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Consumer demand: on its dependence on prices, disposable income, wealth and the rate of interest

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In this paper a household is assumed to maximize utility over a planning cycle of some years while taking into account a budget restriction stating that total consumption, over the whole cycle, equals total (expected) income plus present minus final wealth, where all amounts are discounted by means of a rate of interest that is assumed to prevail for the whole planning cycle. The utility function is asymmetric: for the present period (i.e., period 1 of the planning cycle) all quantities consumed per budget item considered enter as arguments, while each other period only is represented by the total amounts of consumption expenditure in that period. We adopt a log-linear specification of the utility function resulting in simple expressions for the quantities consumed in period 1 per budget item in terms of present (disposable) income, expected incomes in the future, present and final wealth and the rate of interest (section 2).

These demand functions are aggregated over all households resulting in a system of equations expressing demands per household in disposable income, prices (both for period 1) and the rate of interest (section 3).

This per capita model is applied to a time series for the Netherlands for nominal as well as for real rates of interest (section 4). The results are compared with some other models (section 5). In the Appendix we present our data.

1. Introduction

In analyzing patterns of household behaviour on the basis of some data set we always need a theory to create some coherence in the mass of

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variables involved. There is no hope that the data speak themselves. Here we are interested in how quantities consumed per household depend on prices, income (per household) and possibly other explanatory variables such as the rate of interest and wealth. The theory we need for this analysis has, in our opinion, to be built in two stages. First, a theory on individual household behaviour has to give us a micro demand system. Secondly, this micro system has to be aggregated into a macro system relating per capita demand to per capita income (and wealth) and prices and the rate of interest.

Having done this we can expect that macro data speak more or less clearly in the sense that parameters of the macro model can be estimated such that explanations, interpretations and tests are possible. But in testing we have to be careful. In our opinion, this can never be done without at least maintaining some basic hypothesis. It seems to us a contradiction to say that it is possible to "test the whole theory" on the basis of a data set that just had to be structured by that theory because of lack of information in itself.

Such a basic hypothesis in this case is the postulate that a consumer always makes his decisions according to an optimization principle. He always tries to act such that some objective function is maximized while taking into account one or more restrictions. Of course, one can be wrong in specifying the objective function (its mathematical form and/or its detailedness) and the restrictions. This can be shown by the results of a test, but such a test does not refute optimizing behaviour as such[1]. We are no fanatics with respect to the belief in the optimization principle, though we admit that, because we observe that people obviously choose from a set of possibilities, there must be some criterion according to which this choice is made. We think that it is virtually impossible to find the "true" objective function and that, moreover, for practical purposes, it does not matter whether modelling a household as a utility maximizer is done from a belief or for more pragmatic reasons. We are convinced, however, that for our purpose (analyzing broad patterns of household consumption and savings behaviour) modelling the consumer as a utility maximizer is the most practical way of theorizing[2]. Even a quick glance in the literature supports this view.

This means that we only consider the household from outside, at some distance. We consider it as a "black box". We observe that some inputs in certain quantities "disappear" in it and we assume that these "reappear" as utility and that this transformation process can be described by a utility function. Given the household's budget, there are numerous input possibilities. Apparently, as we already said above, the household makes a choice and, therefore, there must be some choice criterion. The best assumption to make about this criterion is, for us, that it is utility maximization under the condition of the budget restriction. Theorizing from this criterion is flexible enough to take into account quite a lot of characteristics, more than many critics wish to admit. At the same time it is workable in the sense that it permits us to "stylize" the household such that, first, it can meaningfully be compared with another household by only comparing two vectors of characteristics that are not too big, and, secondly, aggregate behaviour can be analyzed on the basis of a macro theory derived from this theory of the individual household by means of a sound aggregation procedure.

This aggregation procedure is dealt with in section 3; it is a substantial part of the paper. The exposition of the micro model precedes in section 2. We try to incorporate savings behaviour and commodity demand into one model by assuming that these decisions are interdependent and, therefore, are determined simultaneously; furthermore, we assumed that the household looks ahead more than one period.

In section 4 we present some empirical results for the period 1949-1966 for Dutch data. We chose this data set because it enabled us to make comparisons (in section 5) with the work of Somermeyer and Bannink (1973) <u>who inspired us to this paper</u>. Finally, we also made comparisons with the so-called Extended Linear Expenditure System of Lluch (1973), who explains consumer demand from prices and disposable income. The latter model as well as our model have the advantage, in comparison with "classical" demand systems, that disposable income figures as an explanatory variable instead of the more or less artificial variable "total amount spent on consumption"; the latter amount can now be determined endogeneously.

We end this introduction with a remark that, unfortunately, is not superfluous. We are aware of the restrictions of our model. It is not suited to analyze the behaviour of an individual household in great detail. But we do not pretend to do that as we are interested in analyzing economy-wide streams of commodities consumed and amounts saved. If one is interested in, say, why people prefer certain brands of matchboxes to other brands, the fact that, existing or not, a household's "true" utility function is actually not to find, precludes adopting an optimization approach like presented in this paper. Here we have to do with the well-known truth "different models for different purposes". Keeping this in mind dissuades researchers from decrying each other.

2. The micro model

We assume that a family plans its consumption for a certain period (to be called period 1) such that consumption in other periods yet to come is also taken into consideration. Aspects of the nearest future (period 1) are taken into account in more detail than those with respect to periods that are more remote in future. Therefore, we assume that the family has a utility function with as arguments the quantities of the different commodities consumed in period 1 on the one hand, and, on the other hand, for the other periods under consideration, the total consumption amounts per period. With the term planning cycle we denote all periods taken together.

Furthermore, we take the mathematical form of the utility function as simple as possible, such that empirical work relating to period 1 can be done without knowledge of the endogenous variables relating to the other periods of the planning cycle.

For no family the sky is the limit. Therefore, (we assume that) they consider a budget restriction for the whole planning cycle: the total amount, over the whole planning cycle, spent on consumption equals total (expected) income plus wealth at the beginning of the cycle minus desired final wealth; all amounts have to be duly discounted, of course. Discounting all amounts to the end of period 1 yields the following planning-cycle budget constraint:

$$\sum_{\ell=1}^{L} C_{\ell}(1+r)^{1-\ell} = \sum_{\ell=1}^{L} Y_{\ell}(1+r)^{1-\ell} + W_o - W_L(1+r)^{1-L}; \tag{1}$$

see also Somermeyer and Bannink (1973, p. 41 and further) for a slightly different budget constraint.

In relation (1) the symbols have the following meaning:

L = number of periods (years, for example) of the planning cycle,

C_{ℓ} = total amount spent on consumption in period ℓ,

r = interest rate supposed by the family,

Y_{ℓ} = (expected) income in period ℓ,

W_o = the family's wealth at the beginning of period 1,

W_L = its desired wealth at the end of period L.

Note that the family considers only one interest rate for the whole cycle and that the interest over period ℓ is assumed to be paid at the beginning of period ℓ+1.

For the family's utility function we propose:

$$U = \sum_{k=1}^{K} \alpha_k \ln (q_k - \gamma_k) + \sum_{\ell=2}^{L} \beta_{\ell} \ln C_{\ell}, \tag{2}$$

where:

K = number of budget items considered in period 1,

q_k = quantity of commodity k consumed in period 1,

and where the α_k, β_{ℓ} and γ_k are parameters with:

$$\sum_{k=1}^{K} \alpha_k + \sum_{\ell=2}^{L} \beta_{\ell} = 1 \tag{3}$$

and

$$\gamma_k < q_k. \tag{4}$$

Furthermore,

$$C_1 = \sum_{k=1}^{K} p_k q_k, \tag{5}$$

where p_k is the price of commodity k in period 1.

About prices and interest rate several assumptions can be made. First, prices can be assumed not to change during the planning cycle; the rate of interest is then a nominal one. Secondly, r might be considered as a real interest rate in which the (expected) nominal rate of interest and the (expected) rate of inflation have been combined. For both assumptions the mathematics is the same; differences arise in the empirical implementation.

Maximizing (2) with respect to (1) and (5) while taking into account (3) and (4) yields:

$$p_k q_k = p_k \gamma_k + \alpha_k \Big(\sum_{\ell=1}^{L} Y_\ell (1+r)^{1-\ell} + W_o - W_L (1+r)^{1-L} - \sum_{k=1}^{K} p_k \gamma_k \Big) \tag{6}$$

for k = 1, ..., K, and

$$C_{\ell'} = \beta_{\ell'} (1+r)^{\ell'-1} \Big(\sum_{\ell=1}^{L} Y_\ell (1+r)^{1-\ell} + W_o - W_L (1+r)^{1-L} - \sum_{k=1}^{K} p_k \gamma_k \Big) \tag{7}$$

for $\ell' = 2, \ldots, L$[3].

Relations (6) can be considered as describing the family's consumption behaviour in period 1, whereas relations (7) describe the ideas, for the time being, that the family roughly has about future consumption. At the end of period 1 the family again applies a maximization procedure as described above with as (new) period 1 the old period 2 on the basis of possibly changed opinions about the length of the planning cycle, prices, interest rate, wealth and present and expected incomes. Hence the relations (7) have to be considered as first "guestimations" of future consumption and not as final decisions. For estimation purposes we, therefore, are only interested in relations (6). Fortunately, these relations do not contain the variables C_2, ..., C_L

that are unobservable in period 1; this is because of the simplicity of the utility function. The unobservability of the expected incomes Y_2, ..., Y_L can be overcome, in principle, by relating them to current income, e.g. by means of known age-income profiles.

On the basis of at least two cross-section data on consumption and income, for periods with different prices and interest rates, equations (6) could be estimated in principle. This will not be done here[4]. Instead we will aggregate (6) into a system of macro relations such that time series of aggregate data can be used in estimation. Using time series requires no assumption about the nature of period L; L is simply the length of the planning cycle. In cross-section analysis one has to make more specific assumptions with respect to L. Somermeyer and Bannink (1973) in their analysis of individual saving take L as the expected duration of the individual's life. In time series analysis, however, one can even give, as a result, a (very rough) idea of the average length of the planning cycles of the individual decision units.

3. The macro model

Indicating a household by h (= 1, ..., H) and assuming that we have at our disposal a time series of T observations of relevant data per household we rewrite, for h = 1, ..., H and t = 1, ..., T, relation (6) as follows:

$$\begin{aligned} p_{kt}q_{kht} = p_{kt}\gamma_{kh} + \alpha_{kh}\Big(&\sum_{\ell=1}^{L_{ht}} Y_{\ell ht}(1+r_t)^{1-\ell} + \\ &+ W_{oht} - W_{L_{ht}}(1+r_t)^{1-L_{ht}} - \sum_{k=1}^{K} p_{kt}\gamma_{kh}\Big), \end{aligned} \tag{8}$$

where we have assumed that the families may have different utility functions and face the same prices p_{kt} and interest rate r_t in each period t. Note that we distinguish between "historic time" (or period of observation), indicated by the symbol t, and periods of the planning cycle, indicated by ℓ.

Relations (8) will be aggregated over h to an aggregate demand system on the basis of some (heroic) assumptions. See also Somermeyer and Bannink (1973, ch. 5); we make variations on their theme. The first assumption is:

$$W_{oht} - W_{L_{ht}}(1+r_t)^{1-L_{ht}} = 0 \tag{9}$$

for all households h and all periods of observation t; existing wealth is carried over to the next planning cycle. Lack of data facilitated the making of this assumption[5]. One might argue that this means that the families do not save or dissave (apart from interest on initial wealth) and that, therefore, the accumulation or decumulation of wealth cannot be described by our model supplemented by assumption (9). Our argument in favour of (9) is that it just stresses the "occasional character" of a family's savings: for most families savings is a residual in our opinion. That, nevertheless accumulation or decumulation occurs is explained by the fact that the families revise their consumption plans and the fact that plans often appear not to be carried out for some reason.

Now we classify all households according to their "age class" a (being a function of the age of all the family's members) and to their social status s. For each a,s-group of families we assume (our second assumption) that there is zero correlation between the α_k parameters of the families on the one hand, and, on the other hand, the present and expected incomes as well as the γ_k parameters of the group's families. This leads to:

$$p_{kt}\bar{q}_{k,ast} = p_{kt}\bar{\gamma}_{k,as} + \bar{\alpha}_{k,as}\left(\sum_{\ell=1}^{L_{ast}} \bar{Y}_{\ell,ast}(1+r)^{1-\ell} - \sum_{k=1}^{K} p_{kt}\bar{\gamma}_{k,as}\right), \tag{10}$$

where the bars mean averages per household within the a,s-group.

For empirical applications based on (10) we have to make a number of additional assumptions in order to arrive at a model with time-independent parameters. Therefore, the third assumption we make is that for all $t = 1, \ldots, T$:

$$L_{ast} = L_{as}, \tag{11}$$

meaning that the length of a planning cycle only depends on a and s, i.e. "planning habits" do not shift in time.

A <u>fourth assumption</u> regards the discount factors $(1+r_t)^{1-\ell}$. We assume that these can be approximated by a quadratic function of the deviation of r_t from its average over all observations $t = 1, \ldots, T$. Defining $\bar{\bar{r}} = (\sum_t r_t)/T$, we state:

$$(1+r_t)^{1-\ell} = (1+\bar{\bar{r}})^{1-\ell}(1+\tilde{r}_t)^{1-\ell} \simeq (1+\bar{\bar{r}})^{1-\ell}\left(1-(\ell-1)\tilde{r}_t + \tfrac{1}{2}\ell(\ell-1)\tilde{r}_t\right), \tag{12}$$

with

$$\tilde{r}_t = (r_t - \bar{\bar{r}})/(1+\bar{\bar{r}}). \tag{13}$$

Further we assume:

$$\bar{Y}_{\ell,ast} = m_{\ell as}\bar{Y}_{1,ast}, \tag{14}$$

for $\ell = 2, \ldots, L_{a,s}$. This <u>fifth assumption</u> states that average prospective income in period ℓ ($\geqslant 2$) within each a,s-group is proportional to average income in period 1 for that group with a ratio that depends on ℓ, a and s, but not on time of observation; see Somermeyer and Bannink (1973, p. 187) who elaborate this assumption.

Inserting (12), (13) and (14) into (10) yields:

$$p_{kt}\bar{q}_{k,ast} = p_{kt}\bar{\gamma}_{k,as} + \bar{\alpha}_{k,as}\Big[\bar{Y}_{1,ast} + \tag{15}$$

$$+ \bar{Y}_{1,ast}\sum_{\ell=2}^{L_{as}} m_{as\ell}(1+\bar{\bar{r}})^{1-\ell}\left(1-(\ell-1)\tilde{r}_t + \tfrac{1}{2}\ell(\ell-1)\tilde{r}_t^2\right) +$$

$$- \sum_{k=1}^{K} p_{kt} \bar{\gamma}_{k,as}].$$

Equation (15) can be simplified in the following way. Defining:

$$A'_{as} = 1 + \sum_{\ell=1}^{L_{as}} m_{as\ell} (1+\bar{\bar{r}})^{1-\ell}, \tag{16}$$

$$B'_{as} = - \sum_{\ell=1}^{L_{as}} (\ell-1) m_{as\ell} (1+\bar{\bar{r}})^{1-\ell}, \tag{17}$$

$$C'_{as} = \tfrac{1}{2} \sum_{\ell=1}^{L_{as}} \ell(\ell-1) m_{as\ell} (1+\bar{\bar{r}})^{1-\ell}, \tag{18}$$

we can rewrite (15) as:

$$p_{kt} \bar{q}_{k,ast} = p_{kt} \bar{\gamma}_{k,as} + \bar{\alpha}_{k,as} [(A'_{as} + B'_{as} \tilde{r}_t + C'_{as} \tilde{r}_t^2) \bar{Y}_{1,ast} - \sum_{k=1}^{K} p_{kt} \bar{\gamma}_{k,as}]. \tag{19}$$

Because the $\alpha_{k,as}$ vary inversely with L_{as} (see (3)) and the A'_{as}, B'_{as} and C'_{as} increase as L_{as} increases, we cannot assume that the A'_{as}, B'_{as} and C'_{as} have zero correlation with the $\alpha_{k,as}$ variables. But if we consider, as the <u>sixth assumption</u>, all correlation coefficients between the A'_{as}, B'_{as} and C'_{as} on the one hand and the $\bar{\alpha}_{k,as}$ for all k = 1, ..., K on the other hand, as being equal then we get finally for all H households together:

$$p_{kt} \bar{q}_{kt} = p_{kt} \bar{\gamma}_k + \bar{\alpha}_k [(A + B\tilde{r}_t + C\tilde{r}_t^2) \bar{Y}_t - \sum_{k=1}^{K} p_{kt} \bar{\gamma}_k] \tag{20}$$

with

$$\bar{q}_{kt} = \frac{1}{H} \sum_{h} q_{kht} \tag{21}$$

$$X = \frac{1}{N} \sum_{a,s} X'_{as} (1+\rho), \tag{22}$$

where X = A, B, C; N is the total number of a,s-groups and ρ is the above mentioned correlation coefficient. With $\bar{Y}_t$ we denote the t-th observation of (labour) income per household in period 1 of the planning cycles; the index 1 is no longer needed; the bars in $\bar{q}_{kt}$, $\bar{\alpha}_k$ and $\bar{\gamma}_k$ denote likewise per household variables and parameters, respectively.

The model (20) is suited for time series estimation. This will be set out below. The number of parameters is as low as possible: 2K+3.

4. Some empirical results

From (20) we derive the following stochastic specification:

$$\frac{p_{kt}\bar{q}_{kt}}{\bar{Y}_t} = \frac{p_{kt}}{\bar{Y}_t}\bar{\gamma}_k + \bar{\alpha}_k(A + B\tilde{r}_t + C\tilde{r}_t^2 - \sum_{k=1}^{K}\frac{p_{kt}}{\bar{Y}_t}\bar{\gamma}_k) + u_{kt} \tag{23}$$

with u_{kt} a disturbance term.

We assume that for all disturbance vectors $u_t = (u_{1t}, \ldots, u_{kt})'$:

$$u_t = Ru_{t-1} + s_t, \tag{24}$$

with R a K×K matrix of coefficients of autocorrelation and s_t a K-vector of normally distributed variates with zero expectation such that:

$$E(s_t s_\tau') = \delta_{t\tau}.\Sigma, \tag{25}$$

where Σ, a K×K matrix, is independent of t and where $\delta_{t\tau} = 1$ if $t = \tau$ and = 0 otherwise. Because the shares in the left-hand side of (23) do not necessarily add up to 1, we can assume that Σ and R are non-singular.

Our data are presented in the Appendix. Together with (23), (24) and (25) and the assumption of normality of the vector s_t they give rise to a log-likelihood function that depends on the parameters of the model and on the elements of Σ and R. This log-likelihood function can be concentrated upon the parameters of the model by first eliminating R and, subsequently, eliminating Σ. The result is:

$$L^* = M - \tfrac{1}{2}(T-1) \log \det DD', \tag{26}$$

where M is a constant and:

$$D = V_{(1)} - V_{(T)}(V'_{(T)}V_{(T)})^{-1} V'_{(T)}V_{(1)}, \tag{27}$$

with $V_{(x)}$ the matrix which results after deleting the x-th row of the T×K matrix V consisting of residuals:

$$v_{tk} = \frac{p_{kt}\bar{q}_{kt}}{\bar{Y}_t} - \frac{p_{kt}}{\bar{Y}_t}\bar{\gamma}_k - \alpha_k(A + B\tilde{r}_t + C\tilde{r}_t^2 - \sum_{k=1}^{K} \frac{p_{kt}}{\bar{Y}_t}\bar{\gamma}_k). \tag{28}$$

For the details we refer to De Boer and Van Daal (1980).

In (28) the $\bar{Y}_t$ denote the disposable incomes per household in the Netherlands for the years 1949-1966. The p_{kt} with k = 1, 2, 3 are price indices (1970 = 100) in these years for (i) food beverages and tobacco, (ii) durables and (iii) other goods and services. The $\bar{q}_{kt}$ denote quantities consumed per household. The $\tilde{r}_t$ are computed according to (12) on the basis of the average yield per year (in per cents) in the period 1949-1966 of Government Bonds. Note that the $\bar{Y}_t$ in the Appendix include also non labor income which may cause some obscurity with respect to the carry-over of wealth to the end of the planning cycles; for the time being we shall ignore this.

In table 1 we present the estimation results:

TABLE 1. The estimates of relation (23).

	estimate	asymptotic standard error
γ_1	3.514	0.252
γ_2	1.471	0.241
γ_3	3.649	0.434
α_1	0.172	0.020
α_2	0.240	0.044
α_3	0.208	0.023
A	1.031	0.086
B	-0.038	0.013
C	0.014	0.007

As an estimate of the matrix R of elements of autocorrelation we found:

$$\hat{R} = \begin{bmatrix} -0.250 & 0.008 & 0.129 \\ -0.460 & 1.124 & -0.970 \\ -0.063 & 0.461 & 0.233 \end{bmatrix} \tag{29}$$

with eigenvalues $0.685 \pm 0.521i$ and -0.262; the determinant of $\hat{R}$ equals -0.194[6].

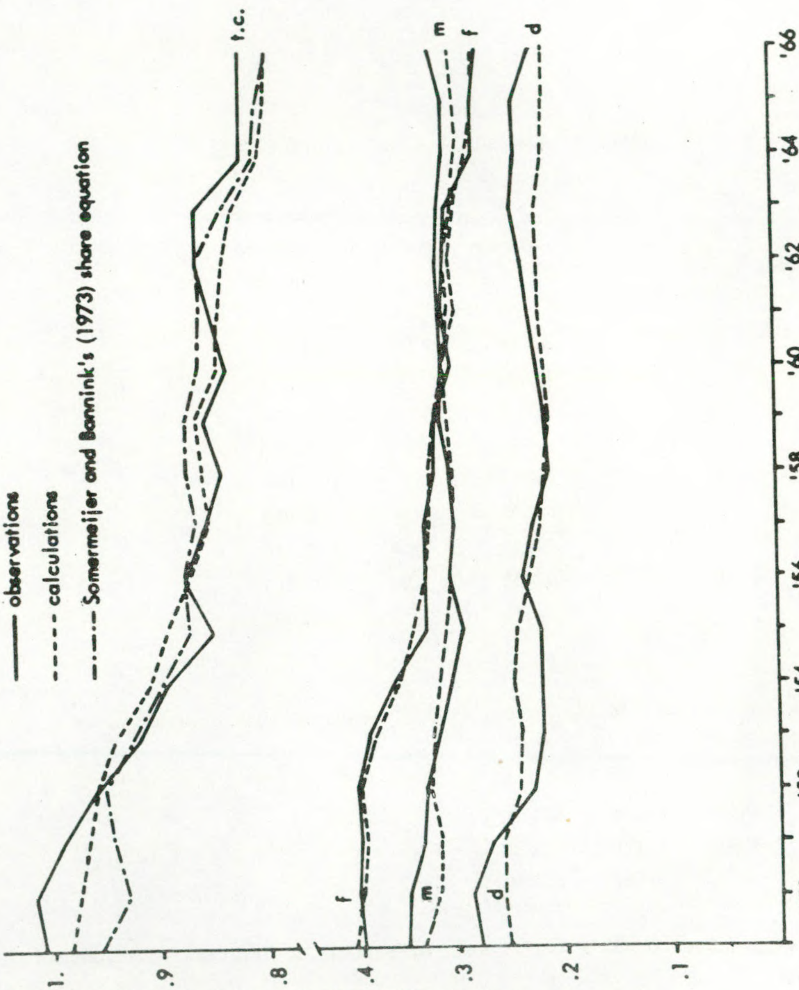

Figure 1. Consumption as shares of disposable income.

f: food, d: durables, m: miscellaneous, t.c.: total consumption.

Figure 1 might give the reader some intuitive idea about the goodness of fit of our model. In the lower part of the figure the consumption of food (f), durables (d) and other goods and services (m) per household are depicted as shares of average disposable income. The uninterrupted lines connect observations and the interrupted lines connect values computed by means of the non-stochastic part of (23) using the data and the estimates of table 1. For durables the similarity between the uninterrupted line and the dashed one is slightly less than for the two other categories distinguished.

Another interesting feature of our model is that it enables us to construct a macro consumption function with (disposable) income as well as prices and the rate of interest as arguments according the a serious theory. In our example this function becomes:

$$\begin{aligned}\bar{E}_t = {} & 2.910\ p_{1t} + 1.118\ p_{2t} + 2.890\ p_{3t} + .639\ \bar{Y}_t - .024\ \bar{Y}_t\tilde{r}_t + \\ & + 0.009\ \bar{Y}_t\tilde{r}_t^2,\end{aligned} \tag{30}$$

where $\bar{E}_t$ means average consumption expenditure per household in year t.

In the upper part of figure 1 we have depicted $\bar{E}_t$ as a fraction of $\bar{Y}_t$. Again, the dashed line connects the calculated ratios and the uninterrupted one the observed values; of course, these graphs are the results of adding-up of the corresponding graphs in the lower part of the figure.

So far we used nominal rates of interest and, therefore, we assumed that households do not take into account expectations about future price changes. Where increases of the price level were the order of the day in the periods of observation, it is not very likely that decision makers completely ignored this phenomenon. So our next exercise was estimating the model (23) on the basis of a slightly changed data set. We replaced for each observation t the nominal rate of interest r_t by $r_t - \dot{p}_t$, where $\dot{p}$ denotes the percentage change per year of the general price level for consumption goods. Using the same stochastic specification we got estimates as reported in table 2.

TABLE 2. Estimates for the model (23) with real interest rates.

	estimates	asymptotic standard error
γ_1	3.611	0.244
γ_2	1.038	0.425
γ_3	3.733	0.363
α_1	0.109	0.047
α_2	0.249	0.086
α_3	0.119	0.045
A	1.118	0.191
B	0.0018	0.0019
C	0.0017	0.0008

Here we found a maximum log-likelihood value that is 2.0 more than that for table 1. The influence of the interest rate is the lowest we found so far and is far from significant. Moreover, the circumstance that the estimates of B and C have the same sign is unsatisfactory.

5. Some comparisons

Now we present a result of Somermeyer and Bannink (1973, relation 7.4.5.1). They found the following macro consumption function (in our notation):

$$\bar{\bar{E}}^s_t = .594\ \bar{\bar{Y}}_t - 0.0125\ \bar{\bar{Y}}_t \tilde{r}_t + .053\ \bar{\bar{W}}_t, \tag{31}$$

where $\bar{\bar{W}}_t$ means average wealth per individual at the beginning of the planning cylce; all double bars denote average per individual. The length of that cycle is assumed to be the expected number of years of life yet to come, whereas final wealth W_L is assumed to be zero for all individuals. The shares of the calculated values of $\bar{\bar{E}}_t^s$ in $\bar{\bar{Y}}_t$ are presented by means of the dotted line in figure 1. The results for (30) and (31) do not differ very much, though the models themselves differ substantially. Note that in (31) the rate of interest has slightly less influence when compared with (30); in both models the terms with $\tilde{r}_t$ contribute at most some hundreds of guilders to total expenditure per household.

Somermeyer and Bannink's aggregate consumption function (31) is the result of aggregating the micro consumption function:

$$C_{ht} = \beta_\ell \Big(\sum_{\ell=1}^{L_{ht}} Y_\ell (1+r_t)^{1-\ell} + W_o - W_L (1+r_t)^{1-L_{ht}} \Big) \qquad (32)$$

in a way that inspired us to obtain (20). Their aggregation procedure is such that the coefficient of $\bar{\bar{W}}_t$ appears as a kind of average over all individuals of the fractions β_h. In cross-section analysis they found individual values of β_h in the order of magnitude between L^{-1} and $2.L^{-1}$. The mean time that adults expect still to live was roughly 25 in the years of observation; this corresponds more or less with the value of the estimate of the parameter of $\bar{\bar{W}}_t$ in (31).

In our results the $\bar{\alpha}_k$ can be considered as some kind of averages over households of the individual values of the α_k. Our estimates of $\bar{\alpha}_k$ add up to .620 when summed over k. This points to a planning cycle of only some years on average.

Because the influence of the rate of interest in (30) appears to be fairly low we compared our results with estimations of the so-called Extended Linear Expenditure System of Lluch (1973):

$$p_{kt} q_{kt} = p_{kt} \gamma_{k*} + \alpha_{k*} (\bar{Y}_t - \sum_k p_{kt} \gamma_{k*}).$$

Our estimates of γ_{k*} and α_{k*} for the data of the appendix are presented in table 3.

TABLE 3. Estimates of the ELES.

	estimate	asymptotic standard error
γ_{1*}	3.765	.124
γ_{2*}	1.682	.191
γ_{3*}	3.996	.251
α_{1*}	.167	.010
α_{2*}	.256	.019
α_{3*}	.203	.020

Here, too, we used a stochastic specification in shares with an additive disturbance term for which relation (24) is assumed to hold. The resemblance of the estimates reported in table 3 and the corresponding estimates in table 2 is striking. The maximum value of the log-likelihood function for our ELES-estimates is 3.3 lower than that corresponding with the results of table 1; this is just such that, at a 90 per cent level of confidence, the model (23) has not to be rejected in favour of ELES. It is a matter of taste, however, whether such a likelihood-ratio argument is to be considered as strong enough for making a choice between both models. The highest maximum value of the log-likelihood function we found was that for (23) with real rates of interest: 5.3 more than that for ELES. This might mean that (23) with real interest rates is "superior" to ELES, but the values of the estimates of A, B and C are disappointing from the point of view of a supporter of (23).

Concluding, one might say that introducing the rate of interest into a demand system, with prices and disposable income as the other explanatory variables, as we did, looks sound from a theoretical point of view. The first confrontation of this new model with a data set are such that a definite answer to the question of its usefullness can only be given after more empirical work.

Footnotes

* I thank P.M.C. de Boer and J. Theeuwes for stimulating discussions and useful suggestions. My special acknowledgments are due to W.H. Somermeyer for more than can be said in a footnote.

1) For a discussion on this controversy see Boland (1981).

2) We prefer considering the household, rather than its individual members, as the basic decision unit from which to start.

3) Note that (6) can be derived for other specifications of the second term of the right-hand member of (2) as well; then, of course, (7) changes.

4) Using some value of r one could construct numerical values for

$$\sum_{\ell=1}^{L} Y_\ell (1+r)^{1-\ell}$$

for individuals of a certain age-class on the basis of assumptions about L and of age-income profiles and the income distribution over age-classes. These values can be aggregated over all age-classes into aggregate total expected income. A time series of such aggregates can be used for estimating (6). Of course, other data and assumptions are needed in addition. This will be set out in a forthcoming paper.

5) In fact, (9) is "over-sufficient" and could, for example, be relaxed to:

$$\sum_{h=1}^{H} \alpha_{kh} \left(W_{oht} - W_{L_{ht}} (1+r_t)^{1-L_{ht}} \right) = 0.$$