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An example of using M/G/∞ queues for decision support in aircraft maintenance

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0. Abstract

Many practical problems can be solved by using concepts that are developed within queueing theory. However, only a fraction of the real life applications of queueing theory are reported in journals that are available for practitioners and scientists. The reasons for this are obvious. On one hand practitioners often don't have the time to write a paper after they have solved their problem. On the other hand the material and data that are used can be confidential which prohibits publication.

The goal of this paper is limited. We will give a practical example of an M/G/∞ queue that was used within aircraft maintenance, we will indicate how such a model can deliver relevant information for decision support. By this we hope to enrich the literature with <u>another</u> practical application of queueing theory.

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1. Introduction

In this paper we will describe an application of queueing theory within aircraft maintenance. The study was part of a bigger project that was carried out with the Royal Dutch Airlines KLM, which started in 1980, to improve the inventory control of spare parts for aircraft maintenance. Research with respect to the determination of optimal spares stocking policies is a well known problem. Nahmias gives a thorough review of the research work that is done [3]. Hausman and Scudder used a simulation model to examine priority scheduling rules in the repair environment of jet engines [1], and Muckstadt described an analytical spares stocking model [2]. An elementary introduction of inventory management can be found in [4]. At KLM about 200,000 different parts are stocked. The spare parts can be divided into several groups, where each group of spare parts has his own characteristics with respect to demand pattern, price, lead time, etc. We will concentrate ourselves to a group that consists mainly of large components like computers, altitude indicators, generators, and coffee machines.

Aircraft regulations require that all components of an airplane are regularly inspected, and revised or repaired if neccessary. In general such an inspection takes place after a given number of flight hours. For this inspection the component is removed from the airplane and immediately replaced by a spare part component. This is done to keep the time an airplane is grounded as short as possible. A removal of a component from the airplane can also occur, in between two scheduled inspections, if the component shows a defect. After a component is removed it is tested and if neccessary repaired or revised. If the component passes the test it is labelled "serviceable" and put on the shelf, waiting to replace a similar component that has to be removed from an airplane.
As we see, the "state" of these components rotates between being in the airplane, being removed, being repaired or overhauled and

being on the shelf. Therefore one refers to these components as rotables.

However, if a rotable is tested it might appear that specific parts have to be renewed to carry out the required repair. These parts are only used once and vary from nuts and bolts to eg. bearings of a generator or a pointer of an altitude indicator. One refers to this group of parts as the group of "consumables". When consumables needed for repairing a rotable are available in stock, the rotable can be repaired immediately and thereafter it will be added to the stock of serviceable rotables, and put on the shelf. However, it is possible that either a required consumable has never been stocked previously or that the article is out of stock. Therefore, it is possible that consumables needed for repairing a rotable are not immediately available, and that the repairment of the removed rotable is delayed. The rotable is then classified as "Temporarily Unacceptable", abbreviated as "TU". When a rotable is declared as TU, the consumables needed for repairment are ordered immediately. So the TU rotable is useless waiting for consumables to be delivered. After the delivery of the consumables the TU will be repaired immediately. Once the TU rotable is repaired, it is finally added to the stock of serviceable rotables. The entire process is given in figure 1.

From the point of inventory control it might be inefficient to have the valuable TU rotables useless waiting for a less valuable consumable to be delivered, since a large number of TU rotables causes that the total stock of rotables has to be larger in order to keep a certain service level. Therefore the management has to evaluate whether it is more profitable to decrease the number of TU rotables or to carry a higher level of spare parts. In the recent past the average number of TU rotables was about 700, representing a value of several millions of dollars. So one created a "management by objectives" project to bring this number down. After about a year they succeeded in having an average of about 400 TU rotables. And the question was whether this was reasonable or if a further decrease could be achieved.

There are a number of possibilities to effect the number of TU rotables. We can think of:

- try to substitute other consumables for consumables that are not available
- emergency purchase of required consumables
- raise the servicelevel of consumables
- expedite the vendor lead times of the consumables.
- etc.

By evaluating the effects of different alternatives it is possible to give management insight in the power of the several types of control. Moreover, one can get insight in the process that is involved and based on this knowledge set a standard for the amount of TU rotables.

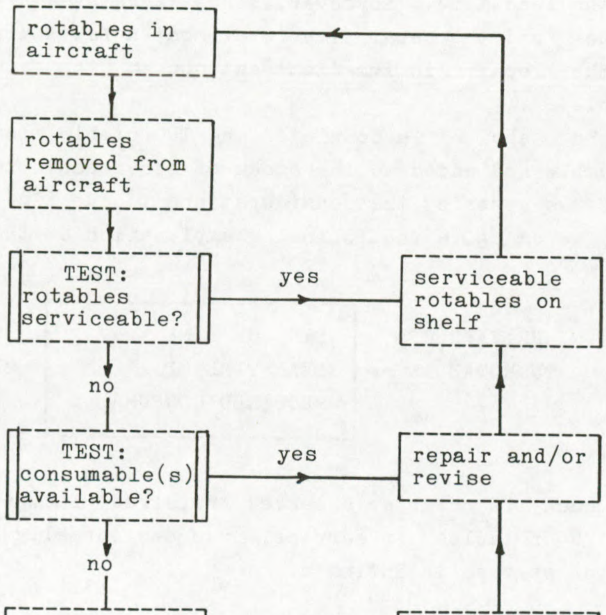

Figure 1. Process of rotables

2.Modelformulation

As mentioned in the introduction we will show that a queueing model can be used to describe the process of TU rotables. One could say that a rotable arrives as TU at the moment that a removed rotable requires a consumable that is not available in inventory. It appeared that the arrival process could be described as a Poisson process. This will be elucidated later on. Once a TU rotable has arrived it will stay TU until it is repaired. So the "service time" of a TU rotable consists of the ordering of the consumables needed for repair, the time until the consumables are delivered by the vendor, and the repairing of the rotable. Hence the service can take quite some time, especially if the consumables have a long lead time. These lead times have the most effect on the servicetime because ordering and repairing is a matter of hours, whereas the lead times can take months. So, an acceptable approximation of the time that a TU rotable stays TU is the lead-time. Moreover, since there are quite a number of repairmen and the repairtime is at most a few hours one can say that the repair is immediate as soon as the consumables are available.

Once the service is completed the TU rotable is declared as serviceable and added to the stock of serviceable rotables.

So, if we restrict the considerations to the time the rotables are TU, we can give the following explanation of the process:

ARRIVALS OF TU ROTABLES	→	SERVICE TIME i.e. LEAD TIME OF REQUIRED CONSUMABLE	→	OUTPUT OF SERVICEABLE ROTABLES

Since each TU rotable is served (repaired) independent of the other TU rotables in service, one can formally say that the number of servers is infinite.

Now by using Kendall's notation we could approximately describe the TU system by a $M/G/\infty$ queueing system. Where M indicates that the arrival process is modelled by a Poisson process. For the service process the actual lead time distribution can be used, which is allowed to be general (G). While the number of servers can, as mentioned before, be modelled as infinite (∞).

Some comments have to be made in order to justify the chosen model. With respect to the arrival process we first investigated whether several TU rotables were caused by the absence of the same consumable. It appeared that over a two year period this has never occurred once. Nevertheless one can imagine practical situations were dependence will occur, so one has to follow this phenomenon closely. The poisson distribution was more or less a consequence of the fysical process that was involved. Since arrivals of TU rotables occurred if one of the required consumables was not available, and the total amount of different consumables was very large it was clear that arrivals occurred at random. The service pattern was mainly determined by the vendor lead times. It appeared that we had to distinguish between some groups of rotables. The characteristics of the groups was on one hand based on the lead times of consumables. On the other hand there appeared to be a difference between mechanical, instrumental and electronical consumables. Even within the group of electronical consumables a subdivision had to be made. For each of the groups a $M/G/\infty$ model was used to get insight in the process and the sensitivity for several parameters.
Let λ be the average number of arrivals of TU rotables for a certain group of rotables and let μ be the mean service time i.e. the average lead time of consumables for that group, then the distribution of the number of TU rotables in the system is given by

$$p(n) = \frac{\left(\frac{\lambda}{\mu}\right)^n e^{-\lambda/\mu}}{n!} \qquad n = 0,1,2,\ldots \qquad (1)$$

where p(n) denotes the probability that there are n TU rotables.

The average number of rotables A(TU) is given by

$$A(TU) = \lambda/\mu \tag{2}$$

The stationary probabilities given in (1) are independent of the distribution of the lead time and depend only on the first moment of this distribution. For a formal proof see [5].
In the next sections we will discuss some practical examples and show how (1) and (2) can be used for sensitivity analysis.

3. Some examples

The information that was neccessary for the modelbuilding could be gathered since the KLM maintenance department introduced in 1979 a computerized administration and registration system named COMPASS. Sub-systems of this system enabled the evaluation of demand and delivery patterns for consumables as well as rotables. In the examples that are discussed in this section we use data that are slightly modified in order to avoid giving confidential information.

For one department three different types of rotables had to be considered, roughly one could say that the groups were, Instruments, Electronics, and Radio & "Accessories". Which explains the name of the department which was IERA. We will indicate the groups by I, E, RA, respectively. For each of the groups the arrival rates were based on historical information. The estimated arrival rates per week are given in table 1.
It turned out that the "service time" of the TU rotables were more or less the same for each of the groups. Since the service time was mainly a function of the vendor lead times which were not directly related to the type of rotable. The mean service time was 8.8 weeks or approximately 2 months, where in the computations a restriction was made to rotables of the B747 and DC10 fleet. This mean service time was considered to be

representative for the three groups.

	arrival rate
group I	34.9
group E	18.5
group RA	9.0

Table 1. Estimated arrival rates per week for each group in the IERA-department.

The expected number of TU rotables in each group can now be computed, and the initial results are given in table 2. Note that the actual number of TU rotables differ considerably from the expected number of TU rotables. However we will give an explanation for this in the next section.

hall	actual level of TU's	expected no. of TU's	standard deviation	expected no. of TU's (no stock-outs)
I	247	307	17.53	85
E	116	162	12.74	80
RA	91	79	8.89	35
tot	454	548	23.41	200

Table 2. Actual and expected number of TU rotables in 2 different situations during the time period jan-sept 1981.

4. Sensitivity analysis

As mentioned in section 2 there are several possibilities to influence the number of TU rotables. One of these possibilities is to raise the service level of consumables. If it would be possible to stock all consumables in a sufficient way, hardly any TU rotable would occur. however the inventory costs corresponding with such a policy will be enormous. So in practice such a policy is impossible. Moreover TU rotables sometimes occurred since the required consumables were never stocked before, for example consumables for new types of airplanes.

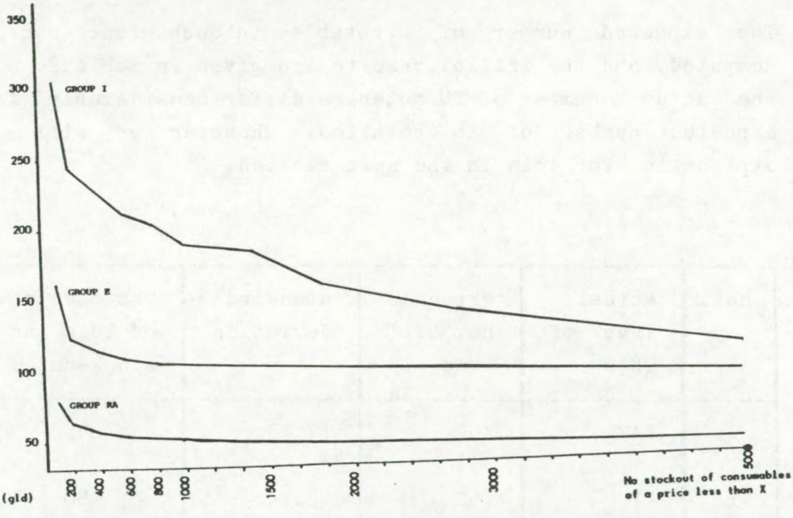

Figure 2. The X-axis is the price (in guilders) of consumables causing TU rotables.
The Y-axis is the expected number of TU rotables given that stockout are not permitted for consumables of a price per unit less than X.

We considered as one extreme the change of the arrival rate if TU rotables where only caused by consumables that were never stocked before, assuming that other consumables had a service level of 100%. In that case the three arrival rates dropped considerably and the expected number of TU rotables dropped correspondingly. In a less extreme situation one could restrict the considerations to consumables wich are relatively inexpensive. In figure 2 we give the expected number of TU rotables as a function of X, in such a way that we supposed that all consumables with a price less than X are stocked and thus cannot cause a TU rotable.
In each of the groups the level of TU rotables will drop considerably up to a price of approximately 400 guilders. This indicates that it was worthwhile for the management to look after possibilities which increase the service level of a class of cheap consumables, and find an equilibrium between the connected costs and the resulting profits.

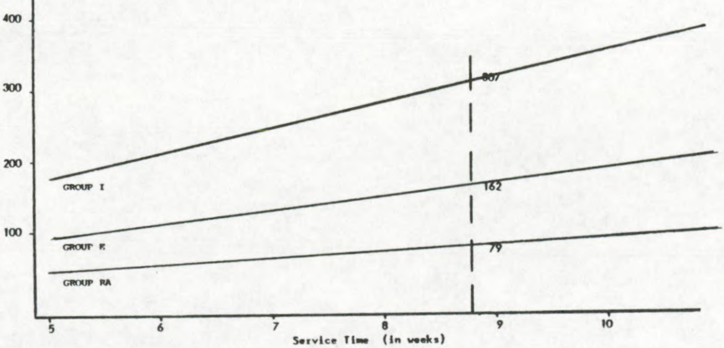

Figure 3. Service time vs Number of TU rotables in IERA.

In figure 3 we give the expected number of TU rotables as a function of the lead time. Note that the slope of linear functions are different for the three groups. This means e.g. that longer lead times in group I cause relatively more TU rotables then in group RA. Based on information as given in figure 3 one can get insight in the amount of inventory cost reduction that can be achieved for the group of rotables by reducing the lead times. This reduction can then be compared with the costs required for reducing the lead times. Based on information as given in figure 2 one gets insight in the reduction that can be achieved if a higher level of spare parts is available. Such insights gave management a better starting point for initiating several activities with respect to inventory control. In fact the actual number of TU rotables as given in table 2 was considerably less then the expected amount, since the lead times as computed on the basis of historical data was to high. This was partly due to the economical recession, which causes that the lead times had dropped at the end of the period under consideration. This meant that part of the success of the MBO project would have occurred any way.

Acknowledgement

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