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DIFFERENTIAL CONSUMER DEMAND SYSTEMS

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Abstract

It is shown in this paper that several well-known systems like the Rotterdam model and the Almost Ideal Demand System can be seen as different parametrizations of the budget share differential equation. Using a third parametrization a new system, called the CBS system, is developed and discussed. Special attention is given to the relative price versions of the systems and in particular to the special case of preference independence. When it comes to estimation, the system approach, where all equations are estimated simultaneously, is discussed, while in case only a limited number of observations is available, an equation-by-equation approach using stepwise methods is proposed. Some estimates for the Netherlands, based on time series for 1951-1977, using various approaches, are presented.

1. Introduction

In this paper we will present several alternatives to the well-known Rotterdam model for consumer demand, with the emphasis on estimation. The Rotterdam model was introduced by Barten (1966) and explored by Theil (1967, 1975, 1980). We will follow closely the notation of Theil (1975), who provides a comprehensive survey of the Rotterdam model. This paper aims at estimation of characteristics of consumer behaviour, not at the theoretical fundamentals of the consumers choice. In particular, we will deal with problems concerning the specification and estimation of the so-called Engel curve (relating the quantities to total expenditures), and the Slutsky coefficients (relating the quantities to changes in prices). In case relatively short time series for a large number of commodities are available, a particular case, called preference independence (see Theil, 1975, 1980), will get special attention. Besides the Rotterdam model, two competing models will be considered, viz. the well-known AIDS model (cf. Deaton and Muellbauer, 1980) and a third model, which we will call the CBS model.

2. The Rotterdam model

Since Theil (1975, 1980) provides an excellent summary and overview of the Rotterdam model, we will pass over the preliminaries and start with the basic equation, in differentials:

$$w_i \, Dq_i = \mu_i \, D(m/P) + \sum_j \pi_{ij} \, Dp_j \tag{2.1}$$

where (2.1) corresponds to eq. (4.3) in chapter 2 of Theil (1975); the notation is basically the same: w_i is the budget share, μ_i the marginal budget share, q_i and p_i are quantities and prices, m is total expenditure, Dx stands for the total differential of log x,

$$Dx = d(\log x) \tag{2.2}$$

while P, the Divisia price index, is implicitly defined by

$$DP = \sum_i w_i \, Dp_i \, , \tag{2.3}$$

and π_{ij} are the so-called Slutsky coefficients, while the indices $i,j= 1,\ldots,n$ refer to the commodities. (Note that we use the operator D to express relative infinitesimal instead of finite changes, in contrast to Theil.) Equation (2.1) is referred to as the absolute price version of the Rotterdam model since price changes are measured in absolute terms. Below, we will give the alternative formulation of the relative price version of the Rotterdam model.

Additionally, we need the total differential of the budget share

$$w_i = p_i q_i / m \tag{2.4}$$

which reads

$$dw_i = w_i \, Dq_i + w_i \, Dp_i - w_i \, Dm \, . \tag{2.5}$$

There are two sets of restrictions on the parameters of equation (2.1). The first set, which we call the set of weak restrictions on consumer demand, follows directly from the budget constraint (adding-up) and the homogeneity of the demand equations. They read

$$\sum_i w_i = 1 \qquad \text{(adding-up)} \tag{2.6}$$

$$\sum_i \mu_i = 1 \qquad \text{(adding-up)} \tag{2.7}$$

$$\sum_i \pi_{ij} = 0 \qquad \text{(adding-up)} \tag{2.8}$$

$$\sum_j \pi_{ij} = 0 \qquad \text{(homogeneity)} \tag{2.9}$$

The homogeneity of the demand equations, stating that demand is invariant to equal proportional changes in all prices and income, might be justified by the theory of a utility maximizing individual, as is usually done.

However, we might also consider it as a simple principle of scale invariance: if we change from dollars to florins, the actual quantities demanded should remain the same.

For the set of strong restrictions on consumer demand we do need the theory of a utility maximizing individual. These restrictions are, besides the weak restrictions,

$$\pi_{ij} = \pi_{ji} \qquad \text{(symmetry)} \tag{2.10}$$

$$[\pi_{ij}] \text{ is negative semidefinite} \qquad \text{(quasi-concavity)} \tag{2.11}$$

Using homogeneity (2.9), we can easily arrive at the alternative formulation of the Rotterdam model, called the relative price version, since price changes are measured relative to some kind of marginal price index P^+, called the Frisch price index, defined implicitly by

$$DP^+ = \sum_i \mu_i \, Dp_i \, . \tag{2.12}$$

This version of the Rotterdam model reads

$$w_i \, Dq_i = \mu_i \, D(m/P) + \sum_j \nu_{ij} \, D(p_j/P^+) \tag{2.13}$$

where

$$\pi_{ij} = \nu_{ij} - (\sum_k \nu_{ik})\mu_j \tag{2.14}$$

Even under strong conditions, $[\nu_{ij}]$ need not be symmetric. It is unidentified since we may add to ν_{ij} a term $\lambda_i\mu_j$ (λ_i being an arbitrary constant) leaving π_{ij} unchanged.

Conversely the parametrization (2.14) of π_{ij} implies homogeneity (see (2.9)) given adding-up, in other words estimating (2.13) always results in estimates of π_{ij} for which homogeneity holds provided that the marginal budget shares add up to one. Notice also that the adding-up condition (2.8) implies

$$\sum_i \nu_{ij} = \phi \, \mu_j \, , \tag{2.15}$$

where $\phi = \Sigma\Sigma\ \nu_{ij}$. If we impose strong restrictions on $[\pi_{ij}]$, in particular symmetry, there exists a symmetric matrix $[\nu_{ij}]$ satisfying (2.14). For this symmetric matrix $[\nu_{ij}]$ then holds of course

$$\sum_i \nu_{ij} = \sum_i \nu_{ji} = \phi\mu_j \ ; \tag{2.16}$$

so

$$\pi_{ij} = \nu_{ij} - \phi\mu_i\mu_j \tag{2.17}$$

where ϕ is the reciprocal of the income elasticity of the marginal utility of income, or the income flexibility for short (cf. Theil (1975), pp. 29-30), while $1/\phi$ corresponds to the so-called Frisch parameter (cf. Frisch, 1959). Also in this case, $[\nu_{ij}]$ is not identified since we may add to ν_{ij} a term $\alpha\mu_i\mu_j$ (α being an arbitrary constant), leaving π_{ij} unchanged. This property may be interpreted in the sense that a monotonically increasing transformation of the utility function affects the matrix $[\nu_{ij}]$, while the Slutsky matrix remains unaltered. Consequently, also ϕ is not identified and depends on the utility function chosen. If we choose an additive utility function (i.e. preference independence), then this important parameter in the utility theory is related to the so-called overall average elasticity of substitution σ^+ (see Sato (1970) and Keller (1980), chapter 4), by

$$\phi = -\sigma^+ \ . \tag{2.18}$$

Since the usual parameters of interest are income and price elasticities, we also give the relationships of these parameters to the ones introduced above. For the income elasticities η_i we have

$$\eta_i = \frac{\partial q_i}{\partial m} \frac{m}{q_i} = \mu_i / w_i \tag{2.19}$$

while the compensated price elasticities η^*_{ij} are

$$\eta^*_{ij} = \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i}\bigg|_u = \pi_{ij}/w_i \tag{2.20}$$

where the derivatives are evaluated holding the utility level u constant. An alternative way (without reference to the utility context) of defining the compensated elasticities η^*_{ij} is by relating them to the uncompensated elasticities η_{ij}

$$\eta_{ij} = \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} = \eta^*_{ij} - \eta_i w_j \ . \tag{2.21}$$

Besides the compensated and uncompensated price elasticities, we may also be interested in the so-called specific (compensated) price elasticities

$$\eta^+_{ij} = \nu_{ij}/w_i \tag{2.22}$$

which give an indication of the price elasticity after correcting for the income and average substitution effects.

A special case deserves some attention now: the case of preference independence (cf. Theil, 1975), where

$$\nu_{ij} = 0 \qquad (i \neq j) \tag{2.23}$$

so that

$$\nu_{ii} = \phi\mu_i \ . \tag{2.24}$$

This case corresponds to the additive utility function, i.e. a utility function which can be written (possibly after a monotonically increasing transformation) as the sum of n functions each containing only one q_i as its argument. Keller (1980) demonstrated that (2.23) holds approximately if the utility function can be (locally) approximated by an affine one-level CES function with elasticity of substitution σ . Preference independence is obviously an interesting case in view of its parsimony in terms of parameters. However, as Deaton (1974) has shown, it is also a highly restricted case since income and price elasticities are strictly tied together, which may be unrealistic. Notice that by imposing the restriction (2.23) the matrix $[\nu_{ij}]$ becomes identified, since adding a term $\alpha\mu_i\mu_j$ would violate (2.23). The interpretation of this property is that additivity of a utility function gets in general lost by a monotonically increasing transformation of this utility function.

Under preference independence, especially the relative price version of the Rotterdam model becomes very simple; substituting (2.23) into (2.13) we have

$$w_i\ Dq_i = \mu_i\ D(m/P) + \nu_{ii}\ D(p_i/P^+) \tag{2.25}$$

with, under the 'strong' restrictions (see equation (2.24)),

$$\nu_{ii} = \phi\mu_i\ .$$

3. Different parametrizations of the budget share differentials

In this section we will show how different demand equations can be found by simply applying different parametrizations to the differentials of the budget shares. The terms 'parametrization' refers to the assumptions made concerning the constancy of certain parameters. We already have met two parametrizations, the absolute and the relative price version, of the Rotterdam model, respectively

$$w_i\ Dq_i = \mu_i\ D(m/P) + \sum_j \pi_{ij}\ Dp_j \tag{3.1}$$

$$w_i\ Dq_i = \mu_i\ D(m/P) + \sum_j \nu_{ij}\ D(p_j/P^+) \tag{3.2}$$

where in the absolute price version assumptions are made with respect to the constancy of the Slutsky coefficients π_{ij} and in the relative price version with respect to the constancy of the specific price coefficients ν_{ij}. Since both parameters are in general a function of total expenditure and prices, the assumption of constancy over the period of observation is in fact an assumption as important as the choice of a model. The same holds for the assumption of constant μ_i: this parametrization which implies linear Engel curves again defines a particular model. Notice that the constancy of ν_{ij} implies the constancy of $\phi\mu_i$ (see (2.15)) and of ϕ and so the constancy of μ_i. Consequently it implies the constancy of π_{ij}. This, however, does not imply that the estimates for π_{ij} from equations (3.2) and (2.14) are the same as those from (3.1), unless (3.1) is estimated under homogeneity-restrictions (2.9).

A first alternative to equations (3.1) and (3.2) is found by using the differential of the budget share as expressed in equation (2.5). We have

$$dw_i = \mu_i \; D(m/P) + \sum_j \pi_{ij} \; Dp_j + w_i \; Dp_i - w_i \; Dm$$

$$dw_i = \mu_i \; D(m/P) + \sum_j \nu_{ij} \; D(p_j/P^+) + w_i \; Dp_i - w_i \; Dm$$

which might be parametrized as

$$dw_i = \beta_i \; D(m/P) + \sum_j \gamma_{ij} \; Dp_j \tag{3.3}$$

$$dw_i = \beta_i \; D(m/P) + \sum_j \xi_{ij} \; D(p_j/P^+) \tag{3.4}$$

where

$$\beta_i = \mu_i - w_i \tag{3.5}$$

$$\gamma_{ij} = \pi_{ij} - w_i w_j + w_i \delta_{ij} \tag{3.6}$$

$$\xi_{ij} = \nu_{ij} - w_i w_j + w_i \delta_{ij} \tag{3.7}$$

and δ_{ij} equals the Kronecker delta:

$$\delta_{ij} = 0 \qquad \text{if } i \neq j$$

$$\delta_{ij} = 1 \qquad \text{if } i = j \; .$$

Again β_i, γ_{ij} and ξ_{ij} should satisfy a number of restrictions, which can be straightforwardly derived from the restrictions on the Rotterdam model. In particular, we mention the weak restrictions

$$\sum_i \beta_i = 0 \qquad \text{(adding up)} \tag{3.8}$$

$$\sum_j \gamma_{ij} = 1 \qquad \text{(homogeneity)} \tag{3.9}$$

while under the strong restrictions there holds, among other things,

$$\sum_j \gamma_{ij} = 0 \quad \text{and} \quad \sum_j \xi_{ij} = \phi\mu_i \; . \tag{3.10}$$

Equations (3.3) and (3.4) provide two parametrizations of the budget-share differentials next to the Rotterdam model. By assuming β_i and γ_{ij} or, alternatively, β_i and ξ_{ij} constant over the period of observation, two new versions arise. The constancy of β_i, instead of μ_i, implies Engel curves of the form

$$w_i = \alpha_i + \beta_i \log (m) \; ,$$

which was used by Working (1943) and Leser (1963, 1976), and which, according to Leser, provides an excellent fit to cross-section data. This Engel curve has recently become known as the PIGLOG Engel curve, which was constructed to be consistent in aggregation over households by Muellbauer (1975, 1976). This property of consistency in aggregation over households also spawned the development of the AIDS (Almost Ideal Demand System) model, which was introduced by Deaton and Muellbauer (1980). As we can easily verify, our absolute price version (3.3) approximately matches the AIDS model in terms of differentials (we say 'approximately' since the exact differential form of the AIDS model involves a definition of DP different from ours; however, empirical results using both versions of DP happen to be nearly the same. See Deaton and Muellbauer (1980)). Subsequently, equations (3.3) and (3.4) will be referred to as the absolute and relative price versions of the AIDS model.

A disadvantage of both AIDS versions is that the concavity restriction cannot easily be translated into a condition on the matrices γ_{ij} and ξ_{ij}, in view of their relation to π_{ij} and ν_{ij} as expressed in equations (3.6) and (3.7). Therefore, another parametrization seems to be in order. Using again the differential of the budgetshare (2.5) and equations (3.6) and (3.7), we can easily arrive at the following two equations

$$w_i \, D(q_i/Q) = \beta_i \, D(m/P) + \sum_j \pi_{ij} \, D(p_j) \tag{3.11}$$

$$w_i \, D(q_i/Q) = \beta_i \, D(m/P) + \sum_j \nu_{ij} \, D(p_j/P^+) \tag{3.12}$$

where the quantity index Q is implicitly defined by

$$DQ = D(m/P) \; . \tag{3.13}$$

These two models, which we will call the absolute and relative price version of the CBS model (acknowledging the support of the Netherlands Central Bureau of Statistics), combine the preferred Engel curve with the simplicity of the Slutsky matrix, including the ease of implementing concavity and other restrictions, such as preference independence.[1)] However, as far as the relative price versions are concerned, one disadvantage of the AIDS and the CBS model over the relative price version of the Rotterdam model should be mentioned. Since β_i is assumed constant, equation (3.5) implies that the marginal shares, μ_i, will in general be non-constant. With constant ξ_{ij} and ν_{ij}, it is then impossible to impose the restrictions (3.10) and (2.10) at all points in time.

As the Rotterdam model, the relative price version of the CBS model seems appropriate for modelling preference independence, which might be of interest in view of the parsimony in the number of coefficients to be estimated

Notes

1) After completing the first version of this paper, professor Theil draw our attention to a model used by him and Suhm (1981) for international consumption comparisons. As our equation (3.11), their model is based on the PIGLOG Engel curve and constant Slutsky coefficients. Besides some minor differences mainly due to differences in context (time series versus cross-section), their model (equation (3.19) in Theil and Suhm, 1981) is similar to our equation (3.11) except that our model is expressed in terms of relative changes while their model is stated in levels.

$$w_i\ D(q_i/Q) = \beta_i\ D(m/P) + \nu_{ii}\ D(p_i/P^+)\ . \tag{3.14}$$

Notice that the left-hand side of the CBS model might be interpreted as the weighted change in the volume share q_i/Q, while the left-hand side of the AIDS model might be rewritten (using equation (2.5)) as

$$dw_i = w_i\ Dw_i \tag{3.15}$$

i.e. as a weighted change in the budget share w_i.

So several demand systems, like the Rotterdam, the AIDS and the CBS model, can be seen as special cases of a general differential equation for the budget share. Under the assumption of a utility maximizing consumer the underlying demand equations in levels can also be assessed. For a derivation and a discussion of the results, see Van Driel (1982).

4. Estimation under weak and strong restrictions

Having dealt with several versions of the demand models, we now come to the estimation of these models. Here, we will focus on two cases: estimation of small systems under strong restrictions on the one hand and estimation of large systems under weak restrictions on the other. The adjectives 'small' and 'large' refer to the number of commodities relative to the number of observations. First, however, we will look at the formulation of the equations in finite changes, since that is the representation used in estimation.

In order to arrive at estimable equations, we convert the equations in differential form to ones in terms of finite changes, according to the usual practice with the Rotterdam model. For a thorough discussion and explanation of the method involved, we refer to Theil (1975, chapter 2). Now we assume that for each finite time period t=1,...,T (usually a year) we have information on prices and quantities and other information derived from these two variables. Then we define

$$\bar{w}_t = (w_{t-1} + w_t)/2 \tag{4.1}$$

$$\Delta w_t = w_t - w_{t-1} \tag{4.2}$$

$$\bar{D}x_t = \log(x_t/x_{t-1}) \qquad t=2,\ldots,T \tag{4.3}$$

Using a local quadratic approximation, we find for the finite-change expressions corresponding to equations (3.1), (3.2), (3.3), (3.4), (3.11) and (3.12), with intercepts added:

Rotterdam model (in absolute prices)

$$\bar{w}_{it}\bar{D}q_{it} = \kappa_i + \mu_i\, \bar{D}(m_t/P_t) + \sum_j \pi_{ij}\, \bar{D}p_{jt} + \varepsilon_{it}\,, \tag{4.4}$$

Rotterdam model (in relative prices)

$$\bar{w}_{it}\bar{D}q_{it} = \kappa_i + \mu_i\, \bar{D}(m_t/P_t) + \sum_j \nu_{ij}\, \bar{D}(p_{jt}/P_t^+) + \varepsilon_{it}\,, \tag{4.5}$$

AIDS model (in absolute prices)

$$\Delta w_{it} = \delta_i + \beta_i\, \bar{D}(m_t/P_t) + \sum_j \gamma_{ij}\, \bar{D}p_{jt} + \varepsilon_{it}\,, \tag{4.6}$$

AIDS model (in relative prices)

$$\Delta w_{it} = \delta_i + \beta_i\, \bar{D}(m_t/P_t) + \sum_j \xi_{ij}\, \bar{D}(p_{jt}/P_t^+) + \varepsilon_{it}\,, \tag{4.7}$$

CBS model (in absolute prices)

$$\bar{w}_{it}\bar{D}(q_{it}/Q_t) = \zeta_i + \beta_i\, \bar{D}(m_t/P_t) + \sum_j \pi_{ij}\, \bar{D}p_{jt} + \varepsilon_{it}\,, \tag{4.8}$$

CBS model (in relative prices)

$$\bar{w}_{it}\bar{D}(q_{it}/Q_t) = \zeta_i + \beta_i\ \bar{D}(m_t/P_t) + \sum_j \nu_{ij}\ \bar{D}(p_{jt}/P_t^+) + \varepsilon_{it}\ , \tag{4.9}$$

after adding a disturbance ε_{it} to each of the equations (for simplicity we have chosen to use the same notation ε_{it} for each although the disturbances will in general be not the same). Besides some straightforward substitutions and the addition of an intercept and time subscripts, the expressions are completely analogous to the ones expressed in differential terms. Similarly, the finite-change expressions in case of 'preference independence' become, with intercepts added (only the relative price versions are relevant here, see equations (2.25) and (3.14)):

Rotterdam model (preference independence)

$$\bar{w}_{it}\bar{D}q_{it} = \kappa_i + \mu_i\ \bar{D}(m_t/P_t) + \nu_{ii}\ \bar{D}(p_{it}/P_t^+) + \varepsilon_{it} \tag{4.10}$$

CBS model (preference independence)

$$\bar{w}_{it}\bar{D}(q_{it}/Q_t) = \zeta_i + \beta_i\ \bar{D}(m_t/P_t) + \nu_{ii}\ \bar{D}(p_{it}/P_t^+) + \varepsilon_{it}\ . \tag{4.11}$$

The intercepts κ_i, δ_i and ζ_i are introduced primarily for econometric reasons; they might pick up some exogenous (and exponential) time trends in the model in levels. A significant estimate might be interpreted as a shift in tastes. Due to adding-up, we have as additional restriction

$$\Sigma\kappa_i = \Sigma\delta_i = \Sigma\zeta_i = 0\ . \tag{4.12}$$

When it comes to estimation of equations (4.4) through (4.9), several ways are open. We will primarily distinguish two, mutually exclusive, alternatives, viz. the system approach and the equation approach. Slightly parallel to this distinction we will consider estimation under strong and weak restrictions. We will start with the system approach. In all cases the usual assumptions concerning the disturbances (including normality and independence over time) are made.

In case of the system approach, the set of equations for all commodities $i=1,\ldots,N$, are estimated simultaneously, using techniques developed for the so-called Seemingly Unrelated Regression (SUR) model. Since the coefficients ν_{ij} in the relative price versions are unidentified when no additional restrictions are imposed, we will focus on the absolute price versions in case of the system approach. In these versions, all the variables on the right-hand side are equal for all commodities and observable, so that the method of Ordinary Least Squares (OLS) will give maximum likelihood estimates satisfying the adding-up restrictions including the restrictions on the intercepts (see Barten, 1969). However, this is only possible if the number of observations (T-1) is greater than the number of commodities plus one, so

$$T > N + 2 \, . \tag{4.13}$$

In practice, T should be much larger than N + 2 in order to provide accurate estimates, in view of the severe multicollinearity in prices often observed.

Imposing homogeneity, symmetry and/or concavity necessitates more complicated estimation procedures. In general, routines for unconstrained nonlinear optimization can be used to maximize the likelihood since the restrictions can be dealt with easily by means of reparametrizations. In particular, if the matrix $\Pi = [\pi_{ij}]$ has to obey the homogeneity, symmetry and concavity restrictions, we estimate the $\frac{1}{2}N(N-1)$ elements of an upper-triangular matrix L of order N×N, compute the diagonal elements from the restrictions (2.9) and compute Π subsequently by

$$\Pi = -L'L \, , \tag{4.14}$$

where L' indicates the transpose of L. Now Π will obey the restrictions mentioned above. However, since the matrix $[\gamma_{ij}]$ is not negative semidefinite, this procedure breaks down for the AIDS model. In this case, only homogeneity and symmetry can be imposed.

Notice that, as usual in ML estimation procedures of systems in share-equations, one of the equations should be deleted in order to circumvent singularity of the contemporaneous covariance matrix.

Summarizing, if the number of observations is large relative to the number of commodities, we might succesfully use the system approach. OLS will then give us estimates satisfying the adding-up constraints. If we impose, furthermore, homogeneity, symmetry and eventually concavity, non-linear methods to maximize the likelihood will in general be in order. Since the absolute price version of the CBS model allows for consistent aggregation over individuals, flexible Engel curves and the imposition of all the restrictions including concavity on the estimates, we tend to prefer it over the other models.

If the number of observations is small relative to the number of commodities, the system approach will, in general, be no longer appropriate. Then, we suggest the equation approach, where each equation is estimated separately without imposing adding-up, symmetry, and concavity, and in the absolute price version, homogeneity. However, since each equation contains all the N price variables, OLS applied to each equation will break down if $T>N+2$, as stated above. Therefore we suggest the following alternative, which could only provide estimates that approximate the maximum likelihood estimates. As a bonus, the estimation procedure becomes very simple. The steps of reasoning are as follows.

First, we concentrate on the relative price version of the CBS model, in view of its Engel curve, its implicit homogeneity and its simple representation in case of preference independence. In particular, estimating the relative price version will provide interpretable parameters (viz. ν_{ij}), which are directly related to the specific price elasticities. Additionally, significances of non-diagonal elements of $[\nu_{ij}]$ are interpretable as significant deviations of preference independence, while zero or non-significant non-diagonal elements easily fit into the canonical form provided by separable additive utility functions.

Second, in order to apply ordinary regression methods, we approximate $\bar{D}P_t^+$, which depends on the unknown marginal shares μ_{it}, by $\bar{D}P_t$, so

$$\bar{D}P_t^+ \approx \bar{D}P_t \ . \tag{4.15}$$

It is well-known that the approximation of the marginal price index P^+ by the price index P_t is in general not valid, in view of the observed discrepancies between the shares w_{it} and the marginal shares μ_{it}. We will not deny this fact. However, what matters here is not the approximate equality of the level of P_t and P_t^+ and not even the approximate equality of the relative changes in P_t and P_t^+, but in fact the correlation between the series DP_t and DP_t^+. So even if the level of the index P_t, given a base year, is substantially different from the level of the marginal index P^+, the correlation might well be above 0.90, implying nearly identical results in terms of the regression coefficients, when running the regressions with $\bar{D}P_t$ as proxy for $\bar{D}P_t^+$. In our empirical work we found correlations in the order of 0.98.

Third, in order to get rid of the large number of exogenous variables, we might use stepwise regression methods to select those price variables $\bar{D}(p_{jt}/P_t)$ into the regression which make a significant contribution to the explained variance. Since the income variable $\bar{D}(m_t/P_t)$ and the own price variable $\bar{D}(p_{it}/P_t)$ for the i-th equation are elements of the most parsimonious version, i.e. that of preference independence, they should be forced into the regressions before the other price variables. In view of practical considerations, the same holds for the constant term. Notice that this procedure implies a simple way to detect significant departures from preference independence in a natural way, i.e. via the ν_{ij} $(i \neq j)$ coefficients. The same procedure might be applied to the relative versions of the Rotterdam and AIDS models; the interpretation in terms of preference independence only holds in case of the relative version of the Rotterdam and CBS model, however.

Although it seems a practical way of estimation, the procedure outlined above has its drawbacks. We already mentioned the substitution of $\bar{D}P_t$ for $\bar{D}P_t^+$: even in terms of correlation, the first one might be a bad proxy for the second one. Furthermore, the equation approach does not allow for inter-equation restrictions such as adding-up (see equations (2.7) and (3.8)), symmetry and concavity. We know that if the right-hand side variables are identical in all the N regressions, the estimated intercepts α_i and income coefficients β_i do add up to zero, in view of the adding-up restrictions side variables, this adding-up property of the estimated coefficients does not hold any longer. Note that this problem also arises in the simple case of preference independence since the own price variable will be different for each equation. In fact, under preference independence the coefficients ν_{ii} should satisfy

$$\nu_{ii} = \phi\mu_i \tag{4.16}$$

with ϕ identical in all equations. As for the other cross-equation restrictions, it is not possible to satisfy this restriction in the equations approach. Finally, the usual stepwise regression methods do not guarantee an optimal set of regressors, since the selection will in general not only depend on the contribution of the variables to the explained variance, but also on the order in which the variables are entered into the regression.

5. Some empirical results

In order to develop some feeling for the order of magnitude of the differences in estimates using the various approaches suggested above, we estimated several models using data for The Netherlands 1951-1977. These data were constructed at the Netherlands Central Bureau of Statistics. As the most detailed level of aggregation, 108 commodities were distinguished. A description of the data and of the way they were constructed can be found in CBS (1982).

In table 1 we concentrate on the Full Information Maximum Likelihood (FIML) estimates of the absolute versions of the Rotterdam, the (approximate) AIDS, and the CBS model as given in equations (4.4), (4.6) and (4.8), respectively. As with all models estimated in this section, an intercept was included in order to cope with possible time trends in levels. We used the RESIMUL program developed by Wymer (1978) for all FIML estimates, including the (asymptotic) standard errors. The FIML estimates were subject to all possible restrictions (adding-up, homogeneity, symmetry and concavity) with one exception: concavity could not be imposed on the AIDS estimates. Besides these rather 'sophisticated' FIML estimates, we also added to table 1 the results using the most simple model of all, the CBS model

TABLE 1 COMPARING INCOME AND COMPENSATED OWN PRICE ELASTICITIES FOR SEVERAL CONSUMER DEMAND SYSTEMS (ELASTICITIES EVALUATED AT AVERAGE SHARES)

	AVERAGE SHARE	INCOME ELAST.				COMP. OWN PRICE ELAST.			
		RDAM	AIDS	CBS	CBS/PI	RDAM	AIDS	CBS	CBS/PI
GROCERIES/DAIRY/BREAD	0.12	0.16*	0.28*	0.14*	0.20*	-0.36∇	-0.05	-0.36∇	-0.07
POTATOES/FRUIT/VEGETABLES	0.04	0.63	0.13*	0.62	0.17*	-0.43	-0.08	-0.43	-0.15
MEAT AND FISH	0.07	0.86	1.04	0.85	0.64	-1.12∇	-0.91∇	-1.10∇	-0.82∇
STIMULANTS AND THE LIKE	0.09	0.38*	0.29*	0.37*	0.46*	-0.44	-0.29	-0.42	-0.57
TEXTILE/FOOTWEAR/LEATHER	0.15	2.67*	2.61*	2.66*	2.43*	-0.51∇	-0.16	-0.52∇	-0.35
AUTOMOBILES	0.01	0.19	1.50	0.21	0.54	-3.30∇	-2.60∇	-3.29∇	-2.93∇
OTHER DURABLES	0.11	2.22*	1.47	2.23*	2.26*	-0.54	0.53	-0.55	-0.73
FUEL/ELECT/GAS/WATER	0.06	0.69	1.12	0.69	0.61	-0.69∇	-0.64∇	-0.69∇	-0.36
OTHER ARTICLES	0.06	1.75*	1.98*	1.75*	1.27	-0.81∇	-0.29	-0.79∇	-1.37∇
GROSS RENT	0.07	0.00*	0.07*	0.01*	0.07*	-0.42	-0.31∇	-0.44	-0.37∇
MEDICAL CARE	0.06	2.24*	0.37	0.63	0.66	-0.42	-0.67∇	-0.40∇	-0.40∇
OTHER SERVICES	0.16	-0.35*	0.53*	0.25*	0.38*	-0.53	0.76∇	-0.52	0.48

TABLE 3 COMPARING INCOME AND COMPENSATED OWN PRICE ELASTICITIES FOR SEVERAL ESTIMATION METHODS APPLIED TO THE CBS-MODEL (ELASTICITIES EVALUATED AT AVERAGE SHARES)

	AVERAGE SHARE	INCOME ELAST.				COMP. OWN PRICE ELAST.			
		PI	STEP	OLS	FIML	PI	STEP	OLS	FIML
GROCERIES/DAIRY/BREAD	0.12	0.20*	0.21*	0.22*	0.14*	-0.07	-0.18	-0.13	-0.36∇
POTATOES/FRUIT/VEGETABLES	0.04	0.17*	0.04*	-0.15*	0.62	-0.15	-0.02	-0.03	-0.43
MEAT AND FISH	0.07	0.64	0.92	1.18	0.85	-0.82∇	-1.13∇	-0.95∇	-1.10∇
STIMULANTS AND THE LIKE	0.09	0.46*	0.53*	-0.05*	0.37*	-0.57	-0.91∇	-1.09∇	-0.42
TEXTILE/FOOTWEAR/LEATHER	0.15	2.43*	2.55*	2.74*	2.66*	-0.35	-0.31	-0.14	-0.52∇
AUTOMOBILES	0.01	0.54	0.87	-0.49	0.21	-2.93∇	-2.91∇	-3.19	-3.29∇
OTHER DURABLES	0.11	2.26*	2.07*	1.45	2.23*	-0.73	-0.21	-0.26	-0.55
FUEL/ELECT/GAS/WATER	0.06	0.61	0.82	1.35	0.69	-0.36	-0.37	-0.49	-0.69∇
OTHER ARTICLES	0.06	1.27	1.85*	2.06*	1.75*	-1.37∇	-0.96∇	-1.01	-0.79∇
GROSS RENT	0.07	0.07*	-0.02*	0.17*	0.01*	-0.37∇	-0.29∇	-0.20∇	-0.44
MEDICAL CARE	0.06	0.66	0.59*	0.63	0.63	-0.40∇	-0.52∇	-0.64∇	-0.40∇
OTHER SERVICES	0.16	0.38*	0.40*	0.59	0.25*	0.48	0.30	0.41	-0.52

*) Significant different from 1 at 5 % level (based on asymptotic standard errors)

∇) Significant different from 0 at 5 % level (based on asymptotic standard errors) under preference independence, equation (4.11), estimated with ordinary least squares applied to each equation separately, taking into account no restrictions other than the implicit homogeneity. Hereafter, the corresponding estimates will be labeled CBS/PI.

All elasticities are evaluated at the average shares over 1951-1977. In table 1, the average shares over the years 1951-1977 and the estimated income and compensated own price elasticities, for twelve groups of consumer expenditures are presented. We concentrate on income and own price elasticities since these are the figures of most practical interest for policy purposes. As can be seen from the table, the order of magnitude of the estimates of the income elasticities are roughly the same. In particular, the very simple CBS/PI model does not seem to give substantially different estimates as compared to the more sophisticated models. The estimated income elasticities for the Rotterdam and CBS model are rather close to each other, despite the differences in underlying Engel curves, with one exception: medical care. The AIDS estimate for the income elasticity of automobiles is substantially larger than the others, which are smaller than might be expected. The figures for the compensated own price elasticities, which should be negative under concavity, diverge slightly for the various models, with some positive estimates for AIDS and CBS/PI as notable deviations of the overall picture. This phenomenon was expected in view of the concavity restrictions imposed on the Rotterdam and CBS/FIML estimates and the lack of these restrictions on the AIDS and CBS/PI model. Again, the Rotterdam and CBS estimates are rather close to each other.

The overall impression from table 1 is that all four models produce reasonable estimates, while no model gives substantially worse or better interpretable results than the others.

In order to compare the fit of the models we computed the sum of the squared residuals $\Sigma\Sigma(w_{it}- \hat{w}_{it})^2$ after reconstruction of the predicted shares $\hat{w}_{it}$ for all models considered in table 1 (see also Theil (1975), subsection 5.5; we preferred the sum of squared residuals over his information accuracy in view of the violation of the adding-up restriction by CBS/PI and the incidental appearance of small but negative predicted shares $\hat{w}_{it}$). Besides the model considered in table 1, we also computed the sum of squared residuals for a naive model ($\Delta w_{it}=0$) in order to provide a bench mark. The results are shown in table 2. The figures indicate a substantial gain of all models (including CBS/PI) over the naive model, as might be expected. The performance of the Rotterdam model is substantially less than that of the CBS/FIML and AIDS models, which are rather close, with a small advantage for AIDS. Surprisingly, the CBS/PI model also performs better than the Rotterdam model, despite its much smaller number of parameters.

Table 2. Sum of squared residuals of budget shares for several models*)

RDAM	AIDS	CBS	CBS/PI	NAIVE
2.88	2.11	2.25	2.45	6.73

*) Figures × 1000

In table 3, we focus on the CBS model and compare the results for four variants ranging from simple single-equation based methods to sophisticated FIML system estimates upon which all theoretical restrictions are imposed. The most simple variant is the relative price version of the CBS model under preference independence, equation (4.11), estimated by simple-equation regression. The relative price version without imposing preference independence, was estimated by forward stepwise regression methods, as described in section 4. The corresponding estimates are labelled CBS/STEP. By using non trivial selection criteria ($F \geq 1$ and Tolerance ≥ 0.7) a subset of the twelve relative price terms was selected into the regression for each equation separately. As suggested in section 4, the income and own price term were forced into the regression before other regressors were selected. Note that in order to identify the elements ν_{ij} in equation (4.9), we deliberately specified restrictive selection criteria (see above), avoiding the selection of all price terms into the regressions. On the average, 4 out of 12 price terms were selected.

Besides these two relative price versions we estimated the CBS absolute price version by applying OLS to each equation (4.9) separately (including all price terms), and as specimen of the more sophisticated methods presented in table 1, FIML to the system of equations (4.9) with all the theoretical restrictions imposed. The first method will subsequently be labelled CBS/OLS, the second CBS/FIML. So all specifications in table 3 are based on the CBS model, while the methods of estimation of the income and price elasticities are in an order of increasing complexity when going from CBS/PI to CBS/FIML. Note that for CBS/PI and CBS/STEP, only homogeneity is implicitly imposed, while is case of CBS/OLS only additivity is implicitly imposed.

Inspection of table 3 reveals hardly larger differences in estimates between methods than in table 1, which indicates that the differences in estimates due to sophistication of estimation methods are not substantially larger than the differences induced by the choice of the underlying model. Somewhat surprising is that the CBS/STEP method, proposed as a 'quick-and-dirty' approximate for the FIML procedure when the number of observations is small relative to the number of commodities, correlate higher with the FIML estimates than the CBS/OLS estimates where all price terms are entered into the regression. In general, the estimates for the CBS/OLS method deviate more from the CBS/FIML estimates than the other methods, including the very simple CBS/PI method. A possible explanation might be that the inclusion of all price terms without restrictions induces instabilities in the estimates, in view of the severe multicollinearity between the time series of prices.

In view of the plausible results from the CBS/PI and CBS/STEP as compared to the CBS/FIML method, we could not resist the temptation to apply these simple methods to our 108-commodities data base. The results can be found in table 4.

6. Conclusions

In this paper we have presented several alternatives to the well-known absolute and relative price versions of the Rotterdam model. We found that by simple substitutions, four other models can be arrived at, viz. an absolute and relative price version of the (approximate) AIDS model and similar versions of a new model, called the CBS model. In particular the CBS model couples a flexible and 'aggregable' Engel curve model to a directly interpretable representation of the price effects through the matrix of estimated price coefficients. By imposing simple restrictions on this matrix in case of the absolute price versions, we can easily arrive at estimates which satisfy all the restrictions from the theory of consumer demand and which, at the same time, provide flexible and 'aggregable' Engel curves. In particular the possibility to impose concavity directly on the matrix of Slutsky coefficients is an advantage over the AIDS model.

The relative price version of the CBS model might be a good starting point in case the number of observations is smaller than the number of commodities. Then, assuming that the price index is highly correlated with the marginal price index (with the budgetshares replaced by the marginal budgetshares as weights), and disregarding the adding-up, symmetry and concavity restrictions, we can arrive at simple models that can be estimated by simple stepwise regression techniques. Additionally, the coefficients of the selected price variables in the regression indicate the departures from preference independence, where all non-diagonal elements are zero.

TABLE 4. INCOME AND COMPENSATED OWN PRICE ELASTICITIES FOR THE CBS-MODEL IN RELATIVE PRICES UNDER PREFERENCE INDEPENDENCE (PI) AND WITH STEPWISE REGRESSION (STEP); ELASTICITIES EVALUATED AT AVERAGE SHARES.

COMMODITY	AVER. SHARE	INCOME ELAST. PI	INCOME ELAST. STEP	PRICE ELAST. PI	PRICE ELAST. STEP
FOODS	0.233	0.32*	1.59	-0.08	0.24
GROCERIES	0.046	0.36*	0.60	-0.28	-0.15
PULSES	0.000	-0.31	-0.43	-0.62	-0.43
RICE GROAT AND OATMEAL	0.001	3.22	1.47	-1.21∇	-0.40
VERMICELLI/MACARONI	0.001	1.43	1.43	-1.04	-1.62∇
PUDDING POWDERS	0.001	1.03	0.29	1.72	3.02∇
TEA	0.002	1.72	-0.48	-0.86∇	-1.31∇
COFFEE	0.011	0.46	1.04	-0.33∇	-0.36∇
COCOA	0.000	2.49	2.30	-0.57∇	-0.91∇
CHOCOLATE SPREAD	0.001	0.07	-0.25	-1.41∇	-1.71∇
SUGAR	0.008	-1.71*	-2.60*	-0.87∇	-1.63∇
PRESERVED FRUIT	0.006	0.91	-0.39*	-0.43	-1.08∇
MARGARINE	0.005	-0.47*	0.10*	-0.02	-0.10∇
COOKING FAT	0.001	0.14	1.09	-0.31	0.39
EDIBLE OIL	0.001	-2.22	-3.04	-1.24	-2.02
VINEGAR	0.000	-1.52	-1.64*	-1.68	-3.68∇
SALT	0.000	1.10	2.37	-1.80	-0.98
OTHER GROCERIES	0.007	1.10	1.68	0.10	0.94
DAIRY PRODUCTS	0.057	0.26*	0.04*	-0.18	-0.28
FRESH MILK	0.015	-0.03*	-0.19*	-0.55∇	-0.57∇
BUTTER	0.005	-1.57*	-3.17*	-1.94∇	-1.61∇
CHEESE	0.011	0.59	0.01*	-0.16	-0.38∇
CREAM AND CONDENSED MILK	0.005	1.94	3.23*	-0.17	-0.15
SKIMMED-/BUTTER MILK	0.001	1.32	1.67	-1.50∇	-1.68∇
BUTTER MILK SOUP	0.001	-0.87*	-1.13*	-0.44	-0.13
YOGHURT	0.002	-0.24*	-0.54*	0.47	0.48∇
CHOCOLATE MILK	0.001	1.16	0.90	-1.21∇	-1.12∇
SPECIAL MILK PRODUCTS	0.003	1.25	2.40*	-0.20	-1.32∇
EGGS	0.007	0.98	-0.70*	-0.25	-0.36∇
MARGARINE	0.005	-0.43*	0.12*	-0.03	-0.14∇
BREAD	0.020	-0.46*	-0.45*	-0.22	-0.24
PATATOES/FRUIT/VEGETABLES	0.039	0.17*	-0.12*	-0.15	0.03
PATATOES	0.006	0.45	1.06	-0.05	0.03
ONIONS	0.000	-0.94	-1.17*	-0.11	-0.09
VEGETABLES	0.009	0.16	0.06*	-0.52∇	-0.28∇
FRUIT	0.016	-0.26*	-1.27*	-0.58∇	-0.57∇
PRESERVED VEGETABLES	0.007	1.84	0.52	-1.62∇	-2.83∇
MEAT AND MEATPRODUCTS	0.066	0.62	1.08	-0.82∇	-1.07∇
ANIMAL FAT	0.000	-18.35*	-0.61	0.13	-0.08
BEEF AND VEAL	0.025	0.64	0.53*	-1.02∇	-1.35∇
PORK	0.019	1.17	1.80*	-0.72∇	-0.85∇
OTHER MEAT	0.002	-1.08	-4.77*	-0.59	1.50∇
PRESERVED MEAT /MEATPROD.	0.017	0.45	1.78*	-1.05	-1.23∇
POULTRY	0.003	1.99	1.46	-2.34∇	-2.41∇
FISH AND FISHPRODUCTS	0.006	1.00	0.06*	-1.18∇	-1.70∇
FRESH FISH	0.004	1.06	-0.07*	-0.61	-1.23∇
SMOKED AND DRIED FISH	0.001	-2.44*	-0.79*	-1.25	-1.11∇
PRESERVED FISH	0.001	2.34	2.79*	-1.21	-0.18
STIMULANTS AND THE LIKE	0.094	0.46*	0.66	-0.57	2.48
CONFECT./PASTRY/CHOCOLATE	0.027	0.21*	0.40*	-0.68	-0.57∇
SUGAR CONFECTIONERY	0.013	0.24	0.14*	-0.98∇	-0.78∇
CAKE	0.001	-0.05*	0.36*	-0.09	0.01
DUTCH RUSKS	0.002	-0.57*	-0.11*	0.13	0.19
BISCUITS/COOKIES/PASTRY	0.008	0.32*	0.64	-0.86∇	-0.92∇
PASTRY COOK PRODUCTS	0.004	0.17*	-0.45*	-0.19	0.64
TOBACCO	0.036	-0.22*	0.02*	-1.00∇	-1.17∇
CIGARS AND CIGARILLOS	0.007	0.71	1.53	-1.34∇	-1.10∇
CIGARETTES	0.022	-0.38*	0.08*	-1.06∇	-1.28∇
CUT TOBACCO	0.006	-0.80*	-0.87*	-0.53	-0.75∇
BEVERAGES	0.028	1.25	1.09	-0.61	-0.10
BEER	0.009	0.81	0.60	-0.18	0.35
OTHER ALCOHOLIC BEVERAGES	0.013	1.04	0.56	-1.83∇	-2.16∇
NON-ALCOHOLIC BEVERAGES	0.006	1.80	2.66*	0.59	0.70∇
ICE-CREAM	0.003	0.60	0.68	-0.59	0.06

*) Signif. different from 1 at 5 % level (based on asymptotic st. errors)
∇) Signif. different from 0 at 5 % level (based on asymptotic st. errors)

TABLE 4. (CONTINUED)
INCOME AND COMPENSATED OWN PRICE ELASTICITIES FOR THE CBS-MODEL IN RELATIVE PRICES UNDER PREFERENCE INDEPENDENCE (PI) AND WITH STEPWISE REGRESSION (STEP); ELASTICITIES EVALUATED AT AVERAGE SHARES.

COMMODITY	AVER. SHARE	INCOME ELAST.		PRICE ELAST.	
		PI	STEP	PI	STEP
DURABLE GOODS	0.273	2.30*	1.12	0.11	0.52
TEXTILE AND CLOTHING	0.128	2.68*	2.62*	-0.26	-0.45▽
MEN'S UPPER WEAR	0.025	2.37*	2.27*	-0.23	-0.87▽
LADIES UPPER WEAR	0.021	1.94*	1.40	-1.25▽	-1.48▽
UNDERWEAR AND NIGHTWEAR	0.010	3.40*	2.60*	-0.15	-0.70▽
RAIN WEAR	0.006	3.06*	3.32*	-0.21	0.19
STOCKINGS AND SOCKS	0.006	2.68*	2.59*	-1.42▽	-2.00▽
FANCY ARTICLES	0.004	2.31*	1.87*	-0.74	-0.49
YARNS	0.003	1.34	0.61	-0.14	0.25
WOVEN MATERIALS	0.014	5.23*	5.48*	1.21	0.37
HOUSEHOLD TEXTILES	0.020	2.44*	2.11*	0.26	0.19
OTHER TEXTILES	0.018	1.98*	2.56*	-1.21▽	-0.72▽
FOOTWEAR	0.016	1.06	0.86	-0.92▽	-0.52▽
LEATHER ARTICLES	0.004	0.62	1.65	-0.75	-0.82
AUTOMOBILES	0.014	0.54	1.17	-2.94▽	-2.91▽
FURNITURE/HOUSEHOLD EQUIP	0.088	2.19*	2.06*	-0.58	0.17
HEATING EQUIPMENT	0.008	3.32*	4.16*	-0.65	-1.44▽
TABLEWARE	0.004	3.12*	2.59*	-1.02	-1.24▽
GLASSWARE	0.002	0.93	0.54	-0.86	-0.57▽
WOOD PRODUCTS FOR HOUSEH.	0.004	1.73	0.48	0.72	-0.09
FURNITURE	0.021	2.29*	1.68	-0.38	-1.66▽
O.ARTICLES F.HOUDEH.USE	0.050	2.02*	1.94*	-0.36	0.04
BICYCLES AND MOTORCYCLES	0.009	3.48*	3.86*	0.86	-0.72
OTHER DURABLES	0.015	1.99	1.63	-0.35	-1.11▽
OTHER GOODS	0.117	1.04	1.15	-0.28	-2.92▽
FUEL	0.030	0.84	0.77	-0.68▽	-0.78▽
SOLID FUEL	0.013	1.36	1.02	0.35	0.27
LIQUID FUEL	0.017	0.17	0.83	-1.34▽	-1.19▽
ELECTRICITY/GAS/WATER	0.030	0.37	0.61	-0.18	-0.29
ELECTRICITY	0.014	0.56	0.69	-0.06	-0.16
GAS	0.011	0.12	0.86	-0.36	-0.41
WATER	0.004	0.48	0.74	-0.50	-0.73▽
OTHER ARTICLES	0.057	1.27	2.14*	-1.37▽	-0.97▽
FLOWERS AND PLANTS	0.008	1.22	0.94	-1.03▽	-1.18▽
PAPER ARTICLES	0.003	2.82*	1.21	-0.53	-1.50▽
COSMETIC PRODUCTS	0.005	1.56	2.54*	-0.96	-0.09
MEDICAL/PHARM.PRODUCTS	0.007	0.84	0.97	-0.61	0.80
GOODS/SERV. BY PUBLISHERS	0.018	0.49	0.10*	-1.33▽	-1.86▽
ARTICLES N.E.C.	0.017	1.81	3.32*	-1.20	-1.53
SERVICES	0.283	0.39*	0.74	-0.08	-0.49
GROSS RENT	0.065	0.07*	0.12*	-0.37▽	-0.27▽
MEDICAL CARE	0.058	0.66	0.53*	-0.40▽	-0.43▽
ENTERTAINMENT SERVICES	0.008	0.72	0.78	-1.30▽	-1.21▽
EXPEND. IN RESTAURANT ETC	0.035	0.13*	-0.04*	-0.27	0.02
TRANP./COMMUNICATION	0.026	-0.12*	-0.27*	-0.43	-0.30
OTHER SERVICES	0.090	0.57	0.55*	0.01	0.16
GOVERNMENT SERVICES	0.001	1.15	0.18	0.96	1.79▽
MAINTENANCE WORK	0.008	1.70	0.85	-1.41▽	-0.23
SERVICES OF CLEANING FIRM	0.001	0.66	0.52	-0.40	0.75
DOMESTIC SERVICES	0.006	-0.66*	-0.68*	-1.57▽	-1.26▽
SERV.BARBERS/BEAUTY-SHOPS	0.005	-0.06*	0.15*	-0.58	-0.15
SERV.OF DYERS /LAUNDRIES	0.005	0.41	0.52*	-1.26	-1.36▽
SERV.OF SWIMMING ESTABL.M	0.001	1.78	0.70	-0.12	-1.01
SHOE REPAIRS	0.002	-0.76*	-0.40*	0.52	-0.30
SERV.OF PHOTOGRAPHERS	0.002	0.22	0.19*	-1.49▽	-1.41▽
SERV.OF METAL-USING CRAFT	0.015	1.69	1.98	0.64	2.05▽
BANKING SERVICES	0.002	1.90	0.60	-0.87	-0.31
INSURANCE SERVICES	0.018	0.15	0.51	-1.06	-0.24
SOCIAL SERVICES	0.010	0.82	1.88	-1.04	-0.78
CONTRIB. INST. OF WORSHIP	0.003	0.16*	-0.43*	-2.25	-1.14
SERVICES N.E.C.	0.010	0.32	1.02	-0.65	-0.78

*) Signif. different from 1 at 5 % level (based on asymptotic st. errors)
▽) Signif. different from 0 at 5 % level (based on asymptotic st. errors)

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