6) We also made estimations without the assumption of autocorrelation, i.e. $R = 0$. The maximum value of the log-likelihood function turned out to be 8.5 smaller. This implies that the hypothesis of autocorrelation in the sense of relation (24) is not rejected at a 95 per cent level of confidence (for 9 degrees of freedom a value of .95 of the distribution function $F(x)$ of the χ^2-distribution is reached at $x = 16.92$ which is slightly less than 2×8.5).

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Appendix

The data: 1 = food etc, 2 = durables, 3 = miscellaneous. Columns (1), (2) and (3) are amounts per household in thousands of guilders. The price index for 1970 is equal to 100. $\tilde{r}$ is calculated according to relation (12) ($\bar{\bar{r}} = 4.09$). $\bar{Y}$ = total disposable income per household in thousands of guilders. $\dot{p}$ = change in per cents per year of the consumer price index.

Sources: Somermeyer and Bannink (1973), Table 7.A.1, Van Daal and Louter (1979), Table 1 and Central Bureau of Statistics (1975), section U.

	(1) p_1q_1	(2) p_2q_2	(3) p_3q_3	(4) p_1	(5) p_2	(6) p_3	(7) $\tilde{r}$	(8) $\bar{Y}$	(9) $\dot{p}$
1949	1.862	1.338	1.673	0.481	0.643	0.376	-0.862	4.79	6.31
1950	2.095	1.516	1.792	0.528	0.716	0.390	-0.862	5.30	9.13
1951	2.285	1.572	1.943	0.581	0.825	0.430	-0.574	5.83	9.62
1952	2.366	1.388	1.983	0.596	0.752	0.455	-0.593	5.98	0.00
1953	2.454	1.444	2.056	0.589	0.735	0.452	-0.824	6.45	0.00
1954	2.636	1.647	2.232	0.606	0.753	0.479	-0.804	7.27	4.56
1955	2.705	1.848	2.437	0.618	0.759	0.490	-0.795	8.24	2.18
1956	2.900	2.071	2.644	0.636	0.743	0.514	-0.151	8.69	0.00
1957	3.014	2.079	2.764	0.673	0.759	0.552	0.704	9.17	6.38
1958	3.028	1.989	2.864	0.672	0.763	0.569	0.281	2.37	2.33
1959	3.112	2.067	2.987	0.688	0.762	0.584	0.089	9.53	0.98
1960	3.185	2.325	3.237	0.690	0.767	0.607	0.089	10.39	3.91
1961	3.384	2.514	3.360	0.649	0.772	0.632	-0.170	10.92	1.57
1962	3.547	2.722	3.639	0.716	0.774	0.653	0.089	11.54	2.16
1963	3.858	3.000	3.978	0.737	0.791	0.690	0.118	12.60	3.63
1964	4.114	3.432	4.458	0.798	0.828	0.739	0.867	14.59	7.58
1965	4.481	3.773	4.873	0.828	0.837	0.780	1.223	15.93	5.32
1966	4.693	3.791	5.451	0.879	0.867	0.826	2.174	16.92	6.35

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Why do Demand Functions Violate Homogeneity Tests?

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I. Introduction

There is overwhelming empirical evidence that systems of Marshallian demand equations based on current prices and current total expenditure when estimated in levels violate homogeneity and suffer from serially correlated residuals.* Typically the introduction of time trends tends to ameliorate but not remove these problems. When estimated in first differences as is the case in the Rotterdam model, especially with the inclusion of intercepts, both problems tend to be reduced - see for example, Theil (1975) who manages to accept homogeneity. However, the stochastic specification of first difference models is difficult to justify on economic grounds and implies a random walk error with unbounded variances in levels - see Davidson, Hendry, Srba and Yeoh (1978) for methodological criticisms of first difference models. Since homogeneity is the most basic requirement of consistency with the budget constraint, this fundamental flaw leaves little point in going on to test symmetry restrictions. There is further evidence, see for example Deaton and Muellbauer (1980b), that the imposition of homogeneity is associated with an increased tendency for the residuals to be positively autocorrelated.

There has unfortunately been no really systematic effort by demand analysts to trace the sources of this fundamental contradiction between theory and facts. Perhaps the difficulty is that there are too many alternative explanations. I shall provide a thumb nail sketch of the theory and list some of the possibilities, most of which were discussed in Deaton and Muellbauer (1980a), D - M for future reference, p.80-82. Exploring one of them in detail is the main subject of this paper.

It is usually assumed that the representative consumer has a utility function

$$u = U\ (v_a(q_t),\ v_{a+1}(q_{t+1}),\ \ldots)$$

*Barten (1969) is the first systematic system test of homogeneity and finds these results.

where q_t is the vector (q_{it}) of purchases under consideration at time t and a is the consumer's age. This utility function is separable in current purchases and U( ) but not v_t( ) could include all kinds of other arguments such as household characteristics, future purchases, leisure and indeed any purchases outside the vector under consideration. Good i is bought at price p_{it} at time t and $x_t = \sum_i p_{it} q_{it}$ is total expenditure. Then the overall utility maximization problem can be broken into finding at one stage the optimal x_t along with making optimal plans for allocation over the life cycle, between goods and leisure etc., and optimally suballocating x_t into $(p_{it} q_{it})$ given p_t, at a second stage. The solution to this second problem, assuming that p_{it} is independent of the amount purchased, is

$$q_{it} = g_{ia} (x_t, p_t)$$

Since the representative consumer would be assumed to have a constant age, the function g_{ia} ( ) would not alter over time.

Homogeneity violations (as well as serially correlated residuals) might be caused by any one or several of the following:

1. Separability Violations

Several types are worth considering:

(a) leisure: if leisure is not separable from goods but the budget constraint is still linear, then x_t should include leisure expenditure and the price vector should include the wage, see e.g., D - M, Ch.4.1. However, estimates by Abbott and Ashenfelter (1976), Phlips (1978)* still find significantly serially correlated residuals and homogeneity tested by Abbott and Ashenfelter fails even in their first differenced

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* To be fair, Phlips's model which is based on habit formation finds significant serial correlation only in the equation for money balances.

specification. If, instead of regarding leisure as chosen on a linear budget constraint, it is regarded as given in the short run whether because of unemployment or costs of adjustment, non-separability implies that the leisure level should enter the demand functions. Blundell and Walker (1981) test and reject separability in a cross-section context. Barnett (1979) uses an ingenious adjustment of the wage to convert it to a shadow price of leisure relevant when households are rationed. He does not test homogeneity but his Rotterdam specification shows no sings of serial correlation. Deaton and Muellbauer (1981) suggest functional forms for estimating such rationed models.

(b) <u>other goods</u>: Deaton (1981) finds that treating housing as a rationed good significantly ameliorates but does not eliminate homogeneity failures and serial correlation in the residuals for non-durable goods.

(c) <u>intertemporal</u>: This would imply that one could not separate consumption function estimation from demand systems estimation since assets, price and income expectations should enter individual demand equations. I know of no study based on non-separability.

2. <u>Aggregation over people</u>

A representative consumer exists only under restrictive conditions (see D - M, Ch.6). In general, aggregate demand equations should depend upon the distribution of total expenditure, demographic variables and their interactions. Omitted variables of this type could, in principle, explain both failure of homogeneity and serially correlated residuals and might account for the significance time trends often take on in demand studies.

3. <u>Aggregation over goods</u>

It is typical to estimate systems for broad categories of goods and use a separability or Hicks aggregation argument (see D - M, Ch.5) to group detailed commodities. I find these arguments empirically fairly plausible.

4. Taste and quality change

I shall discuss habit formation below. Changes in the relationship between market goods and their utility yielding characteristics are likely to have trend like effects and cannot plausibly explain homogeneity failures when trends are included.

5. Uncertainty about prices or budgets

As long as consumers do not make systematic forecasting errors, it seems implausible that behaviour under uncertainty could explain homogeneity failures. However, systematic errors, as for example in Deaton's (1977) model of the savings ratio under rational money illusion, could do so. Whether such errors occur and if so how large they are is a question of some controversy.

6. Functional form

Flexible functional forms approximate unknown preferences perfectly at a point. The approximation may become poor for extreme values of relative prices and total expenditure in real terms. In most aggregate time series the data variation is probably not extreme enough to make this a cause of homogeneity failure. However, non-flexible functional forms such as the Linear Expenditure System tend to show more evidence of mis-specification and for these the restrictiveness of functional form may play a part in homogeneity failures.

7. Durable goods and habits

When goods are durable it is the services derived from their stocks rather than purchases which yield utility, as has long been recognized. In some ways habits are like negatively durable goods: an increase in current purchases adds to the habit which raises the effective cost in the future instead of, as with durables, adding to the stock and hence lowering the effective cost in the future. As we shall see, the implication of the alteration in the relevant concept of utility yielding quantities is to alter the definition of prices and total expenditure and this can account for homogeneity failures and serially correlated residuals in demand equations.

In the next section, this seventh hypothesis will be considered in some detail. I shall attempt to show fairly precisely what kind of mis-specifications of conventional demand systems result when goods are durable or habit forming. As we shall see, this hypothesis can account for some facts not easily explained by the other six.

In section III, I develop the implications of habit and durable goods for the specification of the consumption function in the light of Hall's (1978) version of the life cycle hypothesis. There is also a brief discussion of the integration of demand systems with the aggregate consumption function.

In section IV, I consider Anderson and Blundell's (1981) adaption of the methodology of Davidson, Hendry, Srba and Yeoh (1978) to systems of demand equations. Homogeneity and symmetry can be accepted on their evidence as long run properties of demand systems.

Appendix 1 considers consequences of a somewhat more general formulation of habit formation and durability. Appendix 2 considers the question of what preference structure implies within period non-homotheticity but intertemporal homotheticity.

II. Durable Goods and Habits

For durable goods, it is usually assumed that the service flow on which utility depends is proportional to the stock S_{it} so that the latter is itself treated as the utility giving quantity. Typically it is assumed that

$$S_{it} = q_{it} + (1-\delta_i) \; S_{it-1} \tag{1}$$

where $0 \leq \delta_i \leq 1$ is the rate of deterioration. When $\delta_i = 1$ the stock is equivalent to the flow and we have a non-durable good. A generalization of this model would replace S_{it} by

$$S_{it} - c_i \; (S_{it} - (1-\delta_i) \; S_{it-1} = (1-c_i)(S_{it} - \alpha_i \; S_{it-1}) \tag{2}$$

say, as the argument in the utility function where c_i represents a marginal adjustment cost equivalent to a service loss from the stock. However, we shall assume $c_i = o$ and hence $\alpha_i = o$ in this section.

Habit formation is usually represented something as follows, see Pollak (1970), Phlips (1972, 1974), and Spinnewyn (1981): the argument in the utility function is given as

$$q_{it} - \phi_i S_{it-1} \tag{3}$$

where $o \leq \phi_i \leq 1$ and S_{it} is defined by (1). The argument in the utility function is then

$$S_{it} - (1-\delta_i + \phi_i) S_{it-1} = S_{it} - \alpha_i S_{it-1} \tag{4}$$

say. The idea of (3) is that the effective satisfaction given by good i is reduced by ϕ_i times the amount of habit which has been developed. Buying more of the good now therefore increases the future cost of satisfying this want.

However, the essential idea of habit formation can be represented even more simply. Suppose $\delta_i \geq 1$ and $\phi_i = -(1-\delta_i)$. Then the argument in the utility function is S_{it}. Every period the value in efficiency units from consuming his purchase q_{it} is reduced by an amount proportional to last period's value in efficiency units. This seems quite natural though it is a special case with $\alpha_i = o$ of (4) in a similar sense to which (2) is a special case of (1). It has the great advantage that the traditional analysis of the demand for durables in an intertemporal setting which has been standard since Cramer (1958) can be immediately applied to habit forming goods but with $\delta_i < 1$ for durables, $\delta_i = 1$ for non-durables and $\delta_i > 1$ for habit forming goods. The standard expression for the user cost of durables therefore applies to habit forming goods.

Spinnewyn (1981) noted that for (4) also, one could define a user cost price and so showed that the analysis of a forward looking consumer who rationally takes into account the effect of his current purchases on his future habit formation was a standard problem of maximization subject to a linear budget constraint. Phlips (1974) and Lluch (1974) had earlier considered the problem to be somewhat intractable.

For the present we assume that the utility function is

$$u = v\,(S_{1t}, \ldots, S_{nt};\ S_{1t+1}, \ldots, S_{nt+1}; \ldots, S_{1T}; \ldots, S_{nT}) \tag{5}$$

where we have abstracted from leisure in different periods by assuming separability in stocks of some overall utility function. The period to period budget constraint is

$$A_j = A_{j-1}\,(1+r_j) + y_j - \sum_{i=1}^{n} p_{ij}\, q_{ij} \tag{6}$$

where A is the asset level, r the interest rate, y non-property income and p_{ij} the price of good i in period j.

As is well known, see for example D - M, Ch.4.2, (6) can be combined into the life cycle budget constraint

$$W_t = \sum_{j=t}^{T} \sum_{i=1}^{n} \hat{p}^{*}_{ij}\, S_{ij} \tag{7}$$

where life cycle wealth $W_t \equiv A_{t-1}\,(1+r_t) + \sum_{i=1}^{n} p_{it}\,(1-\delta_i)\, S_{it-1} + \tilde{y}_t$ (8)

and where $p^{*}_{ij} = p_{ij} - (1-\delta_i)\, p_{ij+1}/(1+r_{j+1})$ (9)

and ˆ represents the discounting operation of multiplying by

$\left(\frac{1}{1+r_{t+1}}\right)\left(\frac{1}{1+r_{t+2}}\right)\cdots\left(\frac{1}{1+r_{t+T}}\right)$ and where

$$\tilde{y}_t = y_t + \hat{y}_{t+1} + \hat{y}_{t+2} \ \cdots + \hat{y}_T \tag{10}$$

which is the discounted sum of expected non-property incomes.

The plans which solve this problem take the form

$$S_{ij} = G_{ij} \quad (\hat{p}^*_j\ W_t) \tag{11}$$

and those at $j = t$ are actually put into practice. Note that the discounted prices correctly measure the marginal cost of variations in S_{ij} taking into account the future benefits of adding to stock and the future costs of adding to habits.

We can define $$\sum_{i=1}^{n} \hat{p}^*_{ij}\ S_{ij} = x_j \tag{12}$$

This is period j's expenditure on stocks. Under weak intertemporal separability

$$S_{ij} = g_{ij}\ (\hat{p}^*{}_j,\ x_j) \tag{13}$$

Thus at t

$$q_{it} = g_{it}\ (p^*_{t},\ x_t) - (1-\delta_i)\ S_{it-1} + \eta_{it} \tag{14}$$

where η_{it} allows for realization errors in carrying out the plan conditional upon x_t. The lagged stock is

$$S_{it-1} = \sum_{\theta=1}^{\infty} (1-\delta_i)^{\theta}\ q_{it-\theta} \tag{15}$$

and is thus computable given δ_i and some initial stocks S_{io}. Similarly

$$x_t = \sum_{i=1}^{n} p^*_{it}\ S_{it} = \sum_{1}^{n} \left[p_{it} - (1-\delta_i)\ p_{it+1}/(1+r_{t+1}) \right] S_{it} \tag{16}$$

where $S_{it} = q_{it} + (1-\delta_i)\ S_{it-1}$. Thus, estimating (14) is a feasible but not easy non-linear estimation problem.

Note that in particular $x_t \neq \sum_{i=1}^{n} p_{it}\ q_{it} = e_t$ which is conventionally measured expenditure in goods. We now want to get an approximate relationship between x_t and e_t.

$$S_{it} = q_{it} + (1-\delta_i)\ q_{it-1} + (1-\delta_i)^2\ q_{it-2} + \cdots \tag{17}$$

Suppose $\delta_i = \delta$, all i. Weighting the S_{it} and q_{it} by some fixed price vector we can define aggregates $\bar{S}_t$, $\bar{q}_t$ such that

$$\bar{S}_t = \bar{q}_t + (1-\delta)\,\bar{q}_{t-1} + (1-\delta)^2\, q_{t-2} + \ldots$$

$$= q_t/(1-(1-\delta)L) \tag{18}$$

where L is the lag operator. We can approximate (16) by

$$x_t \simeq (\bar{p}_t - (1-\delta)\,\bar{p}_{t+1}/1+r_{t+1})\,\bar{S}_t = \bar{p}_t\,\bar{S}_t\,(1-(1-\delta)/(1+r^*_{t+1}))$$

$$\simeq \bar{p}_t\,\bar{S}_t\,(\delta + r^*_{t+1})$$

where r^*_{t+1} is the expected real rate of interest. With r^*_{t+1} constant, (18) implies

$$x_t/\bar{p}_t - (1-\delta)\,x_{t-1}/\bar{p}_{t-1} = (\delta+r^*)\,q_t = (\delta+r^*)\,e_t/\bar{p}_t \tag{20}$$

Now let us suppose that (13) is linear so that

$$p^*_{it}\,S_{it} = \Sigma\alpha_{ik}\,p^*_{kt} + \beta_i\,x_t + \eta_{it}$$

is the true model. What sort of relationship does (20) imply between q_{it} and e_t? Let us apply the transformation (20) to (21).

Then $(p^*_{it}/\bar{p}_t)\,S_{it} - (1-\delta)\,(p^*_{it-1}/\bar{p}_{t-1})\,S_{it-1}$

$$= \sum_k \alpha_{ik}\left[(p^*_{kt}/\bar{p}_t) - (1-\delta)\,p^*_{kt-1}/\bar{p}_{t-1}\right] + \beta_i(\delta+r^*)\,e_t/\bar{p}_t$$

$$+ \eta_{it}/\bar{p}_t - (1-\delta)\,\eta_{it-1}/\bar{p}_{t-1} \tag{22}$$

Suppose prices move in proportion and are expected to do so. Then

$$p^*_{it} = p_{it}\left[1 - (1-\delta)\,\bar{p}_{t+1}/\bar{p}_t\,(1+r_{t+1})\right] \simeq p_{it}\,(\delta+r^*_{t+1})$$

Then at a constant real rate of interest, (22) becomes

$$(p_{it}/\bar{p}_t)\,(S_{it} - (1-\delta)\,S_{it-1}) = \delta\sum_k \alpha_{ik}\,(p_{kt}/\bar{p}_t) + \beta_i\,e_t/\bar{p}_t$$

$$+ (1/\delta+r^*)\,(\eta_{it}/\bar{p}_t - (1-\delta)\,\eta_{it-1}/\bar{p}_{t-1}$$

Thus

$$p_{it}\, q_{it} = \delta \sum_k \alpha_{ik}\, p_{kt} + \beta_i\, e_t + (1/\delta + r^*)\, (\eta_{it} - (1-\delta)(\bar{p}_t/\bar{p}_{t-1})\, \eta_{it-1}) \qquad (23)$$

Apart from the moving average error term, this is identical to a conventional demand function.

However, suppose that goods do not have the same δ_i. One would expect (20) to remain approximately valid with δ now representing a weighted average of δ_i's. With $\delta_i \neq \delta$, (23) now reads

$$p_{it}\, (q_{it} - (\delta_i - \delta)\, S_{it-1}) = \sum_k \delta_k \;\; (\delta_k + r^*)/(\delta_i + r^*) \;\; \alpha_{ik}\, p_{kt}$$

$$+ (1/\delta_i + r^*)(\eta_{it} - (1-\delta)(\bar{p}_t/\bar{p}_{t-1})\, \eta_{it-1}) \qquad (24)$$

On the given assumptions about prices, the conventional linear Marshallian expenditure functions now suffer from two mis-specifications. Firstly, a moving average error and secondly, the omitted variable $p_{it}\, (\delta_i - \delta)\, S_{it-1}$. The first gives a negatively correlated component to the residuals. For a good of above average durability, the second mis-specification gives a positively auto-correlated component. If durability is average $\delta_i - \delta = o$ and the second mis-specification does not arise. If the good is non-durable $\delta_i - \delta = 1 - \delta$ and $S_{it-1} = q_{it-1}$. Since q_{it-1} and q_{it-2} are likely to be positively autocorrelated, this will give a positive autocorrelated residual component. However, for a habit forming good where $(1-\delta_i) < o$ S_{it-1} will tend to be negatively correlated wit S_{it-2}. Then both mis-specifications will give negatively autocorrelated residual

In this context, it is noteworthy that out of 8 commodity groups (excluding "durables"), Deaton and Muellbauer (1980b) p.319 find that drink and tobacco, which is conventionally thought to be the most habit forming category, is the only one with significantly (and indeed strongly so) negatively autocorrelated residuals.

The assumptions about prices needed to get (24) are not very realistic. To (24) need to be added further terms which reflect discrepancies between actual behaviour and these assumptions. These include $p_{kt}/\bar{p}_t - p_{kt-1}/\bar{p}_{t-1}$, $E_{t-1}[p_{kt}/p_{kt}(1+r_t) - 1/1+r^*_t]$ and $E_t[p_{kt+1}/p_{kt}(1+r_{t+1}) - 1/(1+r^*_{t+1})]$, all k, where $E_t(\ )$, $E_{t-1}(\ )$ make explicit the date at which forecasts are made. All three terms reflect the effect of non-proportional price movements. Finally $E_t(r^*_{t+1}) - r^*$ and $E_{t-1}(r^*_t) - r^*$ reflect the effects of variable real interest rates. It seems reasonable that most of these effects would be picked up by unrestricted rate of change terms for each good's price. The omission of such terms therefore seems a third mis-specification of conventional expenditure equations.

III. Implications of durable goods and habits for the consumption function

To explore the question of what the implications of the analysis so far are for the aggregate consumption function, we must begin by asking whether the usual assumptions about demand systems and the consumption function are mutually consistent. Demand systems always reflect non-homothetic preferences at the level of each period. Life cycle models of the consumption function usually assume homotheticity of life cycle preferences with respect to the period groups. Extending Gorman's (1959) price aggregation theorem, it is shown in Appendix 2 that the two sets of assumptions are consistent only if

(a) life cycle preferences are additive over the period groups

(b) within period preferences for each group belong to the PIGL class.

The PIGL class of preferences, see Muellbauer (1975, 1976), implies that the period t group cost function can be written in the form

$$x_j^\alpha = a_j^\alpha + u\,[(H_j a_j)^\alpha - a_j^\alpha] \tag{25}$$

where $\alpha \to o$ implies the PIGLOG class,

$$\ln x_j = \ln a_j + u \ln H_j \tag{26}$$

and a_j is linear homogeneous in p_j, H_j is homogeneous of degree zero in p_j. Moreover, as Appendix 2 shows, $\alpha < 1$ for concavity of life cycle preferences. Empirical evidence suggests values of α of zero or less which is consistent with concavity. But it is a remarkable result that when within period Engel curves are linear and life cycle preferences are homothetic and separable in periods, then consumption in any period is a perfect substitute for consumption in any other - with the unfortunate consequences entailed by this! More generally, the elasticity of intertemporal substitution is $1/1 - \alpha$ so that the parameter α governs both the shapes of Engel curves and the degree of substitution through time.

These results abstract from durability or habit formation but extend naturally for the redefined goods and budget constraint which are then appropriate. They have the implication, if the integration of demand systems and the consumption function are pursued on the usual assumptions, that functional forms of the PIGL class, for example AIDS, see Deaton and Muellbauer (1980b), be chosen to fit demands of the type (14).

Recently Hall (1978) had deduced some interesting implications for the life cycle consumption function of the assumption that expectations are formed rationally. In particular, for a single non-durable consumption good, he shows that consumption is a random walk with drift. The model implies that given last period's information set, the optimal forecast of current consumption is last period's consumption plus drift.

Hall's model can be explained as follows. Assume a single non-durable non-habit forming good and write (6) as

$$A_{t-1} = A_{t-2}\,(1+r_{t-1}) + y_{t-1} - p_{t-1}\,q_{t-1} \tag{27}$$

and (8) as

$$W_t \equiv A_{t-1}\,(1+r_t) + \tilde{y}_t$$

Then $\dfrac{W_t}{1+r_t} - W_{t-1} = A_{t-1} + \dfrac{\tilde{y}_t}{1+r_t} - A_{t-2}\,(1+r_{t-1}) - \tilde{y}_{t-1}$

$$= y_{t-1} + E_t\left(\frac{y_t}{1+r_t} + \frac{y_{t+1}}{(1+r_t)(1+r_{t+1})} + \ldots\right) - E_{t-1}\left(y_{t-1} + \frac{y_t}{1+r_t} + \frac{y_{t+1}}{(1+r_t)(1+r_{t+}}\right.$$

$$- p_{t-1}\,q_{t-1}$$

$$= \varepsilon_t^* - p_{t-1}\,q_{t-1} \tag{29}$$

where under rational expectations ε_t^* is the new information on permanent labour income arriving between t-1 and t. Assuming homothetic preferences so that

$$p_t\,q_t = k_a\,W_t \tag{30}$$

where k_a depends on age and real interest rates,

$$\frac{p_t\,q_t}{k_a(1+r_t)} - \frac{p_{t-1}\,q_{t-1}}{k_{a-1}} = \varepsilon_t^* - p_{t-1}\,q_{t-1}.$$

Since $x_t = p_t\,q_t$,

$$(x_t/p_t) = \frac{k_a}{k_{a-1}}\,(1+r_t^*)(1-k_{a-1})(x_{t-1}/p_{t-1}) + \varepsilon_t \tag{31}$$

where $\varepsilon_t = \varepsilon_t^*\,k_a\,(1+r_t^*)$ and $1+r_t^* = (1+r_t)\,p_{t-1}/p_t$.

With a constant real rate of interest, (31) is a random walk with drift which can be written as

$$(x_t/p_t) = (1-\gamma)(x_{t-1}/p_{t-1}) + \varepsilon_t. \tag{32}$$

With many commodities, as long as aggregate expenditure is still proportional to wealth, a relationship equivalent to (32) holds where p_t is the price index a_t which arises in the PIGL class of within period preferences.

If goods are durable or habit forming it can be shown that (32) continues to hold with $x_t = \sum_i p_{it}^* S_{it}$ and the price index $\bar{p}_t$ a function of the vector p_t^*. However, in (20) we obtained an approximate relationship between x_t and e_t which together with (32) implies:

$$e_t/\bar{p}_t = (1+\gamma)\, e_{t-1}/\bar{p}_{t-1} + (\varepsilon_t - (1-\delta)\, \varepsilon_{t-1})/(\delta+r^*) \tag{38}$$

Thus, while it remains true that by the theory $(1+\gamma)\, x_{t-1}/\bar{p}_{t-1}$ remains the best forecast of $x_t/\bar{p}_t$ given the information set available at $t-1$, the same is not true for conventionally measured expenditure: the reason is that at $t-1$, ε_{t-1} is known.

One interesting application of these ideas is in overcoming an endogeneity bias which potentially arises when equations such as (14) are to be estimated. Current expenditure whether x_t or e_t contains the current decision variables q_{it} and hence may be correlated with the disturbances η_{it}. In principle, equations such as (32) offer a way of obtaining instruments which should be uncorrelated with the η_{it}'s.

To end on a note of caution, there are a number of objections to models of the Hall type. These include the ommission of transitory consumption from (30) whose presence would have entailed moving average error components in (32) and additional ones in (38). More fundamentally, serious questions can be raised about the expectations assumption itself and about the assumption of no asymmetries in credit markets on which most versions of the life cycle hypothesis rest.

IV. An empiricist approach to short run dynamics

One of the implications of the specification analysis of section II was that comparing steady states, the conventional demand functions remain valid. In other words, in the long run but not the short run one should expect the properties of homogeneity and symmetry to be valid. Thus one might think of estimating functions which allow reasonably general short run dynamics but imply a long run solution of the conventional demand function form. This is very much the approach of Davidson, Hendry, Srba and Yeoh (1978) in the context of the consumption function: the long run solution there is a constant average propensity to consume out of income but one that depends on the steady state growth rate.

Anderson and Blundell (1981a, 1981b) have applied these general ideas to systems of demand equations. In the context of AIDS which they use, their approach can be explained as follows: Let

$$w^*_{it} \equiv \alpha_i + \sum_j \gamma_{ij} \ln p_{jt} + \beta_i \ln (x/P)_t \tag{39}$$

where w^*_{it} is the "steady state budget share" and is specified in the usual AIDS form where $\Sigma\, \alpha_i = 1$, $\sum_i \gamma_{ij} = o$, all j and $\sum_i \beta_i = o$

and $\gamma_{ij} = \gamma_{ij}$ for symmetry. Then posit for $i = 1, \ldots, n$

$$\Delta w_{it} = \sum_{j=1}^{n} a_{ij} \;\Delta\ln p_{jt} + a_{io}\;\Delta\ln (x/P)_t + \sum_{j=1}^{n-1} b_{ij}\;(w_{jt-1} - w^*_{jt-1}) \tag{40}$$

In a steady state all the Δ- terms are zero and $w_{it-1} = w^*_{it-1}$. Note that the last term which provides the equilibrating force in the system is summed over 1 to n-1. This is because $\sum_1^n w_{it-1} = 1$ so that one share is redundant. Adding up requires $\sum_i a_{ij} = o$, $j = o, 1, \ldots, n$ and $\sum_i b_{ij} = o$, $j = 1, \ldots, n-1$.

The general specification allows one to test a number of special cases.

(a) <u>The static model</u>: here $b_{ii} = -1$, $b_{ij} = o$ all $i \neq j$, $\gamma_{ij} = a_{ij}$ all i,j and $a_{io} = \beta_i$, all i.

(b) <u>The first difference model</u>: here $a_{ij} = \gamma_{ij}$, all i,j and $a_{io} = \beta_i$, all i but $b_{ij} = o$ all i,j.

(c) <u>The shares partial adjustment model</u>: here $a_{ij} = o$, all i and $j = o, 1, \ldots, n$.

The static model with a first order vector regressive process is another nested hypothesis.

On 5 non-durable categories for annual per capita Canadian data Anderson-Blundell are able to accept homogeneity and symmetry for the general form (40) but reject the static model, its autoregressive version and the partial adjustment model. They do not test the first difference model. Of the $\Delta \ell n\ p_{jt}$ terms, 7 out of 20 are significantly different from zero, while 8 out of 16 $(w_{jt-1} - w^*_{jt-1})$ terms are significantly different from zero.

Unfortunately, the formulation is so different from the durable goods/habits model discussed in section II, that there is little scope for interpreting their results specifically in those terms. Although there is no substitute for exploring such theory specific models, it is certainly much easier to estimate dynamizations of AIDS and other demand systems of the Anderson-Blundell type. However, for medium or large systems, the durable goods/habits model is <u>much</u> more parsimonious than systems such as (40).

To this implicit criticism of Anderson-Blundell an answer is possible though they do not discuss the issue. What we need is a structure of the

a_{ij} and b_{ij} which preserves adding up but is much more parsimonious. One might imagine that the key effects are the diagonal effects. Then it would make sense to make all the off-diagonal effects identical. Thus $a_{ij} = -a_{ii}/n-1$ and $b_{ij} = b_{ii}/n-1$, all $i \neq j$ which are all easily testable propositions. Then (40) has only around n more parameters than the durable goods/habits model and becomes a practical proposition for medium to large systems. Systems of the type (40) can therefore be recommended as a useful tool for empirical research.

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