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A Supplementary Method for Consumer Demand Analysis and Welfare Comparison Applied on United Kingdom and West German Data Sets

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Abstract

In consumer demand analysis the expenditure systems estimated are often related to the Hicks-Allen utility maximization model. In this paper a method is described with which one can determine in a general, revealed preference-like way if arguments can be found against the hypothesis of consistent and transitive choice behaviour as implied by Hicks-Allen utility maximization. This method is applied on two average United Kingdom and two average West German consumers. To discover eventual differences in preference structure between an average United Kingdom and an average West German consumer a spliced data set is constructed for these two consumers on a 23 commodity group aggregation level. The general choice analysis is also applied on this average United Kingdom-West German consumer. The results do not indicate a rejection of the hypothesis of consistent and transitive choice behaviour for all average consumers and, hence, argue in favour of the usual practice in demand analysis to estimate expenditure systems related to Hicks-Allen utility maximization. The choice analysis also yields a great deal of information about the preference ordering of the chosen packets. So an ordinal welfare comparison of a substantial part of the United Kingdom and West German packets also results.

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1. Introduction

In applied demand analysis the expenditure system to be estimated is often related to the Hicks-Allen utility maximization model. In Maks (9) and (10) it has been proposed, before estimating an expenditure system, to determine in a general, revealed preference-like way if arguments can be found against the hypothesis of consistent and transitive choice behaviour as implied by Hicks-Allen utility maximization. If this is *not* the case one has an argument in favour of the estimation of expenditure systems related to this theory. In this paper the general method of choice analysis is applied on United Kingdom and West German data sets. These sets contain prices and quantity indexes per capita related to commodity groups on two levels of aggregation (i.e. 23 and *ca.* 40 commodity groups). We do *not* prefer to assume that these data sets describe the choice behaviour of *one* average consumer on two levels of aggregation. This point of view is based on the fact that the index types used in this study do not fulfill the requirements of such a consistent aggregation.[1] It is to be noted that this applies for all well-known index types. So, in the rest of this paper an average consumer is identified with a given data set on a specified level of aggregation.

For a 23 commodity groups classification data are constructed for both an average United Kingdom and an average West German consumer. So the question arises whether we can observe differences in the preference structure between the two or, in contrast, we can find an argument in favour of the hypothesis of seeing them as *one* average consumer. To obtain an insight in this matter the choice analysis is applied on an average United Kingdom-West German consumer, related to a data set constructed by splicing the quantity index data for both countries. The choice analysis detects eventual inconsistencies and intransitivities between United Kingdom packets on the one hand and West German packets on the other. For the non-inconsistently and non-intransitively chosen bundles the procedure yields a great deal of information about the preference ordering of the chosen packets. So, if one accepts the hypothesis of one average United Kingdom-West German consumer, this preference ordering can be interpretated as an ordinal welfare comparison of the chosen United Kingdom and West German packets. Evaluating this approach against the usual

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[1] For a more elaborated argument, see Maks (10), esp. pp. 29-35.

one, based on comparison of indexes of total consumption, the following is to be noted. If our method detects an inconsistently chosen pair of packets, one has a good case for questioning the point of calculating and comparing indexes of total consumption related to this pair of packets.

This latter point is elaborated in section 2, after the exposition of the general analysis of choice behaviour. In section 3 the construction of the data sets is reported, whereas in section 4 the results of the application of general choice analysis on these data sets are considered.

2. The General Method of Choice Analysis

In this section the general method of choice analysis that leads to a rejection or a confirmation of the hypothesis of consistent and transitive choice behaviour as implied by Hicks-Allen utility maximization is dealt with.

A utility function $\phi(q)$ is assumed to exist with domain $\{q \varepsilon R^n / q > 0\}$. The symbol R^n denotes euclidean n-space and q can be interpreted as a (n x 1)-vector of quantities of the n goods. The Hicks-Allen maximization model can be summarized as follows. A consumer unit is supposed to maximize the utility function subject to his budget constraint: $p'q \leq \mu$. The symbol μ denotes income, and p a (n x 1)-vector of prices of the n goods. The utility function is assumed to be twice continuously differentiable and strictly quasi-concave with positive marginal utilities. As a result the maximization probles has an unique solution. The solution is found on the budget constraint and demand functions can be deduced. Suppose that a given data set D consists of r price and quantity vectors and describes for some consumer unit the chosen commodity vector q^i for each price vector p^i of period i, i = 1,...,r. Based on this data set, a matrix E can be constructed, for which the elements are defined as follows:

$$e_{ij} = -1 \quad \text{iff } p^{i'}q^j - p^{i'}q^i < 0$$

$$e_{ij} = 0 \quad \text{iff } p^{i'}q^j - p^{i'}q^i = 0$$

and

$$e_{ij} = +1 \quad \text{iff } p^{i'}q^j - p^{i'}q^i > 0 \qquad (i,\ j = 1,\ 2,\ldots,r)$$

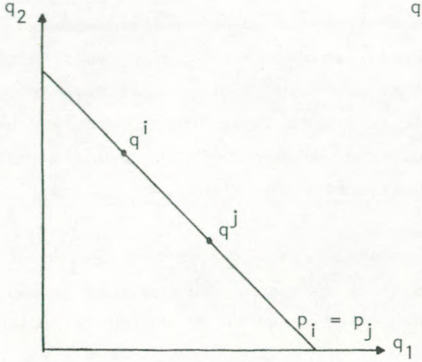
An analysis of the E matrix allows one to determine the subsets of D with the maximum number of commodity packets which corroborate the model formulated above. This reveals, to a large extent, the preference relations which exist between the packets of such a subset.

If for the moment we confine our attention to pairs of commodity packets and their related price vectors, an examination of E leads to three possibilities. First, we can find that $e_{ij} = e_{ji} = -1$ or that $e_{ij} = 0$ and $e_{ji} = -1$. These configurations indicate inconsistent choice behaviour unless both $p^i = p^j$ and $q^i = q^j$, and at least one of the commodity packets involved cannot belong to the same maximum subset of D which does not contradict the model.

A few diagrams can be of help to illustrate this contention. Along the axes in the figure 1 through 4 quantities q_1 and q_2 are measured of two goods. The vectors q^i and q^j denote the packets of these goods chosen in period i and j respectively. The price vectors p^i and p^j are placed next to the budget line of the period to which they correspond. If one accepts Hicks-Allen utility maximization the implication "if $p^{i'}q^j \leq p^{i'}q^i$ then $\phi(q^i) > \phi(q^j)$" is true, unless $p^i = p^j$ and $q^i = q^j$. Applying this implication in the figures 1 and 2 we obtain $\phi(q_i) > \phi(q_j)$ *and* $\phi(q_j) > \phi(q_i)$. This is clearly inconsistent, so the theory of Hicks-Allen utility maximization cannot hold for both choices.

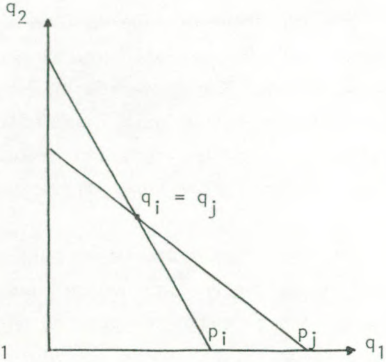
A second possibility is $e_{ji} = 1$ and $e_{ij} = -1$ or 0. Such a configuration enables one to deduce, again based on our choice model, that q^i is preferred above q^j and that they are consistently chosen. However, it remains to be seen if these two packets can be placed in a non-intransitive relation to other packets of D, and consequently whether they can belong to one of the maximum subsets of D we seek. As a final possibility one can find $e_{ij} = e_{ji} = 1$. In this case, the packets are not inconsistently chosen but one cannot directly conclude what the preference relation between them is. At first sight, one would say that the packets are incomparable. But, as we shall presently show, a further comparison with other packets may indirectly reveal a preference relation. These two possibilities are illustrated in the figures 3 and 4 respectively. The situation in figure 3 shows that $p^{i'}q^j > p^{i'}q^i$ *and* $p^{j'}q^i > p^{j'}q^j$. So, a conclusion regarding the preference relation between q^i and q^j cannot be drawn. In figure 4 we see that $p^{i'}q^j < p^{i'}q^i$. So, one can conclude, accepting Hicks-Allen utility maximization, that $\phi(q^i) > \phi(q^j)$.[1] So $\phi(q^j) > \phi(q^i)$ *and* $p^{j'}q^i \leq p^{j'}q^j$ are false. Hence we must have $p^{j'}q^i > p^{j'}q^j$ and this is confirmed by $e_{ji} = 1$.

[1] It is to be noted that the implication of Hicks-Allen utility maximization used here is closely related to the main assumption of Samuelson's revealed preference theory. See Samuelson (12), (13) and (14). For a precise analysis of the relation between the two theories, see Kihlstrom (5).



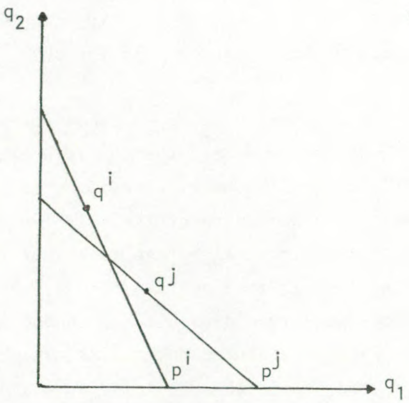
$e_{ij} = e_{ji} = 0$

figure 1



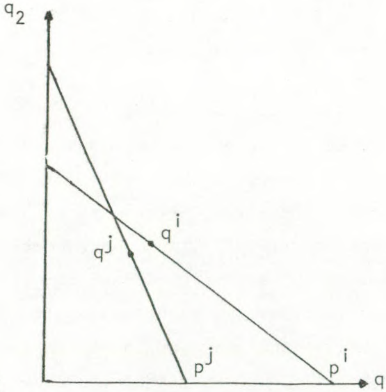
$e_{ij} = e_{ji} = 0$

figure 2



$e_{ij} = e_{ji} = 1$

figure 3



$e_{ij} = -1 \quad e_{ji} = 1$

figure 4

To simplify the exposition we now firstly replace every off-diagonal-zero in the matrix E bij a "-1". Alle inconsistent cases are now coded by $e_{ij} = e_{ji} = -1$ and all consistent and strongly ordered pairs by $e_{ij} = -1$ and $e_{ji} = 1$. So, this substitution does not change the information in E pertaining to the preference relation between the packets concerned. Moreover we assume that in the data set leading to E no cases prevail $p^i = p^j$ and

$q^i = q^j$. In the analyzed data sets such a configuration never occurred.

The next step in the method is the permutation of the period indexes. For one pair of indexes, this amounts to the interchanging of the corresponding pair of rows and of columns in E. This technique turns out ot be of great help in the determination of the subsets of D with the maxmum number of commodity packets corroborating Hicks-Allen utility maximization. The technique is due to Koo.[1] In this study, Koo's procedure is slightly modified, because we want to retain in E information pertaining to cases of inconsistency and incomparability. The period indexes should be permuted in such a way as to obtain square sub-matrices of maximum dimension along the main diagonal of E, starting from the upper left-hand corner. These square matrices must display the property that the lower triangle is composed solely of "1" 's and the upper triangle solely of "-1" 's and "1" 's. We shall denote such a sub-matrix by T^j. Here the superscript j identifies different T-matrices obtained from some particular E. It is rather obvious that the commodity packets of a subset of D corresponding to a T^j are chosen in a way which does not contradict the requirements for consistent and transitive behaviour in our model. Furthermore, we can use such a T^j matrix for the derivation of the preference relations. This may be clarified by considering the following simple example. Suppose that a matrix P is obtained from a (9 x 9)-matrix E by permuting the order of the periode indexes to get (2, 8, 6, 7, 5, 4, 9, 1, 3). The matrix P is shown in table 1, bordered by the

Matrix P

t	2	8	6	7	5	4	9	1	3
2	0	1	1	-1	-1	1	1	-1	-1
8	1	0	1	-1	1	-1	1	-1	-1
6	1	1	0	-1	1	-1	-1	-1	-1
7	1	1	1	0	-1	-1	-1	-1	-1
5	1	1	1	1	0	1	-1	1	1
4	1	1	1	1	1	0	-1	1	-1
9	1	1	1	1	1	1	0	1	-1
1	1	1	-1	-1	-1	-1	-1	0	-1
3	1	1	1	1	-1	1	1	1	0

Table 1

[1] See Koo (6, 7) and Koo and Schmidt (8).

permuted period indexes. Within P we discover the (7 x 7)-matrix T^1, with the above mentioned properties. It can be seen that the elements p_{38}, p_{83}, p_{48} and p_{84} have the value "-1". Because p_{38} and p_{83} relatie to q^6 and q^1, it is revealed that the choice of q^1 is inconsistent when compared with the packets q^6 and q^7. We further note that $p_{59} = 1$ and $p_{95} = -1$, so q^3 is preferred above q^5. And $p_{57} = -1$ and $p_{75} = 1$ reveal that q^5 is preferred above q^9. Using the transitivity property of the utility function we deduce that q^3 is preferred above q^9. But $p_{79} = -1$ and $p_{97} = 1$ indicates the reverse: q^9 is preferred above q^3. Hence, we signal intransitive choice behaviour.

A "1" in the upper triangle of a T matrix means that one cannot directly conclude what the nature of the preference relation is between the packet of period i, indicated by the row index, and the packet of period j, as indicated by the column index. The transitivity property of the utility function can sometimes be used to indirectly reveal a preference relation, resulting in the replacement of such an upper triangle "1" by a "-1". This can be done if the "1" under consideration is *not* connected with the main diagonal zero by a row consisting of "1" 's, *and* under the condition that the packet indicated by the column index of at least one of those "-1" 's has a preference relation with the packet of period j. Application of this procedure to the matrix P of table 1 results in the "1" 's belonging to p_{16}, p_{17}, p_{27}, p_{25} and p_{35} being replaced by "-1" 's. A simple example in a diagram and a corresponding P-matrix may illustrate this point.

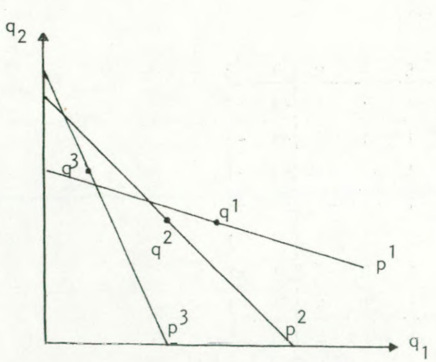


figure 5

Matrix P corresponding to

figure 5

t	1	2	3
1	0	-1	1
2	1	0	-1
3	1	1	0

Table 2

In figure 5 we see that q^1 is preferred above q^2 and q^2 above q^3, so we conclude that q^1 is preferred above q^3. In table 2 the value of $p_{13} = 1$. So in this example $i = 1$ and $j = 3$. This indicates that one cannot directly reveal the preference relation between q^1 and q^3. But this "1" is connected with the main diagonal zero by a row consisting of one "-1", indicating the preference relation: q^1 above q^2. Now q^2 has a preference relation with the packet j: q^2 is preferred above q^3. So, we can replace the "1" in P_{13} by a "-1".

It can easily be verified that T^1 is of maximum dimension. However, if e.g. we exchange q^3 and q^9 we obtain after sufficient permutation a new T matrix of the same size. This demonstrates that the packets of some D belonging tot a T do not necessarily form a unique maximum subset. This creates no serious problem; on the contrary, it allows the researcher to choose that subset best suited to his inquiry. If we consider the relations within T^1 it is obvious that the period indexes 2, 8 and 6 can be permuted to create other T matrices. The same applies to the indexes 4 and 5. The T matrices thus formed are related in the sense that they correspond to the same subset of D. For this reason permutations within a T matrix have no consequence for our construction of the preference orderings. To illustrate this point, the reproduction of T^1 is given in table 3 and in figure 6. In table 3 the longest

Preference ordering belonging to T^1
8[2]6]8
7
4[5
9

Table 3

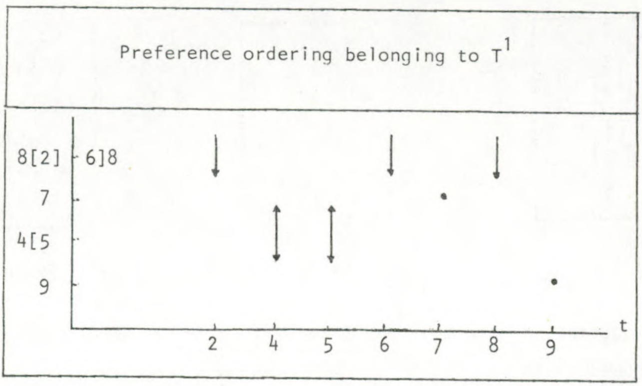


figure 6

column gives a largest subset of bundles of T^1 that can be strongly ordered. The period-indexes in this column, as ranked above, indicate the placement of the packet in this strong ordering. The bracket "[" denotes that for the bundles it seperates no preference relation could be deduced. So the brackets in figure 3 and table 6 indicate that the pairs q^8 and q^2, q^2 and q^6, q^6 and q^8 and the pairs q^4 and q^5 were found to be inorderable in T^1, even after the application of the replacement procedure. At the same time the bracket indicates the ranking of the packets, not contained in the largest, strongly ordered subset, with respect to the packets of this subset, lying above and under the bracket.

It can be seen that, even in our simple example, several other strong orderings of the same size can be made. The information to produce them is contained in table 3. It is also obvious that every feasable permutation of the period indices within T^1 can be summarized in exactly the same way as in table 3. Therefore, this method of summarizing the information about the preference relations and all the feasable permutations within a T matrix is rather convenient. Diagrams like the one given in figure 6 contain the same information along the vertical axis, while along the horizontal axis all the period indices of the non-excluded packets are ranked chronologically. This enables us to plot the preference ordering in a graph for easy analysis.

There is one more complication that needs to be dealt with. If a matrix T^i leads to configuration as depicted in table 4, it is possible that one finds that q^9 is preferred above q^8 or that these packets are incomparable.

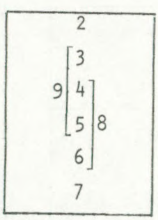


Table 4

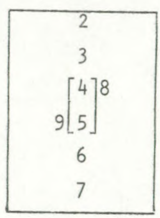


Table 5

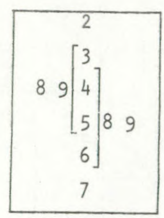


Table 6

It is important to note that the T^i matrix could not have indicated that q^8 is preferred above q^9. If this should have been the case the replacement procedure, as described above, could have been applied to produce the configuration of table 5. For, if the involved T matrix shows q^8 is preferred above q^9, q^3 above q^8 and q^9 above q^6 then one has immediately q^3 above q^9 and q^8 above q^6.

To obtain unambiguity for bundles not contained in a longest, strongly ordered, column we shall use their placement in the table to indicate their preference relation with each other. A packet is preferred above all packets placed beneath it. If, however, a T matrix shows incomparability between packets not placed in the longest column, then these cases are indicated by the use of second brackets. So, in the example given in table 6 one finds that for q^9 and q^8 no preference relation could be deduced

Finally a few conclusions are to be added regarding the calculation of indexes of total consumption. With i and j as symbols denoting respectively the base and current period the Laspeyres quantity index of total consumption can be written as

$L_q^{ij} \equiv \frac{p^{i'} q^j}{p^{i'} q^i}$ and the Paasche index as $P_q^{ij} \equiv \frac{p^{j'} q^j}{p^{j'} q^i}$

Remembering of the definitions of the elements of E makes the following equivalences obvious:

$$e_{ij} = 1 \text{ iff } L_q^{ij} > 1 \quad (1)$$

$$e_{ji} = 1 \text{ iff } P_q^{ij} < 1 \quad (2)$$

$$e_{ij} = -1 \text{ iff } L_q^{ij} < 1 \quad (3)$$

$$e_{ji} = -1 \text{ iff } P_q^{ij} > 1 \quad (4)$$

From these equivalences we conclude that if a pair of packets is consistently chosen ($e_{ij} = 1$ and $e_{ji} = -1$ or vice versa) both indexes indicate the same ordering as found through the general choice analysis. However, if one observes an inorderable pair ($e_{ij} = 1$ and $e_{ji} = 1$) or an inconsistently chosen pair ($e_{ij} = -1$ and $e_{ji} = -1$) the indexes contradict each other. In the latter case a Paasche index prevails with even a greater value than the Laspeyres index. It is well known that other index types, as e.g. the Fisher, Törnquist and Theil index, produce values somewhere between the Laspeyres and Paasche index. So, if one has a data set with a lot of inconsistently or intransitively chosen pairs and/or inorderable pairs, the calculation of indexes of total consumption leads to orderings of dubious value. For the obtained ordering depends on and may differ with the index type.

3. Construction of the Data Sets

For West Germany we had time series data of consumer expenditures on 46-commodity groups, both in current and fixed prices with 1962 as base year, extending from 1950 until 1967, as constructed by Rau.[1] The Rau classification scheme for the commodity groups is given in table 1. Implicitly Rau also gives a series of population estimates for each year for West Germany. These were used to divide the expenditures of each group in current as well as in fixed prices. From these we calculated a set of partial Paasche price and partial Laspeyres quantity per capita indexes for each commodity group and we rescaled this set by choosing 1950 as base year. Finally a series of "prices" was obtained by multiplying the Paasche price indexes of a group with the related nominal expenditures of the base year. We may clarify the calculation in the following identity:

$$p_t'q_t \cdot \pi_t^{-1} \equiv \frac{p_t'q_t \cdot \pi_t^{-1}}{p_o'q_t \cdot \pi_t^{-1}} * \frac{p_o'q_t \ \pi_t^{-1}}{p_o'q_o \ \pi_o^{-1}} * p_o'q_o \cdot \pi_o^{-1}$$

where the subindexes t and o denote the current and base period respectively, p is a price vector and q a quantity vector of the commodities of a group and the scalar π contains the population estimate. So the partial Laspeyres quantity per capita index for a group is

$$L_q^{ot} = \frac{p_o'q_t \ \pi_t^{-1}}{p_o'q_o \ \pi_o^{-1}}$$

and the "price" (P) of a group is to be written as

$$P = P_p^{ot} * p_o'q_o \cdot \pi_o^{-1} = \frac{p_t'q_t}{p_o'q_t} * p_o'q_o \ \pi_o^{-1}$$

From this we see that multiplication of the "price" with the quantity per capita index for each group results in the current expenditures per capita for that group.

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[1] Rau (11).

Rau's Commodity Groups Classification			
No.		No.	
1.	meat	26.	durable domestic utensils
2.	fish	27.	floor covering, housing repair and maintenance
3.	eggs	28.	domestic services
4.	milk, cheese	29.	other goods for housekeeping
5.	butter	30.	maintenance and repair of household goods, clothing and footwear
6.	fats, oils	31.	motorvehicles, bicycles
7.	bread, baked goods	32.	fuel and oil for motorvehicles
8.	potatoes	33.	maintenance and repair of motorvehicles
9.	vegetables	34.	public transport
10.	fruit	35.	communications
11.	jams, sweets, sugar	36.	personal needs
12.	other foods	37.	health
13.	non-alcoholic beverages	38.	radio, television, phonograph, accessories, piano, repairs of these goods
14.	coffee, tea	39.	other durable recreational and educational goods
15.	alcoholic beverages	40.	books, magazines, newspapers
16.	tobacco	41.	other recreational and educational goods
17.	clothing	42.	education
18.	footwear	43.	art, sport, entertainment
19.	rent	44.	donations
20.	electricity	45.	personal accessories
21.	gas	46.	other services
22.	coal		
23.	other fuel		
24.	furniture, household textiles		
25.	heating, household and cooking appliances		

Table 7

The Central Statistical Office of the United Kingdom publishes consumer expenditures data in current and fixed prices. These cover a number of decades, and are based on a rather detailed commodity groups classification.[1] When using these data for the period 1950 through 1970 at least two problems arise. Firstly, the Office frequently revises formerly given figures. This we tried to solve by taking the figures of the last edition of a National Income and Expenditure Blue Book that publishes data for a given year. Second, there are minor alterations in the commodity groups classification during the period considered. Therefore, we determined the most detailed classification which could be maintained throughout the period and simply added consumers' expenditures in current and fixed prices in the troublesome groups. We thus obtained slightly aggregated data in the derived classification scheme. Table 8 lists this scheme. Next, we calculated, in the way described above, a set of prices and quantity per capita[2] indexes for all commodity groups of this classification, with 1950 as base year.

The spliced data set is constructed with the help of data from Gilbert.[3] He gives real consumption per capita indexes, classified in 25 commodity groups, for both the United Kingdom and West Germany, related to the United States for 1950 and based on average European price weights. From these data one can calculated indexes relating the 1950 real consumption per capita for the United Kingdom and West Germany for eacht of 23 commodity groups, as shown in table 9 and clarified in the identity:

$$\frac{p'_{ae}\, q_{uk}}{p'_{ae}\, q_{gr}} \equiv \frac{p'_{ae}\, q_{uk}}{p'_{ae}\, q_{us}} \;\Big/\; \frac{p'_{ae}\, q_{gr}}{p'_{ae}\, q_{us}}$$

where p_{ae} stands for the vector of average European price weights and q_{uk}, q_{gr} and q_{us} are the vectors of quantities of goods, consumed per capita in 1950 in respectively the United Kingdom, West Germany and the United States and the quotients are the real consumption per capita indexes. The index for group 23 is obtained by adding up the expenditures in dollars of the groups health, education and miscellaneous as given in Gilbert's table 23 for both countries,[4] and then calculating their ratio. To be able to use the data of

[1] See Central Statistical Office (2).

[2] The mid-year estimates of the United Kingdom population were taken from Central Statistical Office (1), Table 1.

[3] See Gilbert (3), p. 78, table 25. Gilbert and associates argue that this table is the best option available in their book for comparison of European countries; see *e.g.* p. 19.

[4] See Gilbert (3), p. 76.

Classification of 42 Commodity Groups for the United Kingdom Expenditures Data 1950-1970			
No.		No.	
1.	bread and cereals	22.	women's, girls' and infants' wear
2.	meat and bacon	23.	motorcars and motorcycles
3.	fish	24.	furniture and floor coverings
4.	oils and fats	25.	radio, electricity and other durable goods
5.	sugar, preserves and confectionary	26.	matches, soap and other cleaning materials, *etc.*
6.	dairy products	27.	books
7.	fruit	28.	newspapers
8.	potatoes	29.	magazines
9.	beverages (non-alcoholic)	30.	chemists' goods
10.	other processed food	31.	miscellaneous recreational goods
11.	beer	32.	railway
12.	cigarettes	33.	postal services
13.	pipe tobacco, cigars and snuff	34.	telephone and telegraph
14.	rent, rates and water charges	35.	cinema
15.	maintenance, repairs and improvements of homes by occupiers	36.	domestic service
16.	coal	37.	other goods and services
17.	electricity	38.	wines, spirits, cider, *etc.*
18.	gas	39.	household textiles, soft furnishing and hardware
19.	other fuel and light	40.	running costs of vehicles
20.	footwear	41.	other travel
21.	men's and boys' wear	42.	other entertainment

Table 8

Indexes of real consumption per capita in the United Kingdom in 1950 with 1950 = 1 for West Germany					
Gilbert group no.	index	Gilbert group no.	index	Gilbert group no.	index
1.	1.335	9.	1.214	17.	2.966
2.	1.411	10.	4.625	18.	1.666
3.	.991	11.	2.034	19.	4.500
4.	1.333	12.	2.266	20.	1.977
5.	1.122	13.	1.586	21.	5.00
6.	.95	14.	1.888	22.	1.978
7.	.68	15.	4.679	23.	1.909
8.	1.187	16.	1.888		

Table 9

table 9 one has to aggregate the partial price and quantity indexes initially calculated for West Germany and the United Kingdom into the 23 group classification. This aggregation was carried out using Theil (15) partial chain indexes, by partitioning the more detailed classifications of both countries as shown in table 10. The obtained nominal expenditure figures and price and quantity indexes for each group were used to calculate the quantity indexes per capita and prices with base year 1950 for both countries using the same population data as mentioned before. So, two data sets of prices and partial quantity indexes per capita, related to the same 23 commodity group classification for both countries, were obtained. For all commodity groups the 1950 quantity index is equal to *one*. To splice both data sets, the United Kingdom partial quantity per capita indexes for each group were multiplied by the corresponding Gilbert real consumption per capita index from table 8. This implies a redefining of a "unit" consumption of each commodity group in the United Kingdom data, so at the same time the prices must be rescaled by dividing them by the same Gilbert index, leaving unchanged the value of the product of price and partial quantity index. There is no need for the splicing of the United Kingdom prices in pounds and the West German prices in marks. This is because the construction of each row of an E matrix requires the spliced quantity indexes, but only the prices

Partition of the German 46 groups and the United Kingdom 42 groups into the Gilbert 23 groups		
Numbers of groups from the slightly aggregated C.S.O. classification	Gilbert's 23 commodity groups classification	Number of groups from Rau's 46 groups
1	1. cereal and cereal products	7
2	2. meats, poultry	1
3	3. fish	2
6	4. dairy products	3,4,5
4	5. fats, oils	6
8	6. vegetables	8,9
7	7. fruits	10
11,38	8. alcoholic beverages	15
5	9. sugar, sugar products	11
9	10. non-alcoholic beverages	13,14
12,13	11. tobacco	16
20	12. footwear	18
21,22	13. clothing, household textiles	17
14	14. housing	19
16,17,18,19	15. fuel, light, water	20,21,22,23
15,24,25,26,39	16. household goods	24,25,26,27,29,38
36	17. household and personal services	28
23	18. transport equipment	31
40	19. operation of personal transportation	32,33
32,41	20. public transport	34
33,34	21. communication	35
31,35,42	22. recreation, entertainment	39,41,43
10,27,28,29,30,37	23. other goods and services	12,30,36,37,40,42,44,45,46

Table 10

of one year of one country. It is important to realize that construction of an E matrix[1] produces the exact same results irregardless of the base year of the data.

We may now list and code the data sets we constructed and which are going to be used in the next section. We shall denote the quantity indexes per capita and the corresponding prices for the United Kingdom for the years 1950-1970 by UK^{42} for the 42 commodity groups and by UK^{23} for the 23 group classification. Analogously, the West German data will be represented by WG^{46} and WG^{23}. The spliced data will be presented by WGUK.

4. Application of the Choice Model on the Data

The starting point for this section is the application of the general analysis of choice behaviour to the data sets UK^{23}, WG^{23} and WGUK. Because we derived these sets from UK^{42} and WG^{46}, similar choice models are applied to UK^{42} and WG^{46} in order to compare the resulting preference orderings.

For UK^{23} we can form 210 different pairs of packets. Analysis of the obtained (21 x 21)-T matrix shows that for all these pairs a preference relation can be deduced. So for UK^{23} there is only one T matrix and all its packets are chosen in a way to fulfill the requirements of transitivity. The derived strong preference ordering is given in table 11. It can be seen that the packets of 1953, 1951 and 1952 are ordered below the one of 1950. This development may be connected with the Korean crisis, and the 1950 level has been recovered in 1954. Since then the bundle of each successive year is preferred above all preceeding years.

Application of a similar model for UK^{42} leads to exactly the same results. The obtained preference ordering for UK^{23} may be called "compatible" with the UK^{42} ordering. So, the Theil index used results in an entirely "adequate aggregation".[2]

For WG^{23} 153 different pairs of commodity bundles can be identified. An (18 x 18)-T matrix shows that for 152 pairs a preference relation can be established. The packets of 1966 and 1967 turn out to be incomparable. For WG^{23} two T matrices are generated and all packets are transitively chosen.

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1 See section 2.

2 See Maks (10) for the definition of these concepts, esp. p. 44.

Preference ordering of UK^{42} and UK^{23}
1970
1969
1968
1967
1966
1965
1964
1963
1962
1961
1960
1959
1958
1957
1956
1955
1954
1950
1953
1951
1952

Table 11

The resulting preference ordering is given in the right column of table 12. It can be noted that there was no impact of the Korean crisis on the German ordering reminiscent of the English case. The incomparability of 1966 and 1967 may be brought into connection with the recessionary character of 1967 for West Germany.

Employment of the procedure on WG^{46} gives a clearer picture in this respect. The packet of 1967 is ranked below 1966. So the WG^{23} ordering is not quite compatible with WG^{46}. The choice of another index type may solve this problem.[1] However, for all other years the obtained preference orderings are

[1] See Maks (10) for an application of this idea, esp. ch. 3.

Preference ordering of WG^{46}	WG^{23}
1966	1966]1967
1967	1965
1956	1964
1964	1963
1963	1962
1962	1961
1961	1960
1960	1959
1959	1958
1958	1957
1957	1956
1956	1955
1955	1954
1954	1953
1953	1952
1952	1951
1951	1950
1950	

Table 12

exactly the same. The left column of table 12 shows the WG^{46} ordering.

From the WGUK data 741 different pairs can be found. The results for WG^{23} and UK^{23} already showed that for 362 pairs a preference relation can be determined, and that for one pair this is impossible. An obtained T matrix reveals that for 310 of the remaining 378 pairs a preference relation can be established. It is to be emphasized that each of these pairs consists of one United Kingdom and one West German packet. Furthermore it is remarkable that T matrices can be derived with a (39 x 39)-size. This implies that all packets of WGUK form a unique largest subset that corroborates the applied choice model. The obtained preference ordering is given in figure 7. Along the horizontal axis the pairs of bundles of both countries of the same year given in chronological order. A packet for a given year, say 1950, is denoted by 50U and 50W for the United Kingdom and West Germany, respectively.

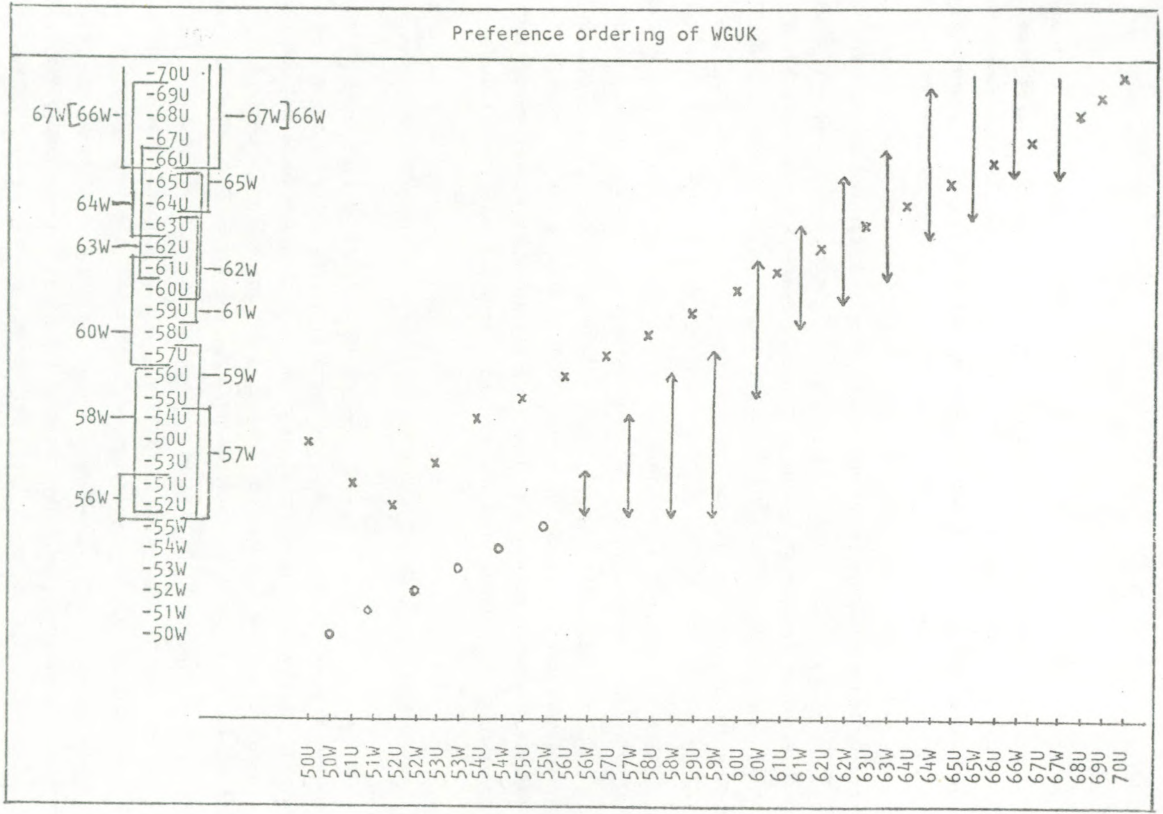
Preference ordering of WGUK
-70U
-69U
-68U
-67U
-66U
-65U
-64U
-63U
-62U
-61U
-60U
-59U
-58U
-57U
-56U
-55U
-54U
-50U
-53U
-51U
-52U
-55W
-54W
-53W
-52W
-51W
-50W
67W[66W-
—67W]66W
-65W
64W-
63W-
-62W
-61W
60W-
-59W
58W-
-57W
56W-
50U 50W 51U 51W 52U 52W 53U 53W 54U 54W 55U 55W 56U 56W 57U 57W 58U 58W 59U 59W 60U 60W 61U 61W 62U 62W 63U 63W 64U 64W 65U 65W 66U 66W 67U 67W 68U 69U 70U

figure 7

UK packets are indicated in the diagram with an "x" and WG packets with an "o" or "↕". The figure shows very clearly the tendency for the WG-bundles to bypass the UK ones in the preference ordering. In the early fifties the WG packets are clearly ranked below the UK ones. But from 1960 onwards one can see that the lowest possible place in the ordering for the WG packet comes closer to the UK one; with, of course, an exception for 67W. This tendency might tempt one to predict that in the early seventies the WG bundles will be clearly ranked above the UK packets of the same year. Because 67 pairs of packets turned out to be incomparable the ordering of these packets resulting from calculation of indexes of *total* consumption may vary with the chosen index type. It seems worthwhile to calculate and compare the various indexes. But the incomparability implies that for all these 67 pairs the Paasche and Laspeyres index will contradict each other.

4. Summary

A general analysis of choice behaviour has been applied to two United Kingdom and two West German average consumers. The United Kingdom consumers were defined on 23 and 42 commodity groups, with yearly observations for the period 1950-1970. The West German consumers were defined on 23 and 46 commodity groups, with yearly observations for the period 1950-1967. The general analysis has also been applied to a spliced United Kingdom-West German data set of 23 commodity groups. The findings confirm the assumption of consistent and transitive choice behaviour for all average consumers. So they argue in favour of the usual practice of applied demand analysis of estimating expenditure systems related to Hicks-Allen utility maximization on this type macro index-data.

Moreover the results corroborate the assumption of one average United Kingdom-West German consumer, behaving consistently and transitively on these 23 groups. Accepting this hypothesis the obtained ordinal welfare comparisons indicate clearly the tendency for West German packets to bypass the United Kingdom bundles. Moreover the results reveal a substantial part of in-orderable pairs. In these cases the calculation of indexes of total consumption diminishes in value

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