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MUDFOLD & UNFOLDING: AN ANALYSIS

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ABSTRACT

This article is a review of W.H. van Schuur's thesis 'STRUCTURE IN POLITICAL BELIEFS, a new model for stochastic unfolding with application to European party activists'. The discussion of this thesis is centered around the MUDFOLD-procedure: the dichotomisation of the data, 'ideal point' items versus 'dominated' items, characteristic monotonicity as a criterion of scalability for a set of items and the null model of MUDFOLD, in particular the suitability of the coefficient of scalability, H. In dichotomising data the metric information is removed from the preference data and the analysis will be a parallelogram analysis. It is claimed that in using dichotomised data MUDFOLD is not suitable for testing the applicability of an unfolding scale. If the J-scale should be rejected by the proposed testing procedure, the item set might contain 'ideal point' items as well as 'dominated' items. There should be a procedure to eliminate the latter. It will be discussed that the H-coefficient might be not the proper test statistic for the null model of stochastic independence of the items. Rejecting the null model of stochastic independence is not equivalent to testing the applicability of the unfolding model. In assessing the suitability of the unidimensional unfolding model the chi-square test of independence should be based on all possible admissible and inadmissible patterns of the J-scale, not exclusively on the patterns in error of all triples of stimuli.

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1. INTRODUCTION

In December 1984 W.H. van Schuur graduated on the thesis 'STRUCTURE IN POLITICAL BELIEFS, A New Model for Stochastic Unfolding with Application to European Party Activists'. In this thesis the political belief structure of more than ten thousand party activists of fifty political parties in nine countries of the European Community are investigated. The data are analysed by means of MUDFOLD ('Multiple UniDimensional unFOLDing') a computer programme developed to unfold preference data. So, on the one hand the thesis is an analysis of the political frame of reference of the political elite of the European Community, on the other hand it is a presentation and an illustration of MUDFOLD. In reviewing this thesis I want to separate these two issues. In section 2 the analysis of political frames of reference will be briefly summarised: I will not go deeper into the politicological section of the thesis. The emphasis of this review will lie on the MUDFOLD-method (section 3) and criticism of MUDFOLD (section 4).

2. ANALYSIS OF POLITICAL FRAMES OF REFERENCE

In Van Schuur's thesis the structure of political opinions in a group of people - party activists - is investigated. The analysis is directed to four kinds of opinions about political problems:

-1- opinions about the importance of political problems
-2- opinions about possible solutions for political problems
-3- opinions about pressure groups (trade unions, employers' organisations, etc.)
-4- opinions about political parties

The 'political belief structure' (the political frame of reference) is defined as the structure within and across these four types of political opinions. There is some structure in the relationships between types of political opinions if e.g. people who think inflation is a more important political problem than unemployment, tend to sympathise with certain employers' organisations. So, structure in the data indicates a certain predictability of the response patterns of individuals.

There are, indeed, objective criteria for the importance of political problems, but they are applied subjectively by people. People (social groups) obviously agree about the objective criteria by which political problems should be judged but they differ in the application of those criteria. In that situation the unfolding model might be suited to describe the structure of opinions about the importance of political problems. Here, the data collection consisted of orderings of the three (out of 15) most important problems. On these data MUDFOLD did not yield a solution; a vector-model did.

Little is known about opinions regarding solutions for political problems. This is true for the degree of structure in the data as well as for the basis for such a structure (e.g. a scale). Such a structure is only found for political elites, for the Netherlands e.g. a progressive-conservative dimension. Here, the data consisted of ratings (five categories), which were dichotomised. For twelve out of the fifteen items a scale was found. The dominance matrix, however, did not show the desired pattern of 'characteristic monotonicity', consequently the unfolding scale was rejected. With the help of 'cumulative scaling' (Mokken scale analysis) two disjoint scales were found, one for the (solution of) more 'progressive' problems and one for the (solution of) more 'conservative' problems. The same structure was found with subgroups of respondents.

Sympathy scores (1-10) for pressure groups and political parties were also dichotomised. For every country a dominant left-right dimension was found. The analysis of sympathy scores with respect to European political parties yielded the same result.

From the analyses with regard to the relations between the four types of political opinions it appears that the left-right (progressive-conservative) continuum is the most important ordering principle.

3. MUDFOLD-ANALYSIS OF PROXIMITY-DATA

In this section we first discuss Coombs' deterministic unfolding model (Coombs, 1964) (3.1), then the MUDFOLD approach (3.2) with the null model (3.3).

3.1 UNFOLDING - THE DETERMINISTIC MODEL

Unfolding is a technique to represent preferences of respondents for a number of stimuli in a multidimensional space. One dimensional unfolding is the technique by which individuals and stimuli are projected on a straight line. The basic assumption of unfolding is that the positions of the stimuli on the continuum are agreed upon, but the preference of individuals for stimuli differs, this preference decreasing as the distance of the individual point to the stimulus point increases. Unfolding data consist of orderings of preference (order k of the n stimuli: 'order k/n' with $k \leq n-1$). The unfolding method results in an ordering of the stimuli and also gives information about the relative size of the <u>distances</u> between the stimuli ('metric information') if $k > 3$. Parallelogram analysis (choose k of the n stimuli: 'pick k/n') only results in an ordering of the stimuli. Unfolding in fact is an extension of parallelogram analysis: with the ordinal information from the orderings, metric information is added to the scale. Given the orderings of preference or the preferential choices of individuals, the aim of unfolding/parallelogram analysis is to test the model and to find the underlying continuum of stimuli and individuals, the 'J-scale'. Locations of individuals are called 'isotonic regions', because a set of points corresponds with an order of preference. From the J-scale 'admissible patterns' of preference can be derived: orders of preference, which correspond to the isotonic regions of the J-scale. Remaining patterns of preference are 'inadmissible' and are not allowed to occur in the ideal case, that of the 'perfect J-scale'. Coombs' unfolding model (Coombs, 1964) is 'deterministic' in the sense that the unfolding model as an explanatory model for orders of preference found, should be rejected as soon as inadmissible patterns of preference occur.

3.2 NON-PERFECT DATA: MUDFOLD

In practice not all individuals supply orderings that can be derived from an unfolding scale. With existing unfolding programmes this results in high stress values and phenomena of 'degeneration'. Van Schuur (1984, p. 52) gives a number of strategies for arriving at a J-scale in the case of non-perfect data:

- dichotomisation of preference data
- a probabilistic representation instead of a deterministic one
- a representation of a maximal subset of stimuli
- a representation of a maximal subset of individuals
- a representation in more dimensions.

Van Schuur chooses for a combination of the first three of the above-mentioned strategies: representation of all individuals on a unidimensional continuum according to their preference for a number of (k) stimuli out of a maximal subset of stimuli. The MUDFOLD-procedure can also be applied in situations in which complete orderings of individuals are not available but only 'ratings', or a limited number of (possibly ordered) choices of individuals ('pick' data or 'rank k/n' data with $k < n-1$). In that sense MUDFOLD is new procedure.

The MUDFOLD-procedure searches a scale by comparing triples of stimuli in their three possible different orders. A response pattern for a triple of stimuli is in error if the two outer stimuli are preferred but the middle one is not. The procedure to find a best possible scale consists of the determination of a unique ordering for a maximal subset of stimuli. For this subset the number of triples in error should be lower than the number of errors expected under a model of stochastic independence of preferential choices. Each stimulus in the J-scale and each J-scale upon itself can be evaluated by means of a scalability coefficient. This scalability coefficient is defined in terms of expected versus observed numbers of errors in response patterns per triple. Moreover, the dominance matrix should display the property of 'characteristic monotonicity' (Van Schuur, 1984, p. 51). For the scalability coefficient for the total scale a lower boundary has been fixed.

3.3 THE NULL MODEL

In order to test whether the frequencies of response patterns found can be accounted for by the marginal frequencies of the separate stimuli ('item popularity'), Van Schuur and Molenaar (1982) devised a stochastic null model. This null model is an adaptation of Goodman's quasi-independence model (1968; see Bishop e.a., 1975; Fienberg, 1980). Under the null hypothesis of 'no structure in the data except for the one that can be predicted from the marginals of the stimuli' and the condition that item choices are stochastically independent, expected frequencies of inadmissible patterns for each triple of stimuli in each of the three possible orders are assessed. The expected and the observed frequences are compared to each other by means of the H-coefficient (Van Schuur, 1984, p. 80). A triple of stimuli is scalable if the H-coefficient exceeds a lower boundary in only one of the three possible orders of the triple (see Van Schuur for other criteria (1984, pp. 83, 84). Stimuli can be added to this scale on the grounds of analogous criteria. So, the null hypothesis of 'no structure in the data' is rejected in favour of the alternative 'an unfolding scale is applicable' if patterns in error appear less frequent than can be expected under stochastic independence.

4. MUDFOLD - A DISCUSSION

In this section the following subjects are discussed: the dichotomisation of ranking data (4.1), parallelogram analysis versus scalogram analysis (4.2), the unfolding model versus the vector model (4.3), characteristic monotonicity (4.4), the null model (4.5), Davison's model (4.6) and the Norpoth data (4.7). In 4.8 a conclusion follows.

4.1 DICHOTOMOUS PREFERENCE DATA VERSUS RANKING DATA

In the dichotomisation of data lies the danger of arbitrariness by the way in which they are dichotomised (see Coombs, 1964, p.230; 'conclusions depend on the method of dichotomisation', according to Davison, 1977, pp. 542-544). Van Schuur describes the advantages and disadvantages of the various possiblities of dichotomisation and the kinds of analyses that result from it. The 'pick k/n' method has the disadvantage that various analyses for sub-groups with a certain k are necessary (because of ties in the data). The 'pick any/n' method has the advantage that the individuals can be taken simultaneously into the analysis. It has the disadvantage that the way of dichotomising e.g. 'rating data' (1... n) is rather arbitrary but does effect the result of the analysis. Van Schuur always makes a well-considered choice or does more than one analysis. It is, however, open to question whether the unwary MUDFOLD-user will proceed just as carefully and expertly. Maybe the MUDFOLD-manual can contribute to this.

Moreover, dichotomisation takes the sting out of unfolding: unfolding without ordinal information is parallelogram analysis.

Apart from the quantity of metric information, methods of collecting data differ in the number of isotonic regions and the width of the isotonic regions in the resulting J-scale. Complete orderings of preference of five stimuli result in a J-scale with 11 isotonic regions, with 'pick 3/5' three groups of individuals with identical orders of preference can be distinguished, with 'order 3/5' eight. For ranking data the discriminability between individuals and stimuli increases with k, for 'pick' data the discriminability decreases with increasing k and an undifferentiated subset of stimuli in the central region becomes larger. Less discrimination between individuals implies less, and hence wider, isotonic regions. In the choice of the method of collecting and analysing data the expected discriminability of individuals between stimuli should also play a part. In analysing data it is always possible to switch from the method of complete rank orderings to a method of 'order k/n' with smaller k or to 'pick k/n' but the reverse is not possible. An example of such a method is the study of Kamenetzky & Smith (1957).

4.2 PARALLELOGRAM ANALYSIS VERSUS SCALOGRAM ANALYSIS

Earlier publications about parallelogram analysis with 'pick k/n' or 'pick any/n' data are a.o. Leik & Matthews (1968) and Coombs & Smith (1973). Coombs and Smith consider the parallelogram (in the context of developmental psychology) as the result of an 'acquisition sequence' (items are constantly added on one side) and a 'deletion sequence' (on the other side items are deleted constantly). If on the other side no items are deleted, we get a triangular pattern (scalogram). A scalogram implies the presence of a parallelogram, the reverse is not true. Under the parallelogram model (ideal points model) the distances from all stimulus points to all individuals' ideal points are compared. In the scalogram model distances of stimulus points to one (external) ideal point are compared. So, the scalogram model is a special case of the ideal points model (Shepard, 1972). Which model is applicable can be deduced from the 'joint frequency matrix', the dominance matrix and the intercorrelation matrix. The intercorrelation matrix contains exclusively positive correlations in the case of a perfect Guttman scale, also negative correlations in the case of an unfolding scale (see Leik & Matthews, 1967; and Davison, 1977).

So with the 'pick any/n' method both a parallelogram and a scalogram can be found, i.e. the items chosen can belong to the ideal-points-model or to the dominance model. With a given set of items and when fitting an unfolding scale, 'dominated' items should be eliminated. When fitting a scalogram model, 'ideal point' items should be eliminated.

4.3 UNFOLDING MODEL VERSUS VECTOR MODEL

With the vector model the order of preference of an individual for stimuli in two dimensions (i.e. two aspects) is found from the order of the projections of the stimuli on the vector of that individual, i.e. the further the projection of a stimulus on that vector, the more preferred that stimulus is. In general a vector solution in p dimensions implies the unfolding model for a suitable ('efficient') subset of the stimuli in (p-1) dimensions (Coombs, 1975; Coombs & Avrunin, 1977). This means that 'dominated' items are removed. For the fact that no solution could be found for (the importance of) political problems, the stimuli are probably to blame. They do not form a homogeneous set in the sense of an 'approach-avoidance efficient set'/'approach-approach efficient set'/'avoidance-avoidance efficient set'. Classification of problems in the above-mentioned sense might then produce a J-scale.

4.4 CHARACTERISTIC MONOTONICITY

The proposed measures for solving those political problems could be scaled both with MUDFOLD and with a cumulative scaling procedure. Van Schuur has, in my opinion, unjustly rejected the MUDFOLD solution on the grounds of lack of 'characteristic monotonicity' in the dominance matrix. The suitability of this measure for 'goodness of fit' has not been proved. In fact 'characteristic monotonicity' is a 'gauge', an ideal pattern that appears in the perfect case. Only a 'sufficient statistic' for the lack of this characteristic monotonicity can act as measure for 'goodness of fit'.

4.5 THE NULL MODEL

The null hypothesis of 'no structure in the data' is rejected in favour of the alternative 'an unfolding scale is applicable' if patterns in error appear less than can be expected under statistical independence. The criticism on this procedure is twofold. First of all: the H-coefficient is not the proper test statistic for the specified null model of 'independence'. The appropriate test statistic in this case is the Pearson chi-square statistic or the likelihood ratio chi-square statistic. If, by means of one of the two, the null hypothesis of 'no structure' can be rejected, it can be tested in an analogous way whether the pattern frequencies fit the alternative model. The tests of 'independence' and of 'fit' of the unfolding model are based on different null models and so they should be applied independently of each other. The way to do this is described in Davison (1979, 1980). The H-coefficient is a descriptive statistic, no test statistic (see also Cureton, 1959). In the second place only expected and observed frequencies for the patterns in error are compared. Rejection of the null hypothesis, however, should be based on the comparison of expected and observed frequencies for every possible admissible and inadmissible pattern of the J-scale triple concerned (see Lewis & Burke, 1949). A positive value for the H-coefficient does not imply that the chi-square value is significant (or visa versa). In 4.7 this criticism is illustrated with an example (the Norpoth-data).

If the null hypothesis is rejected the conclusion of Van Schuur is, that the unfolding model is applicable. For this alternative hypothesis Van Schuur did not define a probabilistic item-response model, nor a probabilistic (multinomial) model for all response patterns, so that Coombs' deterministic parallelogram model has to be applied here. Under the 'pick k/n' model there are (n-k+1) possible isotonic regions. However, scale values are credited to individuals by averaging the scale values of the selected stimuli (Van Schuur, 1984 p.87) For the 'pick 3/5' model there are seven possible person's values with this method but only three possible isotonic regions. This suggests a larger discriminability between individuals than is justified with a 'pick 3/5' method. Scale values (in Coombs' model) are isotonic regions for individuals, not points. A rationale for the allocation method used is missing. A probabilistic model (for all possible response patterns) under the alternative hypothesis would have justified this.

4.6 DAVISON'S MODEL

Davison (1979, 1980) gives an outline of a procedure comparable to the one of Van Schuur and Molenaar, however, the testing procedure takes place in two phases. In the first phase a null hypothesis of 'pure quasi-independence' is tested (no structure in the data except for the one that is predictable from the marginal frequencies). If this hypothesis can be rejected with the usual test statistic of the model, the null hypothesis of 'modified quasi-independence' (the structure in the data agrees with the predictions from the poned J-scale) is tested.

If this hypothesis cannot be rejected we have to conclude that the J-scale put forward is applicable. The quasi-independence model and the 'iterative proportional fitting procedure' he describes are the same as those of Van Schuur and Molenaar (and GLIM, ECTA and BMDP4f).

There is, however, an important difference in the application of the quasi-independence model by Davison on the one hand and Van Schuur and Molenaar on the other. Van Schuur applies parallelogram analysis, Davison unfolding. In the case of parallelogram analysis, the null hypothesis of 'no structure in the data' amounts to testing whether the frequencies observed can be predicted from the item marginals. Under the quasi-independence model this means testing the null hypothesis whether the frequencies for the preference patterns found can be predicted from the popularity (marginal frequencies) of the separate items. In the case of unfolding (Davison's method) the null hypothesis is tested whether the successive choices of an individual are independent. Not only a large number of admissible patterns is expected under the unfolding model, but also dependencies between successive choices. The second chosen stimulus should be found next to the first chosen stimulus on the J-scale, so the first and the second choices cannot be independent.

Each stimulus can occur as first, second etc. choice. Under the quasi-independence model it is ascertained whether the orders of preference can be predicted by 'first choice', 'second choice', 'first plus second choice', etc. If so, the choices of respondents are 'quasi-independent'. Only if there is interaction between the successive choices of individuals the null hypothesis of quasi-independence can be rejected. In view of the applicability of Davison's method by means of existing computer programmes, there is, in case of ranking data, no reason whatsoever to proceed to dichotomisation of data before it is clear that ordinal unfolding analysis will not have the desired result. In 4.7 MUDFOLD and Davison's procedure will be compared by a data set consisting of ranking data.

With the Davison method it is also possible to test the applicability of the cumulative scale model. Presumably this can be done with MUDFOLD, too. The tests of 'pure quasi-independence' and 'modified quasi-independence' for a parallelogram model and a scalogram model are identical, under the last model only the subset of admissible patterns is defined in a different way. The interpretation in the one case is in terms of 'proximity' and in the other case in terms of 'dominance'. This will hold only for 'pick any/n' data.

4.7 THE NORPOTH DATA

On page 187 Van Schuur describes his results with Norpoth's data (1979): complete orderings of preference of German voters in 1969 and 1972 for the five political parties DKP, SPD, FDP, CDU/CSU and NPD. By means of MUDFOLD (via dichotomisation of the data) a J-scale that contained the three largest parties (SPD, CDU/CSU and FDP) could not be found in either case.

As an illustration of the criticism of the MUDFOLD-null model the results of the Norpoth 1972 data set for the three most important political parties SPD ('S'), CDU/CSU ('C') and FDP ('F') are given below. The best J-scale for the three parties was F-S-C. By means of complete orderings for these parties we can test two null hypotheses:

1) H_{01}: the frequencies for the six possible orderings can be predicted from the 'popularity' of the stimuli (first choice of the voters, i.e. the chosen party);

2) H_{02}: the successive choices of individuals are independent.

Every individual chooses one out of six possible orders, so the six permutations of the political parties form an exhaustive set of mutually independent response categories. Under H_{01} expected frequencies for all permutations can be assessed by means of a quasi-independence model. A statistic which contains (under H_0) all the information from the data ('sufficient statistic') is the Pearson chi-square statistic. The value of this statistic is assessed by comparing the expected and the observed frequencies of all permutations. The H-coefficient is assessed exclusively out of the inadmissible patterns CFS and FCS (marked with '*'). The results for the 1972 data by means of the BMDP4f (quasi-independence model)-programme are shown below:

TABLE 1: The NORPOTH 1972 data for SPD, FDP and CDU/CSU: observed and expected frequencies under two null models.

		NULL MODEL			
		H_{01}		H_{02}	
permutation	obs	exp	$(obs\text{-}exp)^2/exp$	exp	$(obs\text{-}exp)^2/exp$
FSC	76	59.5	4.58	91.2	2.53
SFC	639	473	58.25	623.8	0.37
SCF	307	473	58.25	322.2	0.72
CSF	468	360	32.40	452.8	0.51
CFS*	252	360	32.40	267.2	0.86
FCS*	43	59.5	4.58	27.8	8.31
Tot.	1785	1785	$\chi^2=190.46$	1785	$\chi^2=13.30$

The chi-square values found for H_{01} and H_{02} are respectively $\chi^2=190,46$ (df=3, p<.000) and $\chi^2=12,91$ (df=1, p=.0003). So both hypotheses H_{01} and H_{02} can be rejected. The frequencies found cannot be predicted from the item marginals (first choices, H_{01}), nor from an additive combination of first and second choices (H_{02}). So there are dependencies (interactions) between the successive choices of individuals. The H-coefficient is assessed via

$$H = 1 - \frac{obs(*)}{exp(*)} = 1 - \frac{252 + 43}{360 + 59.5} = 0.3 \qquad (H_{01}) \text{ en}$$

$$= 1 - \frac{252 + 43}{267.2 + 27.8} = 0 \qquad (H_{02})$$

So a significant chi-square value does not imply a significant H-coefficient.

When comparing (summed) observed frequencies and expected frequencies of 'admissible' versus 'inadmissible' response patterns, we get the following results:

PATTERNS	Observed	Expected		$(exp-obs)^2/exp$	
		H_{01}	H_{02}	H_{01}	H_{02}
admissible	1490	1365.5	1490	11.35	0
inadmissible	295	419.5	295	36.95	0
TOT.	1785	1785	1785	48.30	0

H_{01}: χ^2=48,30 (df=1, p<.000) ; H=0,3
H_{02}: χ^2=0 (df=1, p=1) ; H=0

The effect of combining all admissible patterns on the one hand and all inadmissible patterns on the other hand in two categories results in a lower power of the test. Irrespective of the quality of the H-coefficient as a test statistic the lumping of patterns into two categories 'admissible' and 'inadmissible' causes the test on independence to become less efficient. So the use of the H-coefficient in case of unfolding is not as efficient as possible.

In any case it can be concluded from the Norpoth example that it is not true that 'no result with dichotomised data' (MUDFOLD) necessarily implies 'no result with a ranking method'.

'Item popularity' with MUDFOLD is defined as the frequency with which items are chosen in the first k positions. So the item popularity is dependent on k. With ranking data the first choice of individuals is sufficient and not dependent on k. In case of the Norpoth data the NPD and DKP were rather impopular and were the last choices of most of the voters. Maybe this is why the estimation of the popularity of the parties by MUDFOLD turned out in such a way that analysis by means of 'pick k/n' (MUDFOLD) produced no scale but analysis by means of 'rank k/n' (BMDP4f, see table 1) did.

4.8 CONCLUSION

By dichotomising data it is possible to test a less restrictive model than that of unfolding, but it would be expected that the less restrictive model would yield a scale easier. With MUDFOLD obviously this is not always the case. In fact the models are incomparable. The null hypotheses differ and so do the data (the marginals) out of which estimations for patterns of preference are calculated. If for no other reason, MUDFOLD is not suitable for unfolding data: with dichotomised data it is not possible to assess the correct expected frequencies under the null model for unfolding. So, Van Schuur's claim that 'rank k/n' data can also be analysed does not seem tenable to me. In any case it is not sure that this produces the same results as a method that does use the rank-information. In fact, every method of preference analysis that uses dichotomised data is a parallelogram method. And so are methods searching for ratio scales for unfolding data, by means of the Rasch model and with dichotomised data. A logistic model that can be used for unfolding data is the Plackett-model (1975). Only if (1) the starting point is orderings of preference of individuals, and if (2) the successive choices of individuals are supposed to be mutually <u>dependent</u>, the analysis is an unfolding analysis and the result an unfolding scale.

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