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CORRELATIONAL STRUCTURE OF THE SUBTESTS OF THE SNIJDERS - OOMEN NON-VERBAL INTELLIGENCE SCALE

Pieter M. Kroonenberg
Vakgroep Wijsgerige en Empirische Pedagogiek
Rijksuniversiteit
Leiden

SUMMARY

Using three-mode principal component analysis on correlation matrices for three age groups of both hearing and deaf children, it is shown that the structure of the subtests is virtually the same in all six groups, and that this structure might be described by a component shared by all tests, and two other components of almost equal importance.

Schuttersveld 9, k. 507
2316 XG Leiden
The Netherlands

1. INTRODUCTION

In this paper we will investigate the correlational structure of the subtests of the Snijders and Snijders-Oomen Non-verbal intelligence scale (S.O.N.) as published in its 1958 version (Snijders & Snijders-Oomen, 1958, 1962). Since then a new version of the S.O.N. has been produced, and a third version is in the process of being developed. English, German, and French versions of the S.O.N. are also available.

Uptil now no detailed investigation of the S.O.N. correlational structure has been carried out. Snijders & Snijders-Oomen (1962, p. 42) report that some factor analyses have been performed on their data, but these have apparently not been published. In Table 1 we give a short characterization of the subtests.

Table 1 *Subtests of the S.O.N.*

	Group	nr.	Subtest	Abbre-via-tion	Parts	Scale
I	Form	1.	Mosaic	MOZA	Mosaic A and B, block patterns	P
		2.	Drawing	DRAW	Copying, finishing a drawing	Q
II	Concrete re-lationships	3.	Combinations	COMB	Puzzles, pictures series A and B	P
		4.	Completion	COMP	Halfs, related pictures, comple-ting pictures	Q
III	Abstractions	5.	Analogies	ANAL	Continuation of series, picture analogies, figure analogies	P
		6.	Sorting	SORT	Sorting chips, sorting cards	Q
IV	Immediate memory	7.	Memory for pictures	MEMO	Memory for pictures, series A and B	P
		8.	Knox blocks	KNOX		Q

The structure of a test consisting of subtests is usually investigated by factor analysis or principal component analysis. In the present case we want to investigate the similarities and the differences between six groups, i.e. three age groups (3-5; 8-11; 14-16) of both hearing and deaf children. Traditionally structures of subtests for such groups are compared by target (or procrustes) rotation, or by factor (component) matching techniques. One paper using both approaches is, for instance, Meyers et al. (1954).

Alternative ways to treat sets of correlation matrices are simultaneous factor analysis for several populations (Jöreskog, 1971), simultaneous procrustes analysis (Ten Berge, 1977), and the perfect congruence approach (Ten Berge, 1982). A fundamental requirement for these methods is that some kind of target matrix is available. We will not go into the relative merits of these methods and the one to be described here.

Here we will analyse simultaneously the correlation matrices of the subtests for each of the six groups (Snijders & Snijders-Oomen, 1962, p. 218, 219) via a three-mode principal component analysis (see e.g. Levin, 1965; Tucker, 1966; Lohmöller, 1979; Kroonenberg & De Leeuw, 1980; or Kroonenberg, 1983a). We will investigate if a common structure is present for all six groups. Necessarily the structure found will be a compromise between the structures for each of the six groups, but the crucial point is whether, and to what extent, the compromise structure is shared by the six groups.

2. THREE-MODE PRINCIPAL COMPONENT ANALYSIS OF CORRELATION MATRICES

Although it is not our intention here to present three-mode principal component analysis in much detail, a few words should be said to enable understanding of what is to follow. We will discuss only those aspects of the technique which are relevant for the present discussion. For a more detailed treatment one may consult Kroonenberg (1983b, especially Ch. 12).

Three-mode principal component analysis is a technique to analyse data which can be classified in three ways. In the present case two of these ways are the same, i.e. subtests. The third way consists of the six groups of children, who each have produced a correlation matrix. Standard (two-mode) principal component analysis produces amongst other things component loadings for the subtests. These loadings provide an indication how the subtests are related. Also in three-mode principal component analysis component loadings are available, but these loadings are now based on the correlation matrices of all six groups jointly. In addition, the relative importance of the components to each of the six groups can be assessed. Thus it is possible to evaluate how each of the groups uses the common relationships between the subtests. If one of the groups should have very little or nothing in common with the other groups it will become clear that this is the case. If, on the other hand, all groups share more or less the same structure this will become apparent as well. The agreement of a group with the common (compromise) solution will be measured by an approximate percentage explained variation, which would arise if the common space was in fact the space for the group. How these quantities are computed will not be explained here, but is worked out in Kroonenberg (1983b, Ch. 12).

3. RESULTS

3.1. *Common subtests space*

In Table 2 the three-dimensional subtest space is presented. The first component reflects the fact that all correlations are moderately positive, i.e. most of them range between .30 and .50. In other words, all subtests measure a common 'trait'. It is interesting to observe that although the values on the first component are roughly equal, there are also some systematic trends present.

Table 2 *Component loadings for all subtests*

Subtest	Short Form		A: Component (x 100)			B: Varimax rotated components (x 100)		
	P	Q	1	2	3			
	group		1	2	3	1	2	3
Mosaic	I		41	-1	-30	48	17	-5
Analogy	III		40	-10	-25	48	7	-2
Combinations	II		39	-16	11	32	-6	28
Drawing		I	37	-29	-30	54	-10	-9
Sorting		III	34	-19	-3	36	-7	14
Completion		II	32	-18	59	5	-20	66
Memory	IV		30	34	58	-14	28	67
Knox		IV	28	83	-23	4	90	-2
% variation explained			45	11	10			

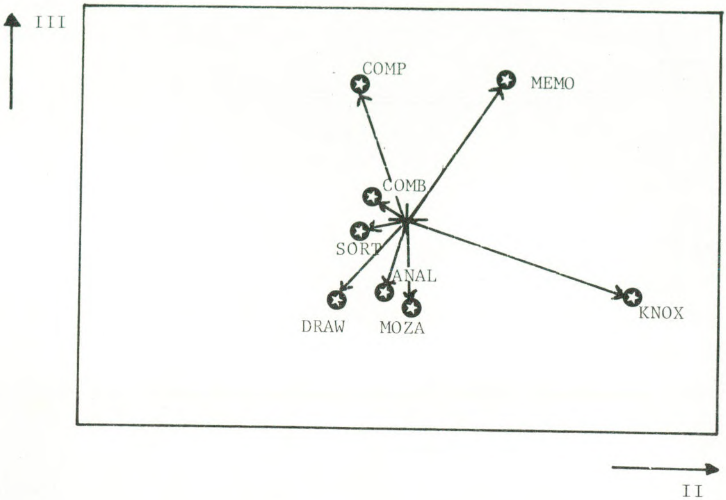
In the first place, there is a systematic difference between the two short forms, P and Q, of the S.O.N. (see Snijders & Snijders-Oomen, 1962, p. 9 for a discussion of the two forms), as all P-subtests have higher loadings than their Q-counterparts. In the second place, the order of the content groups of subtests within the P and Q scales is the same for the two short forms. This suggests that if a short form is to be administered P is the preferred one, because of its greater homogeneity. The amount of variation explained by the first component is 45%. Snijders & Snijders-Oomen (1962, p. 42) quote unpublished averages for separate factor analyses of 36% for the hearing and 41% for the deaf children. Their values were, however, obtained using factor analysis with communality estimates, as pointed out to me by a reviewer.

The second and third components are of roughly the same importance; they explain 11% and 10% of the variation respectively. Snijders & Snijders-Oomen (1962, p. 42) state that factor analyses showed some vague second factor which was not the same in all subgroups. As we will see in more detail later the instability results from the approximate equal importance of the second and third components as expressed by their eigenvalues. This near-equality of the eigenvalues implies that the components define together a plane in which their orientation is more or less arbitrary, as is demonstrated later on in Fig. 2.

For a qualitative description of the structure of the subtests it is most useful to investigate the plane spanned by the second and third component (Fig. 1), rather than the loadings on the components themselves. After all, the orientation of the second and third components is rather arbitrary, and the subtests have almost equal loadings on the first component. When investigating such a plane it should be realized that this plane reflects what is left after the common variation as reflected by the first component has been removed. In three dimensions the structure looks somewhat like the ribs of an umbrella.

The arbitrariness of the orientation of the axes in the plane precludes an unambiguous interpretation of the components without further substantive knowledge. The structure itself is, however, unambiguous, and may be characterized by the positions of the subtest vectors. Thus over and above the common first component *drawing*, *analogies*, and *mosaic* have much in common, as do *completion* and *sorting*. *Knox blocks*, *memory for pictures*, and *combinations* are relatively distinct.

Fig. 1. *Subset loadings for simultaneous analysis* (second versus third component)



3.2. *Differences between the groups: simultaneous analysis*

Table 3 shows the approximate percentages explained variation each group attached to the common components from the three-mode analysis. Also included are the percentages explained variation of the components of the separate principal component analyses per group. The latter will be discussed in the next subsection.

Table 3 *Relative importance of the components*

A. *Relative importance of the common components to each group* approximate percentage explained variation

	3-5				8-11				14-16			
Hearing	45	9	10	64	41	10	11	62	43	11	11	65
Deaf	47	10	10	67	45	11	9	65	50	11	9	70

B. *Relative importance of the components from the separate analyses per group** percentage explained variation

	3-5				8-11				14-16			
Hearing	45	11	9	65	41	12	11	64	43	12	11	66
Deaf	47	11	10	68	45	11	9	65	50	11	9	70

* Note: The percentages in part A of the table refer to the same components; those in part B are not necessarily the same as they result from separate analyses.

For each group there are some slight non-orthogonalities for the common axes, but they are too small for interpretation, and are, therefore, not presented here. From Table 3A we may draw the following conclusions:

- On the whole the relative importance of the components is the same in all groups. In other words the loadings based on all six correlation matrices jointly form a fair representation of the structure between the subtests for each of the groups, regardless age or hearing.
- The general intelligence component is somewhat more important to the deaf than to the hearing children. It is slightly less important to the 8-11 year olds both for the hearing and the deaf.
- No serious age trends are present for any of the groups, and the relative importance of the second and third components is the same and stable over the six groups.
- The total amount of variation explained is approximately equal in all groups, with a slight edge for the deaf children.

3.3. *Differences between groups: separate analyses*

It is instructive to compare the results from the previous subsection with those from separate analyses per group. In line with the previous discussion, the first components are given separately in Table 4, while plots are presented of the second versus the third components. The principal component analyses were performed using the BMDP suite of programs (BMDP4M, Dixon, 1981).

From the percentages explained variation, already given in Table 3 it becomes clear that the separate analyses hardly can explain more variation than the joint analysis did. In other words the amount of the structure of the subtests which can be captured in three components was for all groups adequately represented by the joint analysis.

Table 4 *'General intelligence' components (x 100)*

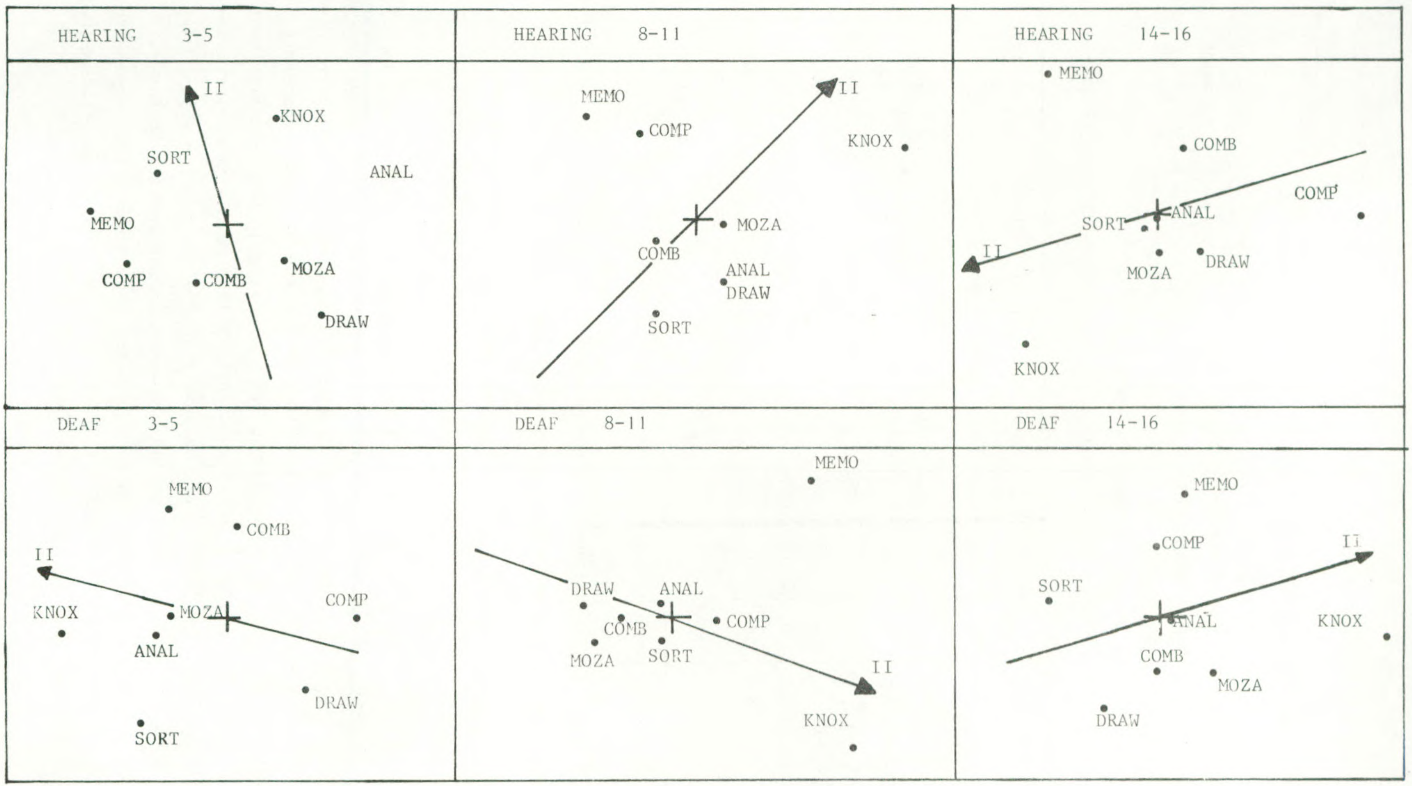
(separate analyses compared to the common three-mode analysis)

Subtest	nr. sub-test	comm. anal.	Hearing			Deaf			Abbrev. in Fig. 2
			3-5	8-11	14-16	3-5	8-11	14-16	
Mosaic	1	41	-2	0	4	-1	-1	-1	MOZA
Analogy	5	40	-7	0	5	-1	-1	-1	ANAL
Combinations	3	39	-0	2	-1	-2	-2	-1	COMB
Drawing	2	37	-4	2	3	-3	-3	1	DRAW
Sorting	6	34	1	1	1	-1	-1	1	SORT
Completion	4	32	3	0	-9	-2	-2	0	COMP
Memory	7	30	5	-2	-4	6	6	0	MEMO
Knox	8	28	6	-5	-5	5	5	-3	KNOX
% explained variance		45	0	-4	-2	2	0	5	

Note: the entries for the separate analyses indicate their difference with the common overall three-mode analysis.

Comparing the first components of the separate analyses given in Table 4 with those of the simultaneous solution given in Table 2 confirms our earlier conclusion about the near identity of the solutions. In Fig. 2, representing the second and third components we have drawns by eye the directions of the common second component (the third would be perpendicular to it), illustrating that the plane defined by these components is generally the same for all groups, but indicating at the same time that the groups differ mainly in which direction they deem slightly more important. This Fig. 2 gives at the same time the explanation why it was difficult to find a stable second component in the earlier factor analysis. It is not enough to inspect just the second and third components by themselves. It is the spatial arrangement which needs to be inspected, especially because the components carry nearly equal weights.

Fig. 2. *Subtest loadings for separate analyses per group* (second versus third components)



Note: component 2 horizontal; component 3 vertical; + indicates origin; all figures have the same scale.

4. DISCUSSION

Conspicuously absent from the above analyses is any mention of mention of transformations (rotations) of the common structure. When using test batteries like the S.O.N., one generally prefers components which show 'simple structures', i.e. one prefers an orientation of the coordinate axes such that each subtest has a high loading on as few, not necessarily orthogonal, components as possible. In this way specific tests can be associated with specific axes which may or may not be correlated.

Also in the present case one could attempt to find such simple structures. A varimax rotation (Kaiser, 1958) gives the result shown in panel B of Table 2, but it is not clear to me whether this varimax solution is a stable one considering the near-equality of the second and third eigenvalues. In other words it is unclear if the varimax solution should be preferred above the principal component one on technical grounds.

In section 3 it was shown that the common component space from the three-mode analysis is equally shared by all groups. This implies that one can obtain a very similar space by analysing the pooled correlation matrix based on the averages from the group correlations. In other cases with large differences between the groups this will not be the case. In certain circumstances, for instance in the test manual of the S.O.N., one might consider presenting only the analysis of the pooled correlation matrix as this analysis will be simpler to explain and understand. In passing one could then note that the representativeness of the structure from the pooled correlation matrix was verified with other means, i.e. three-mode principal component analysis.

5. CONCLUSION

The structure of the subtests of the 1958 S.O.N. is practically identical for all the age groups investigated both for deaf and hearing children, and the structure is of roughly equal importance to each group. In other words the designers of the S.O.N. succeeded in constructing adequate parallel procedures for their target groups. In the same token, the S.O.N. cannot be used for investigating changes in the nature of intelligence in children, if such changes exist.

Apart from the substantive conclusions, it is evident that three-mode principal component analysis can be a useful technique for simultaneous analysis of information from several groups to investigate their differences and common characteristics.

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