3	7441.8	35351.5	13973.9	66897.7	30299.7	6919.5	4113.2	25953.6	12694.0	2194.6	5579.2

			CAP.ACC.			ACTIVITIES ACCOUNTS						INDIRECT TAXES				R.O.W.	TOTALS
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
609.2	6016.0	1556.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	44434.3
229.9	2156.1	395.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	20947.0
96.3	730.7	198.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	14231.4
46.6	378.5	84.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3077.5
29.6	186.1	24.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3731.3
172.5	299.6	101.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7441.8
-	-	-	-	-	-	2730.3	442.1	128.1	7718.4	576.5	12266.2	4592.8	6897.1	-	-	-	35351.5
-	-	-	-	-	-	7093.0	296.5	28.6	1722.4	58.0	2465.7	923.2	1386.4	-	-	-	13973.9
-	-	-	-	-	-	23233.6	2143.7	512.9	10811.8	754.9	28443.6	4571.1	-	-	-	-3591.9	66897.7
-	-	-	-	335.0	-	-	-	-	-	-	-	-	-	-	-	79.5	30299.7
-	-	-	-	35.1	-	-	-	-	-	-	-	-	-	-	-	18.2	6919.5
-	-	-	-	15.0	-	-	-	-	-	-	-	-	-	-	-	10.8	4113.2
-	-	-	-	161.6	-	-	-	-	-	-	-	-	-	-	-	68.1	25953.6
-	-	-	-	1629.7	-	-	-	-	-	-	-	-	-	-	-	33.3	12694.0
-	-	-	-	22.1	-	-	-	-	-	-	-	-	-	-	-	5.8	2194.6
-	-	-	-	37.9	-	-	-	-	-	-	-	-	-	-	-	14.6	5579.2
-	-	-	-	25.4	-	-	-	-	-	-	-	-	-	-	-	4.4	1689.4
-	-	-	-	69.9	-	-	-	-	-	-	-	-	-	-	-	26.0	9886.6
-	-	-	-	307.0	-	-	-	-	-	-	-	-	-	-	-	6.3	2406.8
-	-	-	-	1284.0	-	-	-	-	-	-	-	-	-	-	-	-	16776.4
262.4	274.3	148.3	3248.5	-	-	-	-	-	-	-	-	-	-	7271.9	3292.7	729.4	21604.1
210.5	-154.6	-102.8	13527.9	7660.8	-	-	-	-	-	-	-	-	-	-	-	5229.4	28660.3
-	-	-	-	-	2636.0	724.5	22.8	5908.5	15041.5	3.6	412.8	701.3	249.8	-	-	2321.5	42281.2
-	-	-	-	-	55.7	18.1	43.9	3.1	1826.4	35.7	173.7	4.6	5.6	-	-	1393.4	3686.0
-	-	-	-	-	-	-	-	549.0	-	-	-	224.5	47.1	-	-	9194.3	10868.7
-	-	-	-	-	13250.1	4066.7	190.9	81.0	24164.6	453.6	9202.3	3664.4	1594.9	-	-	2656.7	103979.9
-	-	-	-	-	6.4	6.6	106.2	4.9	576.1	43.5	221.1	85.3	55.7	-	-	-	204.0
-	-	-	-	-	12633.4	3573.2	725.6	576.0	18371.7	112.1	12214.4	233.2	668.3	-	-	2932.2	69797.0
-	-	-	-	-		6.0	2.0	1.1	90.7	9.8	416.9	79.2	162.0	-	-	-	15291.0
-	-	-	-	9961.6	78.8	23.0	10.9	3.3	158.0	4.0	88.9	11.4	9.9	-	-	17.7	11123.2
-	-	-	-	-	-	-220.6	-346.3	3072.2	3486.7	-11.7	1169.0	113.0	9.6	-	-	-	7271.9
-	-	-	-	-	-	116.5	4.3	-	3160.3	-	8.1	-	3.6	-	-	-	3292.7
-	32.4	-	-	58.9	-	910.3	43.4	-	16851.3	-	2714.3	87.0	33.2	-	-	-	21149.8
1689.4	9886.6	2406.8	16776.4	21604.1	28660.3	42281.2	3686.0	10868.7	103979.9	204.0	69797.0	15291.0	11123.2	7271.9	3292.7	21149.8	-

are entered in row 22 and cols. 10 to 21. Furthermore, on the intersect with col. 33 the balance of payments deficit provides the balance with gross investments demand, which is captured in col. 22 and rows 23 to 30.

The row and columns of the activities account form the well-known input-output structure. Reading row-wise we encounter the different final demand categories. Reading the columns of the activities account from top to bottom we encounter factor payments intermediate costs,indirect taxes, and imports.

The last columm to consider is the rest of the world account.

3. Estimation

In order to obtain consistency within the SAM the account aggregates in Table 1 were made consistent with the National Account Statistics as published by the Banco de la Republica (BR).The main reason to use the national accounts as the binding blocks of this statistical application is the fact that national accounts form the reference framework for national policy making. Besides, the model which is behind the matrix had to work with time series based on national accounts.

Use of BR sources in constructing the aggregates in Table 1 implied rescaling statistics from other data sources to fit into these aggregates but use the other sources to fill the inner structures of the aggregates, as done in Table 1. In particular, there are two main types of other data sources concerned.

The first type of data used is the input-output table based on the manufacturing survey and published by Departamento Administrativo Nacional de Estadistica (DANE). This gives slightly higher final demand and factor cost, about 2% more, although BR and DANE show differences regarding certain categories of final demand of between plus and minus 6% [3].

The second type of data used is the Household Income and Expenditure Survey of DANE, 1971. This overestimates total household income and expentidure by margins of 6% and 14%; respectively. The survey estimates have been downscaled accordingly.

The data of the household income and expenditure survey were used to fill the inner structure of the cross accounts of households/factors and wants/household. In filling the incomings in the household account it was necessary to keep at zero the receipts of households from other households and from firms and to assume that household receipts from the rest of the world

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3) Details are found in discussion paper, cf. Cohen and Jellema (1986).

are distributed on household types in proportion to each household group income. These assumptions reflect the general lack of data on income transfers between household groups. These assumptions cleared the way for using taped cross-tabulations of factor income by household group to fill the entries of households/factors, indeed, after rescaling to fit the national accounts. As for the outgoings the survey provided the required data to fill consumption expenditure and direct taxes on household groups, after rescaling to fit the national accounts. The difference between income and expenditure for each household group constitute entries in the capital account. Furthermore, groups with negative residuals, i.e. dissavings, were assumed to incur no outgoings to the rest of the world. Outgoings are proportionately distributed among households with positive residuals, i.e. savings, on the basis of their income.

Finally, the sub-matrix which converts private consumption categories belonging to the wants account to final demand categories in the activities account has been made consistent by applying the RAS-method to a converter matrix obtained from DANE and the already found column and row totals of private consumption and final demand categories, respectively.

4. Predictions

In the input-output analysis an endogenous vector of sectoral production, P, can be predicted from a matrix of input-output coefficients, A, and a vector of exogenous final demand, F, as in eq. 1.

$$P = AP + F = (I-A)^{-1} F \tag{1}$$

The SAM can be used similarly, with the obvious difference that the SAM contains more variables and relationships. To transform the social accounting matrix into a predictive model along the above lines requires performing several steps.

First, the accounts of the SAM need to be subdivided into endogenous and exogenous and regrouped accordingly so that the exogenous accounts would fall to the right bottom of the endogenous accounts.

Following an established convention, which coincides with the focus of the paper, the endogenous accounts would count four categories:

1. Wants, rows and columns 1 to 6
2. Factor incomes, rows and columns 7 to 9
3. Households and firms, rows and columns 10 to 20
4. Production activities, rows and columns 23 to 30

These endogenous accounts form a 28 * 28 submatrix within the regrouped SAM, containing all the flows from endogenous accounts to endogenous accounts.

The outgoings of other accounts constitute a 28 * 5 submatrix to the right which contains flows of sectoral export and investment demands and income transfers from the rest of the world and government. These are exogenous outgoings and can be summed into one exogenous vector.

To the bottom of the endogenous accounts is a submatrix that contains the outgoings of the endogenous accounts into the other accounts, i.e. imports, taxes and savings. These residual balances need not be treated further here.

Secondly, the flows in the endogenous accounts need to be expressed as average propensities of their corresponding column totals. Thus each flow in the 28 * 28 matrix is divided by its respective column total to give the matrix of average propensities, denoted by A.

As a result of the above manipulations the SAM takes the form of Table 2. Note that the A matrix appears in a partitioned form to facilitate a decomposition of the multipliers in the next section. The vector of row totals, y, represents the endogenous variables. While the vector x represents the exogenous variables.

The vector of endogenous variables y can now be solved from eq. 2

$$y = Ay + x = (I-A)^{-1}\ x = M_a x \qquad (2)$$

where M_a is the aggregate multiplier matrix.

Given A, and x for the period 1970-75, in constant prices of 1970, the predicted values of the 28 endogenous variables for 1970-75 are obtained. Table 3 gives the predicted and realized growth rates over 1970-75 for selected variables. The list is not comprehensive as the only readily available data for which the predictive ability of the SAM can be compared relate to gross domestic product by production activity.

The same reasons which underlie the poor ability of the traditional input-output model to predict the value added apply also for the SAM. Furthermore, the generation of additional endogenous accounts by making use of constant coefficients in the SAM reduce the ability of the SAM to function as a predictive tool. Table 3 demonstrates some of these facts. The SAM predicts a total GDP over five years of 20 percent, or about 3.7 percent per annum, compared to a realized growth of 4.9 per cent per annum. One should note, of course, that macro-economic forecasts for the year 1971-75 have been specially weak for most countries.

Predictions of the institutional incomes are underestimates, as well. However, the predicted growth differences between the household groups provide important information which is otherwise not available. In particular, it is noted that the rural households have benefited relatively more than the urban households. Among the rural population the rural land-owners have had the lowest income growth. The incomes of the urban households show a more

Table 2 The SAM in the form of $y = Ay + x$

expenditures / receipts		1. Wants account	2. Factors account	3. Institutions account	4. Activities account	exogenous Account: Government, Capital & rest of the world	Totals
endo-genous	1. wants			A_{13}		X_1	Y_1
	2. factors				A_{24}	X_2	Y_2
	3. Institutions		A_{32}	A_{33}		X_3	Y_3
	4. Activities	A_{41}			A_{44}	X_4	Y_4
exo-genous	Others	residual balance					
Totals		Y_1'	Y_2'	Y_3'	Y_4'		

Table 3. Realized and predicted growth rates of the GDP by sectors and income variables, in constant prices of 1970 over the period 1970-75, in percentage.

GDP by sector	Y	Realized	Predicted	Income by Institution	Y	Predicted
Agriculture	21	17.08	22.01	U. Employed	10	20.56
Mining	22	72.90	24.30	U. Workers	11	21.09
Coffee	23	12.67	32.02	U. Capitalist	12	20.44
Industry	24	34.15	19.01	U. Self empl.	13	20.46
Energy	25	34.02	20.60	U. inactive	14	20.47
Modern Services	26	35.28	17.38	R. Employed	15	27.49
Personal Services	27	32.53	21.83	R. Workers	16	27.49
Government services	28	35.50	26.94	R. Capitalist	17	23.62
Total		27.01	20.21	R. Self empl.	18	27.54
				R. inactive	19	27.59
				Firms	20	21.31

harmonious growth. Of course, in view of the systematic errors in the prediction of GDP by sector, there are also bound to be systematic errors in the prediction of institutional incomes. For instance, the lower realized GDP growth for agriculture would imply lower income growth for the rural households than the SAM predicts.

5. Multiplier analysis

The case for the SAM as an analytical tool does not lie so much in its predictive ability, as was shown above, as in the study of the underlying economic structure via an analysis of its inverse multipliers and their decomposition, which will be the focus of this section.

Recalling eq. 2, the aggregate multiplier matrix M_a can be decomposed into three multiplier matrices M_1, M_2, M_3, as in eq. 3.

$$y = A_y + x = (I-A)^{-1}x = M_a x = M_3\ M_2\ M_1 x \tag{3}$$

In terms of the SAM, M_1, which is known as the transfer multiplier, captures effects resulting from direct transfers within endogenous account (i.e. the effect of an impulse in demand for sector i on the production of sectors i and i', or, in other words, the Leontief multipliers). The open-loop effects, M_2, capture the interactions among and between the endogenous accounts (i.e. from the production account to factor. institution and wants accounts). The closed-loop effects, M_3, ensure that the circular flow of income is completed among the endogenous accounts (i.e. from production activities to factors to institutions and in the form of final consumption back to activities, and again and again).

The formal derivation of the decomposed multipliers proceeds with separating out matrix $\tilde{A}$ from A, provided that $\tilde{A}$ is of the same size as A and that $(I - \tilde{A})^{-1}$ exists.

$$\begin{aligned} y &= Ay + x \\ &= (A-\tilde{A})\ y + \tilde{A}y + x \\ &= (I-\tilde{A})^{-1}\ (A-\tilde{A})\ y + (I-\tilde{A})^{-1}\ x \\ &= \qquad A^{*}\ y + (I-\tilde{A})^{-1}\ x \end{aligned} \tag{4}$$

Here, $(I-\tilde{A})^{-1}$ refers to the transfer multiplier, i.e. Leontief multipliers, M_1. Derivation of M_2 and M_3 proceeds further as described in the appendix. Because the multiplier matrix M_a and its partition into M_1, M_2 and M_3 can be extensive, it is instructive to limit the presentation here to the aggregate and decomposed impacts of exogeneous injections in sectoral activities. Of course, it is possible to pursue the impact of institutional transfers, but this is left for another occasion.

Table 4. SAM Aggregate Multipliers by Type of Activity

COLOMBIA	21	22	23	24	25	26	27	28
1. Food	0.8808	0.9411	0.6124	0.6252	0.8638	0.8059	0.8700	1.0221
2. Other goods	0.4077	0.4460	0.2852	0.2962	0.4173	0.3855	0.4141	0.4846
3. Housing	0.2686	0.3022	0.1893	0.2004	0.2859	0.2539	0.2815	0.3275
4. Health	0.0597	0.0650	0.0417	0.0432	0.0608	0.0561	0.0605	0.0712
5. Education	0.0712	0.0802	0.0502	0.0531	0.0760	0.0700	0.0746	0.0865
6. Other services	0.1437	0.1608	0.1011	0.1061	0.1510	0.1396	0.1482	0.1701
7. U. labour income	0.4983	0.6235	0.3634	0.4678	0.7529	0.6200	0.7571	1.1372
8. R. labour income	0.3474	0.2783	0.2262	0.1908	0.2177	0.2109	0.2549	0.3384
9. Capital income	1.5329	1.7075	1.0747	1.0505	1.4194	1.4032	1.3411	1.1634
10. Salary earners /u.	0.5680	0.6583	0.4035	0.4366	0.6352	0.5814	0.6173	0.7157
11. Wage earners /u.	0.0968	0.1211	0.0705	0.0909	0.1462	0.1204	0.1470	0.2209
12. Employers /u.	0.0814	0.0934	0.0576	0.0608	0.0868	0.0810	0.0839	0.0917
13. Self employment /u.	0.5075	0.5833	0.3595	0.3810	0.5466	0.5077	0.5285	0.5349
14. Family helpers /u.	0.2129	0.2459	0.1510	0.1621	0.2344	0.2159	0.2274	0.2588
15. Salary earners /r.	0.0520	0.0485	0.0350	0.0316	0.0392	0.0383	0.0413	0.0459
16. Wage earners /r.	0.1374	0.1101	0.0895	0.0755	0.0851	0.0834	0.1008	0.1338
17. Employers /r.	0.0390	0.0397	0.0267	0.0251	0.0325	0.0320	0.0324	0.0321
18. Selfemployers /r.	0.2350	0.2191	0.1550	0.1425	0.1768	0.1731	0.1864	0.2077
19. Family helpers /r.	0.0504	0.0463	0.0338	0.0303	0.0373	0.0364	0.0397	0.0451
20. Firms	0.3551	0.3955	0.2490	0.2433	0.3288	0.3251	0.3107	0.2695
21. Agriculture	1.5737	0.6004	0.9710	0.5976	0.5835	0.5337	0.6379	0.6819
22. Mining	0.0348	1.0495	0.0248	0.0483	0.0560	0.0366	0.0384	0.0417
23. Coffee	0.0224	0.0241	1.0688	0.0160	0.0224	0.0208	0.0373	0.0308
24. Industry	1.6098	1.7025	1.1272	2.4351	1.7949	1.5808	1.8056	1.9301
25. Elect., gas & w.	0.0336	0.0669	0.0243	0.0332	1.0581	0.0363	0.0410	0.0449
26. Modern services	0.9332	1.1488	0.7095	0.9055	0.9709	2.0243	0.9183	1.0742
27. Pers. services	0.2901	0.3213	0.2038	0.2143	0.3056	0.2842	1.3023	0.3611
28. Govt. services	0.0195	0.0240	0.0140	0.0163	0.0219	0.0195	0.0204	1.0235

Table 4 gives accordingly the relevant aggregate multipliers within M_a. Specifically, they fall into four compartments.

$M_{a,14}$, $M_{a,24}$, $M_{a,44}$ corresponding respectively to the impacts of injections in activities (subindex 4) on wants (subindex 1), on factors (subindex 2), on institutions (subindex 3), and on activities (subindex 4).

Taking up the <u>first compartment</u>, Table 4 rows 1 to 6, the impact of allocations to activities on the wants account, it is striking to note the relatively high impact of services on food, which surpasses that of agriculture on food. The dominating impact of services, as compared to other sectors, is generally established for other wants categories, too. As can be expected, in terms of impact, the consumption of food is followed by that of other goods, housing, education, and health, reflecting their decreasing shares in consumption expenditure.

Taking up the <u>second compartment</u>, rows 7 to 9, which relates to the impact of allocations on the factor accounts, it is found that, row-wise, labour income is highly affected by expansion in services activities. Other sectors with significant effects are mining and agriculture. Capital income is mostly affected by expansion in the agricultural activity, and equally so or followed by mining and services. Column-wise, the results show the multiplier ratio of labour income to capital income in Colombia to be highest in government services and lowest in electricity and in agriculture which are capital intensive and land extensive, respectively. Industry takes an intermediate position with a multiplier ratio of labour income to capital income around 0.6.

The attention can be shifted now to the analysis of the third compartment, $M_{a,34}$, leaving the analysis of $M_{a,44}$ to the next section. A decomposition of $M_{a,34}$, into its transfer, open and closed multiplier effects requires an analyses of only three submatrices, as in eq. 5.

$$M_{a,34} = M_{3,33} * M_{2,34} * M_{1,44} \tag{5}$$

overall= closed * open * transfer

Tables 5, 6 and 7 give $M_{1,44}$, $M_{2,34}$ and $M_{3,33}$, respectively.

Table 5 which contains $M_{1,44}$ captures the well-known transfer within the input-output accounts. Row-wise, transfer effects are particularly important in the sectors of industry, modern services and agriculture; due to lesser linkages, the multipliers for the other sectors do not exceed .05 disregarding the initial injections. Column-wise, the highest impacts are produced by injections in industry and coffee. The lowest relates to agriculture. The first column of table 5 shows, for instance, an initial injection in agriculture of 1.0 to result in an addition in agriculture of 4.11%, mining 0.36%, industry 15.6%, energy 0.16%, modern services 14.11%, and so on. The original injection of 1.0 leads to a total increase of 1.3459. These transfer effects, will be traced through the rest of the system, $M_{2,34}$ and $M_{3,33}$ in order to illustrate the working of the system.

Table 6 which presents $M_{2,34}$ captures open-loop effects. The highest open-loop effects are those in the column of agriculture for rural households. This pattern is the result of the high concentration of rural factor income in agriculture and the dominating link between rural factors and rural households. In a similar way, the mining sector benefits the urban employers and capitalists while the government sector benefits the other urban households. The lowest open-loop effects are to be encountered in the column of the coffee sector, in which an impulse of 1.0 yields only .0494 of total endogenous institutional income.

The closed-loop multipliers as captured in $M_{3,33}$, Table 7, are associated with the consumption patterns of the households. The increases in income resulting from open-loop effects are used mainly to purchase consumer goods, which increases output, and in its turn, increases factor income that is paid out as institutional income. Reading Table 7 rowwise, and excluding the initial injections and the few exceptions, it appears that the closed-loop

Table 5.: $M_{1,44}$ Transfer Effects in Sectoral Accounts.

	21	22	23	24	25	26	27	28
Agriculture 21	1.0414	0.0275	0.6002	0.2071	0.0524	0.0417	0.1077	0.0600
Mining - 22	0.0036	1.0156	0.0030	0.0258	0.0245	0.0074	0.0070	0.0050
Coffee - 23	0.0000	0.0001	1.0532	0.0001	0.0001	0.0001	0.0156	0.0047
Industry - 24	0.1560	0.1299	0.1133	1.3908	0.3336	0.2279	0.3495	0.2241
E.G.W. - 25	0.0016	0.0316	0.0018	0.0098	1.0249	0.0057	0.0083	0.0071
Mod.Serv. - 26	0.1411	0.2754	0.1543	0.3262	0.1512	1.2676	0.1078	0.1293
Pers.Serv.- 27	0.0012	0.0023	0.0012	0.0033	0.0062	0.0079	1.0062	0.0157
Gov.Serv. - 28	0.0010	0.0036	0.0010	0.0027	0.0023	0.0020	0.0015	1.0015
Total	1.3459	1.4862	1.9228	1.9658	1.5957	1.5603	1.6036	1.4474

Table 6.: $M_{2,34}$ Openloop Effects from Sectors to Institutions.

	21	22	23	24	25	26	27	28
U.Empl. - 10	0.1608	0.1896	0.0162	0.0537	0.1980	0.1672	0.1870	0.2326
U.Worker - 11	0.0125	0.0233	0.0023	0.0144	0.0549	0.0341	0.0583	0.1204
U.Capit. - 12	0.0247	0.0282	0.0024	0.0071	0.0260	0.0233	0.0239	0.0244
U.Selfempl- 13	0.1519	0.1743	0.0147	0.0451	0.1652	0.1463	0.1524	0.1632
U.Inactive- 14	0.0617	0.0719	0.0061	0.0196	0.0722	0.0621	0.0675	0.0794
R.Empl. - 15	0.0224	0.0153	0.0009	0.0029	0.0078	0.0089	0.0095	0.0108
R.Worker - 16	0.0663	0.0318	0.0010	0.0066	0.0112	0.0140	0.0239	0.0493
R.Capit. - 17	0.0155	0.0130	0.0009	0.0024	0.0075	0.0084	0.0073	0.0043
R.Selfempl- 18	0.1012	0.0690	0.0041	0.0132	0.0350	0.0401	0.0429	0.0489
R.Inactive- 19	0.0220	0.0145	0.0008	0.0028	0.0071	0.0032	0.0092	0.0113
Firms - 20	0.1273	0.1347	0.0109	0.0241	0.0857	0.0944	0.0692	0.0000
Total	0.7663	0.7656	0.0603	0.1919	0.6706	0.6070	0.6565	0.7456

Table 7.: $M_{3,33}$ Closedloop Effects between Institutions.

	10	11	12	13	14	15	16	17	18	19	20
U.Empl. - 10	1.4689	0.5094	0.4550	0.4792	0.4543	0.4578	0.5220	0.3583	0.4952	0.4904	0.0000
U.Worker - 11	0.0983	1.1059	0.0988	0.1001	0.0945	0.0951	0.1066	0.0745	0.1017	0.0998	0.0000
U.Capit. - 12	0.0652	0.0709	1.0645	0.0666	0.0632	0.0637	0.0729	0.0499	0.0691	0.0685	0.0000
U.Selfempl.- 13	0.4088	0.4446	0.4047	1.4179	0.3965	0.3996	0.4567	0.3132	0.4329	0.4292	0.0000
U.Inactive - 14	0.1740	0.1891	0.1724	0.1778	1.1686	0.1699	0.1939	0.1332	0.1839	0.1822	0.0000
R.Empl. - 15	0.0339	0.0374	0.0331	0.0348	0.0329	1.0338	0.0394	0.0264	0.0371	0.0369	0.0000
R.Worker - 16	0.0815	0.0908	0.0789	0.0839	0.0789	0.0822	1.0966	0.0640	0.0909	0.0905	0.0000
R.Capit. - 17	0.0269	0.0295	0.0263	0.0276	0.0261	0.0266	0.0308	1.0208	0.0291	0.0290	0.0000
R.Selfempl - 18	0.1531	0.1690	0.1493	0.1573	0.1487	0.1526	0.1778	0.1192	1.1676	0.1669	0.0000
R.Inactive - 19	0.0325	0.0360	0.0317	0.0334	0.0316	0.0325	0.0379	0.0253	0.0357	1.0355	0.0000
Firms - 20	0.2600	0.2841	0.2555	0.2563	0.2532	0.2555	0.2947	0.2003	0.2785	0.2774	0.0000
Total	2.8030	2.9667	2.7803	2.8449	2.7485	2.7743	3.0293	2.3856	2.9217	2.9063	

multipliers are fairly constant. This can be interpreted as the result of similar expenditure and savings patterns over households. The closed-loop multipliers are generally much higher than either the transfer or open-loop multipliers, which reflects the fact that consumption is larger than other categories of final demand. An open-loop effect of 1.0 into any household creates between 2.3856 and 3.0293 of total institutional income. The national impact for transfer effects ranged between 1.3459 and 1.9658, while that for open loop-effects varied between .0603 and .7663, in Tables 5 and 6, respectivily. Being higher than the other multipliers and given their low variance, the closed-loop multipliers tend to dampen the effects of the transfer and open-loop multipliers.

Table 8 summarizes the above interactions in a compact form. Columns 1 and 2 give the combined effects on the incomes of rural worker households and all others of the transfer and open-loop multipliers following the exogenous initial injection in agriculture of one million pesos. Columns 3 and 4 complement the picture by introducing closed loop effects. Column 5 gives the

Table 8. Effects of +1 in agricultural activity on institutional incomes, Colombia.

Transfer Effect coefficient for Agriculture = 1.041

	Open loop coefficients Agriculture	Sum open loop effects all sectors	Closed loop closed coefficients		Sum lated loop effects	Simu- % %	Actual
			R small	Others			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Rural worker hh.	.066	.072	1.097	.076	.137	5.8	3.5
All others	.7	.765	1.792	2.704	2.198	94.1	96.5
Total	.766	.837	2.889	2.780	2.335	100.0	100.0

overall effects, which when summed, result into the overall multiplier for rural workers, as was found in Table 4, $Ma_{,34}$ (16,21), i.e. .137 units. Similarly, the overall multipliers can be obtained for other household groups and firms, resulting in a total overall multiplier effect of 2.335 units.

These results suggest that the marginal share of benefits to rural workers from agricultural expansion amounts to about 5.8 per cent (column 6). Since the income share of rural workers in 1970 amounted to 3.5 per cent (column 7), it can be expected that an injection in agriculture has the effect of enhancing the relative position of rural workers in the income distribution.

The percentage distributions of the multiplier benefits - the adapted multipliers - are pursued further in table 9. The table is selective as it gives adapted overall and openloop multipliers of initial injections in agriculture, mining, energy and government services. These four sectors happen to have the highest multiplier effects. The table gives also adapted closed-loop multiplier for the urban self-employed and the rural workers, which are the largest population groups in rural and urban locations, respectively. To assess the marginal effect of the adapted multipliers on income distribution, column 1 of Table 9 gives the actual income shares in 1970, while column 2 gives the actual household shares in all households and column 3 gives the actual income per household relative to an average of all households of 1.0. These columns indicate that lower rewards for urban salaried employers, capitalists, self-employed, inactive and rural capitalist (in that decreasing order) and/or higher rewards for urban workers and the rural population represent a movement towards more equality at the individual household level.

The overall multiplier of an injection in agriculture promotes a redistribution of income from urban to rural population groups, the multiplier of mining points in the same direction but less significantly. The multiplier of energy distributes relatively more to urban than to rural households. The multiplier of the government sector appears to benefit household groups in various degrees at the cost of firms.

The adapted multipliers can be partially decomposed in terms of adapted open - loop and closed-loop multipliers. The relative redistributionary effects of the open-loop multipliers as represented by the adapted $M_{2,34}$ appear to be very significant. Agricultural benefits are shown to be shared among rural and urban populations in proportions of .53 and .30, while the actual income shares are distributed in the proportions of .69 to urban and .15 to rural. $M_{2,34}$ shows injections in mining to promote equality too, while those in energy and government increase inequality. These significant redistributionary effects, whether they are positive or negative, are muffled by the closed loop effects, $M_{3,33}$, which hardly deviate from the actual income distribution. Table 9 shows the multiplier effect for the two largest urban and rural population groups to divide among all urban and all rural according to the actual shares of .69 and .15. About the same results are obtained from whatever group one starts. The gains obtained on the factor account by one group are lost via consumption to other groups.

We may recall the discussion on the vanishing income redistribution effects [4]. In the present context vanishing effects are the result of the

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4) For instance Taylor and Lysy (1979).

Table 9. Percentage distribution of the multiplier benefits of the SAM and actual percentage distributions in 1970.

Household		Situation 1970			Adapted $M_{a,34}$			
		Income shares	Household shares	Relative income per household	21 Agriculture	22 Mining	25 Energy	28 Govt. Services
U. Employees	10	26.36	19.02	1.39	24.3	25.7	27.0	27.5
U. Workers	11	5.86	8.82	.66	4.1	4.7	6.2	8.5
U. Capitalists	12	3.60	1.40	2.57	3.5	3.7	3.7	3.5
U. Selfemployed	13	22.67	20.12	1.13	21.7	22.8	23.3	22.4
U. Inactive	14	11.04	10.16	1.09	9.1	9.6	10.0	9.9
R. Employees	15	1.63	2.98	.55	2.2	1.9	1.7	1.8
R. Workers	16	3.48	12.59	.28	5.9	4.3	3.7	5.1
R. Capitalists	17	1.37	1.32	1.04	1.7	1.6	1.4	1.2
R. Selfemployed	18	7.32	18.82	.39	10.1	8.6	7.5	8.0
R. Inactive	19	1.79	4.77	.38	2.2	1.8	1.6	1.7
Firms	20	14.87			15.2	15.4	14.0	10.3

Household	Adapted $M_{2,34}$				Adapted $(M_{3,33}-I)$	
	21 Agriculture	22 Mining	25 Energy	28 Govt. Services	13. Urban Selfemployed	16. Rural workers
U. Employees	21.0	24.8	29.5	31.2	26.0	25.7
U. Workers	1.6	3.0	8.2	16.2	5.4	5.3
U. Capitalists	3.2	3.7	3.9	3.3	3.6	3.6
U. Selfemployed	19.8	22.8	24.6	21.9	22.7	22.5
U. Inactive	8.1	9.4	10.8	10.7	9.6	9.6
R. Employees	2.9	2.0	1.2	1.5	1.9	1.9
R. Workers	8.7	4.2	1.7	6.6	4.6	4.8
R. Capitalists	2.0	1.7	1.1	.6	1.5	1.5
R. Selfemployed	13.2	9.0	5.2	6.6	8.5	8.8
R. Inactive	2.9	1.9	1.1	1.5	1.8	1.9
Firms	16.6	17.6	12.8	0	14.4	14.5

Note Col. 1 = $Y/\Sigma Y$ where Y is income. Col. 2 = $P/\Sigma P$ where P is number of households, Col. 3 = Col. 1/Col. 2. Col. 4 to 7, 8 to 11, and 12 and 13 are adapted from Tables 4, 6 and 7.

interaction between three factors, (1) relatively weak transfer effects of agriculture which is the potential sector for a sustained positive redistributionary effect, (2) sector-factor links producing multipliers which are not very significant and insufficiently discrimininant, and (3) very significant leakage from poorer to richer household groups - and otherwise - via their expenditure patterns.

In general, agricultural multipliers show more progressive redistributionary effects than industrial multipliers. It is also shown that the aggregate multipiers of injections in an activity on all activities is higher for agriculture than for industry (Table 4, last quarter), so that, as far as these two sectors are concerned, progressive resitribution and higher growth can go hand in hand. The results direct attention to the presence of degrees of freedom in the selection of balanced socio-economic development policies in spite of the existence of countervailing mechanisms which cause parts of the redistribution and growth potentials to vanish.

Finally, the fourth compartment of Table 4 which shows $M_{a,44}$ gives the aggregate multipliers of injections in activities on activities. In similarity with the previous discussion these aggregate multipliers can be decomposed into their transfer, open-loop and closed-loop effects. But since open-loop effects are not applicable here, an analysis of the differences between the aggregate multiplier and that part of them which forms the transfer effects is sufficient to appreciate the nature of the remaining part which forms the closed-loop effects.

The aggregate multipliers of $M_{a,44}$ can be confronted now with the previously discussed transfer effects of activities on activities , $M_{1,44}$ found in Table 5. The latter represents the simpler inverse of Leontief.

As can be expected, first, the SAM contains more linkages than the Leontief with the result that $M_{a,44}$ is substantially higher than $M_{1,44}$. Secondly , due to the heterogeniety of the linkages the structural pattern of $M_{a,44}$ is also different from $M_{1,44}$.

The first point can be illustrated from Table 10 which gives the frequency distribution of the size of the aggregate multipliers and the transfer effects, or the SAM-inverse and the Leontief inverse, respectively. The percentage of elements with negligible sizes which form the great majority in the Leontief-inverse is significantly reduced in the SAM-inverse reflecting the incorporation of many more indirect effects and additional interdependencies in a social accounting framework. Summing up elements < 0.2 gives a percentage of 86 in Leontief and only 50 in SAM.

The second point can be illustrated from Table 11. Sectors are ranked according to the Leontief total column multipliers in the order of industry

1.97, mining 1.49, services 1.45 and agriculture 1.35. The contribution to production activity and their ranking are significantly different in the SAM total column multipliers: services 5.19, mining 4.94, agriculture 4.52 and industry 4.26. Policy-making regarding allocations to activity with the object of achieving highest growth would have made wrong decisions in a structural analysis based on the Leontief framework as compared to a SAM framework. The results should not be interpreted to mean that if Colombia, for instance, would have expanded in the past, relatively more in agriculture, mining or services than in industry it would have necessarily achieved a higher overall growth. For one thing, the exogenous expansion potential both domestically and in the rest of the world- denoted by x - was and is probably lower for the non-industrial sectors than for industry. Besides, both the SAM and the input-output do not consider limits on the supply side which are likely to be more demarcated for agriculture than for industry in most developing countries.

Table 10. Size distributions of the off-diagonal elements of the SAM-inverse and the Leontief-inverse in percentages; i.e. aggregate multipliers and transfer effects, respectively

element size	SAM/ aggre. multip.	LEONT/ trans effect
< 0.050	46.4	66.0
0.051 - 0.100	3.6	3.6
0.101 - 0.150	0.0	10.7
0.151 - 0.200	0.0	5.4
0.201 - 0.250	3.6	5.4
0.251 - 0.500	8.9	7.1
0.501 - 1.000	21.4	1.8
< 1.000	16.1	0.0

Source: Tables 4 and 5.

Table 11. Own multipliers and total column multipliers in the SAM-inverse and the Leontief-inverses i.e. aggregate multipliers and transfer effects, respectively

sector	SAM/ aggre. multip.	LEONT/ trans effec
Agriculture		
1. own	1.57	1.04
2. total	4.52	1.35
Mining		
1. own	1.05	1.02
2. total	4.94	1.49
Industry		
1. own	2.44	1.39
2. total	4.26	1.97
Services		
1. own	1.02	1.00
2. total	5.19	1.45

Source: Tables 4 and 5.

As is well known, labour use and capital use per unit of additional production can be multiplied by the contribution to production activities to give the employment and investment effects. It is obvious that, given the above, the impact of alternative allocations to activities on the marginal use of labour and capital would be less meaningful when they are derived form the partial framework of Leontief's transfer effects than when they are derived from a more general framework which incorporates SAM's aggregate effects.

7. Conclusions

In the construction of the SAM for Colombia, as was experienced in other case studies, varied procedures had to be followed to accommodate the available data within the framework.

Although use of the SAM for predictive purposes suffers from the same limitations of input-output applications, a SAM-model generates values for income variables which are otherwise not simple to trace.

In addition to its use for purposes of calibrating economy-wide models, it is demonstrated that the SAM forms an appropirate framework for a static analysis of the structural properties of the socio-economic system. The obtained multipliers are found to be meaningful. To illustrate we sum here a few selected results on the aggregate multipliers and their decomposition into transfer, open-loop and colosed-loop effects.

The results show the presence of slightly progressive mechanisms in Colombia.The progressiveness could have been higher if not for the weak correspondences between activities, factors, and housholds,and the very similar consumption patterns among richer and poorer household groups in Colombia.

In general, agricultural multipliers show more progressive restributionary effects than industrial multipliers. It is also shown that the growth effect is higher for agriculture than for industry, so that as far as these two sectors are concerned progressive redistribution and higher growth are not in conflict. Other results direct attention to the presence of degrees of freedom in selecting balanced socio-economic development policies in spite of the existence of countervailing processes which cause parts of the redistribution and growth potentials to vanish.

Furthermore, multiplier results obtained from the SAM differ appreciably from those derived from that part of the SAM which corresponds to the Leontief inverse; this in addition to the fact that the SAM framework provides information on more dimensions of welfare than the Leontief model. This leads to concluding that when a development problem involves significant linkage effects regarding income and expenditure, such as the analysis and planning of investment and growth strategies, it can be expected that results from the input-output model will be inferior to those from the social accounting framework. Although obvious, it is emphasised that this conclusion does not deny the recognised usefulness of the input-output model in other analytical and planning contexts than those described in the paper.

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Appendix - Multiplier decomposition

Specifically, the SAM can be written as a partitioned coefficient matrix as in table 4.

$$A = \begin{bmatrix} o & o & A_{13} & o \\ o & o & o & A_{24} \\ o & A_{32} & A_{33} & o \\ A_{41} & o & o & A_{44} \end{bmatrix} \qquad (1)$$

A_{13} represents the intersection between wants and households and firms, A_{24} for those between factors and activities, etc. From A separate $\tilde{A}$ and invert to obtain M_1, as in eq. 2.

$$\tilde{A} = \begin{bmatrix} o & o & o & o \\ o & o & o & o \\ o & o & A_{33} & o \\ o & o & o & A_{44} \end{bmatrix} \quad M_1 = (I - \tilde{A})^{-1} = \begin{bmatrix} I & o & o & o \\ o & I & o & o \\ o & o & (I-\tilde{A}_{33})^{-1} & o \\ o & o & o & (I-\tilde{A}_{44})^{-1} \end{bmatrix} \qquad (2)$$

It is noted that $(I-A_{44})^{-1}$ is nothing more than the Leontief-inverse from the simple sectoral models. It translates original exogenous impulses in final demand into sectoral output. It does not take into account the impact of the composition of endogenous final demand. $(I-A_{33})^{-1}$ fulfills the same role with regards to institutions. It calculates the first round effect of an exogenous increase in institutional income through the transfer mechanisms between the different institutions.

As a result of the separation in eq. 2, we have A*

$$A^* = \begin{bmatrix} o & o & A^*_{13} & o \\ o & o & o & A^*_{24} \\ o & A^*_{32} & o & o \\ A^*_{41} & o & o & o \end{bmatrix} \quad \text{where:} \quad \begin{aligned} A^*_{13} &= A_{13} \\ A^*_{24} &= A_{24} \\ A^*_{32} &= (I-A_{33})^{-1}A_{32} \\ A^*_{41} &= (I-A_{44})^{-1}A_{41} \end{aligned} \qquad (3)$$

A* shares some of the properties of a permutation matrix. It contains only one block of non-zero entries within each set of rows and each set of columns. Raising such a matrix to the k-th power does not alter this property, it only shifts the position of each block. Since all blocks shift at the same time, there are only four permutations possible with different positions of the blocks.

Given k=4, one can obtain M_2 and M_3 as specified in eqs.4 and 5 , respectively.

$$M_2 = \begin{bmatrix} I & A^*_{13}A^*_{32} & A^*_{13} & A^*_{13}A^*_{32}A^*_{24} \\ A^*_{24}A^*_{41} & I & A^*_{24}A^*_{41}A^*_{13} & A^*_{24} \\ A^*_{32}A^*_{24}A^*_{41} & A^*_{32} & I & A^*_{32}A^*_{24} \\ A^*_{41} & A^*_{41}A^*_{13}A^*_{32} & A^*_{41}A^*_{13} & I \end{bmatrix} \qquad (4)$$

$$M_3 = \begin{bmatrix} (I-A^*_{13}A^*_{32}A^*_{24}A^*_{41})^{-1} & o & o & o \\ o & (I-A^*_{24}A^*_{41}A^*_{13}A^*_{32})^{-1} & o & o \\ o & o & (I-A^*_{32}A^*_{24}A^*_{41}A^*_{13})^{-1} & o \\ o & o & o & (I-A^*_{41}A^*_{13}A^*_{32}A^*_{24})^{-1} \end{bmatrix}$$

(5)

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