KM 8(1982)
pag 138-144

NETHERLANDS CENTRAL BUREAU OF STATISTICS

Department for Statistical Methods

P.O. Box 959, 2270 AZ VOORBURG, The Netherlands

MARSHALL AND GORMAN'S MARKOV CHAIN MODEL

FOR OCCUPATIONAL PRESTIGE

Jos Dessens *)

Wim Jansen *)

Albert Verbeek *)**)

Abstract

This report scrutinizes the Markov chain model for occupational prestige proposed by Marshall and Gorman. It is shown that the results of their application are highly unstable.

*) Department of Theory and Methodology of Sociology, Sociological Institute, University of Utrecht.

**) The Netherlands Central Bureau of Statistics.

The views expressed in this paper are those of the authors and do not necessarily reflect the policies of the Netherlands Central Bureau of Statistics. We thank Angela Wubbels for helpful comments on a preliminary version.

BPA no.: 12932-82-M1
September 24, 1982

Proj. M1-81-314
First draft

1. Introduction

The common procedure of measuring occupational prestige requires judges or respondents to make an ordering on a number of jobs, or to make a number of pairwise comparisons. Marshall and Gorman (1975) have proposed a much simpler procedure. It only takes two survey questions:

- "What do you consider your main job or occupation?"
- "If you had your choice of jobs, or could do the kind of work you really want to do, what would it be?"

The answers are conveniently represented by an r×r contingency table (f_{ij}), with

f_{ij} = number of respondents with main job i, and preferring to have job j, $(i,j=1,\ldots,r)$.

r = number of categories of jobs.

If this table is standardized row wise, to make the rows add to 100%, it can be regarded as a transition matrix, $M=(m_{ij})$. In formula

$$m_{ij} = 100{*}f_{ij}/(f_{i1} + \ldots f_{ir}) =$$

= percentage of respondents with main job i that prefer job j.

If $g=(g_1,\ldots g_n)'$ is the frequency distribution of current main jobs, then the vector-matrix product $g'M$ is the distribution of jobs as preferred.

2. Marshall and Gorman's algorithm

Marshall and Gorman argue as follows. Suppose the Good Fairy flicked her fingers and fulfilled everyone's occupational wishes. Then again after a while certain people would get dissatisfied, and they would want to change. Suppose She grants this again, and again, and again. After each transition we order all occupations according to their frequencies. As soon as this ordering stabilizes after a number of transitions, the Good Fairy is relieved from her task, and this ordering is taken as the 'final' ordering of occupational prestige.

Marshall and Gorman claim that this final ordering can be interpreted as the ordering by prestige. The rationale seems to be that the prestige of a job is one of the major determinants for preference; the more people prefer a job, the more prestigeous it is. Although this reasoning is not beyond criticism, we will not discuss it here, because this interpretation is not essential for our paper.

The following remark is closer to the core of our discussion, as it touches the interpretation of the final equilibrium of a Markov chain. Even if this equilibrium is uniquely determined, it is dynamic rather than static. When equilibrium is reached after many transitions, each new transition will feature a number of people actually changing jobs. Only at the aggregate level of classes of occupations there is stability. The inflow into each occupation is equal to the outflow. In other words: people change, but society as a whole does not.

In order to make this procedure computable, Marshall and Gorman assume that all transition matrices will be the same as the one at step one. Note that the transition matrices at later steps can hardly be measured empirically. This Markov chain assumption may be unrealistic, but it certainly leads to a model that is of interest in its own right.

3. Empirical study

Marshall and Gorman distinguish 28 categories of jobs: $r=28$, M is a (28×28)-matrix, with 784 cells. They interviewed 680 respondents. So the matrix of observations had very many zeroes, and many transition probabilities were estimated rather poorly. This is a vague indication for possible trouble in the analysis. Their population consisted of those who worked at least 30 hours per week, and for whom both the present and preferred occupation were codable.

The unstability of their procedure is investigated by studying the effect of a small perturbation of the observations. After all, another sample of 680 respondents most certainly would have produced a different observed transition matrix. Our first try was to make a minor change for only 2 of the 680 respondents. The effects on the ordering are considerable. We moved 2 respondents with i=j=3, one to the (i=3, j=2)-cell, the other to the (i=3, j=28)-cell. This modification is not totally unrealistic because the categories 2,3, and 28 are rather close in rank:

2 = architects and engineers;

3 = fee for service professionals;

28 = non-medical helping professions.

Table 1 shows the changes in rank, caused by this minor modification.

Table 1. Changes in rank caused by modifying 2 observations out of 680

occupational category	rank by Marshall and Gorman	rank after modification
3 fee for service professionals	1	6
college professionals	2	1
teachers, except college	3	2
writers, artists and entertainers	4	4
28 non-medical helping professions	5	3
farmers	6	7
managers	7	5
registered nurses	8	9
2 architects and engineers	9	8
proprietors	10	10
health technicians and therapists	11	12
financial sales and wholesalers	12	11
(Other ranking unchanged, except:)	..	..
general clerical workers	21	22
food service workers	22	21

4. Explanation of the instability

Having made the Markov chain assumption, a well developed mathematical theory becomes available. In this section we mention two important results and their consequences, See e.g. Roberts (1976) for technical terms for Markov chain theory.

First, repeated fulfilment of everyone's wishes yields the following sequence of frequencies:

present	g'
after 1 step	$g'M$
after 2 steps	$(g'M)M=g'M^2$
after 3 steps	$g'M^3$
...	...

It is well-known that this sequence generally converges to the left eigenvector of M, corresponding to the largest eigenvalue (which is 1 for a transition matrix). See e.g. Wilkinson (1965), 9.3. Let us denote this eigenvector by h'; i.e.

$$h'M = h'$$

and

$$h' = \lim_{n\to\infty} g'M^n$$

(independent of g'!). So, if all frequencies in this limit (i.e. all components h_i, $i=1,\ldots,r$ of h) are different, then the ordering produced by Marshall and Gorman is the same as the ordering given by h. I.e. occupation i ranks higher than occupation j if and only if $h_i>h_j$.

Now consider the situation that some occupations, say i and j, are equally frequent in the limit. I.e. $h_i=h_j$. Especially $h_i=h_j=0$ may occur rather frequently. Marshall and Gorman brake this tie by not going quite to the limit. They stop at $g'M^n$ as soon as 'the order stabilizes'. I.e. they stop if for all $m\geq n$ the order given by $(g'M)^n$ is the same as the order given by $(g'M)^m$. Quite likely this procedure will break all or most ties. In the Marshall and Gorman data we find that the resulting Markov chain has only one 'absorbing state', namely category i=3 (fee for service professionals). Thus, the third component of h equals N, and all other components of h are zero:

$$h_3 = N$$

and

$h_i = 0$ for $i=1,2,4,5,\ldots,r$.

This gives a very uninteresting order, and some way to break the many ties is certainly needed. Marshall and Gorman's algorithm does break all ties, but the result does depend on the initial state g, whereas h does not depend on g. This is somewhat unsatisfactory since Marshall and Gorman's ordering is supposed to reflect people's true wishes, rather than the accidental equilibrium established on the job market.

Secondly, from matrix calculus we have the following result about the uniqueness of h. There is a unique eigenvector corresponding to the largest eigenvalue if and only if the largest eigenvalue has multiplicity one. (For Markov chains this is true precisely for the class of 'regular Markov chains', see e.g. Roberts (1976) 5.5, 5.6 and exercise 5.5.18.) But in this particular data set the five largest eigenvalues of the observed transition matrix $\hat{M}$ are

1.00, .99, .85, .84, .83 .

Note that the first two eigenvalues are nearly equal; the largest eigenvalue nearly has multiplicity two. We conjecture that this means that any probability vector in the plane spanned by the first two eigenvectors is the leading eigenvector of a matrix M^* that differs only very little from $\hat{M}$. I.e. small changes in M may rotate the leading eigenvector over 90 degrees; the result is as unstable as can be. This is the main explanation of the unstability displayed in table 1.

The observed instability can be attributed to two independent causes. The first possible cause is that the population value of the transition matrix may have a second eigenvalue close to the first. The second cause is related to the number of observations. The observed transition matrix and its second eigenvalue may deviate from the population values and more so, if the number of observations per cell is small.

5. Conclusions

The Markov chain assumption is hard to justify empirically. Also the interpretation of the dynamic stability of the limit distribution as the real order of occupational prestige is not beyond discussion. But if these assumptions are accepted, one should test the stability of the procedure by checking that the second eigenvalue of the observed transition matrix is much smaller than 1. Marshall and Gorman's data fail this test.

References

Marshall, K.P. and B.L. Gorman, 1975, Occupational prestige from preference orientations via the Markov chain: an alternative to respondent rating techniques. Social Science Research 4, pp.41-64.

Roberts, F.S., 1976, Discrete mathematical models, with applications to social, biological and environmental problems (Prentice Hall, Englewood Cliffs, New Jersey).

Wilkinson, J.H., 1965, The algebraic eigenvalue problem (Clarendon Press, Oxford).