THE MEASUREMENT OF QUADRATIC PREFERENCE FUNCTIONS WITH SMALL SAMPLES

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A.H.Q.M. Merkies
and T.E. Nijman *)

Interfaculteit der Actuariële Wetenschappen en Econometrie
Vrije Universiteit, Amsterdam

1. Introduction

In an earlier paper Merkies and Nijman (1980) tried to establish preference functions for seven Dutch political parties. This was done by what Johansen (1974) calls direct interviewing. The participating fractions of the Lower Chamber of the Dutch Parliament were asked to evaluate 16 alternative policy programs y^i $(i=0,1,\dots,15)$, consisting each of particular target values given to five policy variables. For each party, the 16 utility indices assigned to these programs together with the 16x5 values of the policy variables constituted the data in a number of regressions to estimate the parameters of the preference functions. It is clear that with such a small number of observations -a number which could hardly be supplemented without jeopardizing the willingness of the parties to cooperate- the estimation results were not very robust. The arbitrary character of the results could be reduced, but could not be completely removed, by taking account of the greater inaccuracy with which less realistic (i.e. less probable) policy programs were evaluated. The lack of robustness was all the more serious when it became apparent that linear functions -with only five parameters to estimate-did not provide an appropriate description of the parties' preferences. Putting restrictions on the parameters helps to reduce arbitrariness. The additional information for obtaining these restrictions can be found in the optimal policy programs for the various parties that are also provided by the inquiry. Although the optimal program for party g -indicated by y_g^0- was already included among the 16 observations that entered the various regressions for party g, its character as an optimum had not been acknowledged.
In this paper we will show how the optimal character of y_g^0 can be put to full advantage if quadratic preference functions are used. The crucial technique is that of mixed regression of the type described by Theil and Goldberger (1961). Results on the basis of the 1977 inquiry are given for the same seven parties that were studied before. Asymmetries included in Merkies and Nijman [1980] are not incorportated here, but this is not essential.
The contents of this paper are as follows. In section 2 we describe the available information, give the details of the estimation problem and show how the difficulty of the insufficiency of the data can to a large extent be overcome by using the information of y^0 for each party. The problem of heteroskedasticity connected with the varying degree of accuracy with which the alternatives were evaluated, which has already been discussed in the

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reference given above, is repeated in short in section 3. The results of the various estimations are then given in section 4 and conclusions and comments follow in a final section.

2. The available information and the estimation problem.

2.1. The Information.

Merkies and Vermaat (1980) gave account of the results of an inquiry in two rounds among political parties and social organizations with respect to their policy aims qualitative as well as quantitative. The political parties that provided useful answers are represented here in table 1 together with the number of seats they held in the lower Chamber of Parliament at the time of the inquiry.

Table 1. The political parties inquired

		Number of seats
P.v.d.A.	- Labour party	53
C.D.A.	- Christian Democrats	49
V.V.D.	- Liberals	28
D'66	- Democrats	8
S.G.P.	- Political Calvinistic Party	3
P.P.R.	- Progressive Radicals	3
G.P.V.	- Calvinistic Political Union	1
Total number of seats of participating parties		145
Total number of seats in Lower Chamber		150

One of the questions in the second round of the inquiry requested the parties to express their socio-economic preferences by attaching a figure ranging from 0 to 100 to each of a number of vector values in five key variables. These variables and their most appropriate definition arose from information obtained in the first round of the inquiry. The specification of the variables and their values in the various alternative policy programs are given in table 2. As the inquiry also yielded a set of values y_g^0 for each party g which this party found optimal, the data sets for the various parties differed with respect to the values of y_g^0 and with respect to the indices U_i. Program y^{15} - also indicated in this paper by $\bar{y}$- consisted of the values of the five key variables that prevailed at the moment of inquiry. Hence, we call $\bar{y}$ the "present" situation.

Table 2. The questionnaire

i	y_1	y_2	y_3	y_4	y_5	U_i
0 "optimal"	..	..	..	..	..	100
1	4	7	92	1	57	..
2	4½	6	92	1	57	..
3	4½	7	90	1	57	..
4	4½	7	92	0	57	..
5	4½	7	92	1	55	..
6	4	7	92	0	57	..
7	4½	6	92	1	55	..
8	4½	7	92	1	59	..
9	4½	7	94	1	57	..
10	4	7	90	1	57	..
11	2½	4	78	-1½	47	..
12	2½	8	92	-1½	57	..
13	4½	4	92	1	47	..
14	3½	8	94	1½	67	..
15 "present"	4½	7	92	4	57	..

y_1 = Registered unemployment as a percentage of the dependent working population.

y_2 = Yearly Percentage increase in the cost of living price-index.

y_3 = Labour share in national income.

y_4 = (Competitiveness) Difference in percentage increase in real unit labour costs (in guilders) between The Netherlands and its competitors.

y_5 = Public share in net national income.

U_i = Utility index.

With respect to the utility indices U_i it should be added that some parties did not complete the whole column, others characterized some programs with qualifications like "revolution" instead of giving an index and also negative values sometimes appeared. All these features have in some sense been taken care of. The latter e.g. is simply corrected by a monotone transformation to a 0-100 scale; the notion "revolution" was given an arbitrary low number (the party involved did not strive for revolution as an aim in itself); and finally the reasons for not completing the list were exactly those that gave rise to the assumption of heteroskedasticity to be dealt with in section 3.

2.2. The estimation.

In Merkies and Nijman's paper the functional form of the preference function was not given. In view of the parsimony required to enable estimation of the parameters, the authors started with linear functions where only five parameters were to be estimated. Then a certain statistical strategy led them to expand on or to shrink from the full linear form. Due to the small number of degrees of freedom they never reached the quadratic form. The latter, however seems to be a more natural function to work with. It provides not only a better approximation than a linear function but it has also the nice property that optimal programs can directly be derived from it. This property will be used to reduce the sensitivity of the parameter estimates, but it has also advantages in other respects. For instance Chossudovsky (1974), who tried to establish party allegiancy by comparing distances from party optima, was seriously hampered by the fact that he had only linear functions at his disposal. Thus he could compute party allegiancy only for the party in power and then only by assuming that the actual and the preferred policy coincided. With quadratic preference functions he could, for each party, have used the optimum that is implied by this function. The argument that the use of a quadratic function provides an additional Taylor approximation term must be treated with care as the choice of the point y^* at which the Taylor expansion must be evaluated is not immediately clear. Possible choices are $y^*=y^0$ and $y=\bar{y}$. Of course as long as the relevant differences such as between y^0 and $\bar{y}$ are of minor importance, the specification of y^*, too, ceases to be important. We considered this to be the case here and we chose simply $y^*=\bar{y}$.

The quadratic preference function is written as

$$U(y) = a_0 - a'y - \tfrac{1}{2} y'Ay \tag{2.1}$$

where a' is a row vector with n linear parameters and A is a positive definite matrix of quadratic parameters. Without loss of generality A can be taken to be symmetric. Although the constant a_0 added to (2.1) is in itself of no relevance, it affects the estimates to be discussed below and is therefore retained here.
For n=2 (2.1) is written as

$$- U(y) = a_0 + a_1y_1 + a_2y_2 + \tfrac{1}{2}a_{11}y_1^2 + \tfrac{1}{2}(a_{12}+a_{21})y_1y_2 + \tfrac{1}{2}a_{22}y_2^2$$

or as

$$- U(y) = [1 \quad y_1 \quad y_2 \quad y_1^2 \quad y_1y_2 \quad y_2^2] \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \tfrac{1}{2}a_{11} \\ \tfrac{1}{2}(a_{12}+a_{21}) \\ \tfrac{1}{2}a_{22} \end{bmatrix} \tag{2.2}$$

To estimate the parameters of (2.1) we have to insert the values y^i and U_i of table 2. As we cannot expect the quadratic preference function to hold exactly, we have to add an error term. From (2.2) it is then seen that the estimation equation becomes

$$U = Y\beta + \varepsilon \tag{2.3}$$

where U is a vector of N indices (= the number of observations), Y is an Nx(k+1) matrix, β is a column vector of (K+1) parameters and ε is a column vector with N error terms. The number K needs clarification. For a full symmetric matrix A there are in general $\frac{n^2+n}{2}$ quadratic parameters to estimate. Together with n linear parameters this leads to $K = \frac{n^2+3n}{2}$. For n=2 we obtain K=5, (cf. (2.2)) and for n=5 we have K=20. With only 16 observations the estimation problem for n=5 cannot be solved. Hence restrictions are required.

2.3. The restrictions.

First we can take all off-diagonal elements of A equal to zero, retaining only the diagonal elements of A and the linear parameters to estimate. This reduces K to 2n. For n=5 the number of parameters is still relatively large in view of the number of observations. So other restrictions are needed. These can be found if it is realised that once we have estimated A and a the optimal value of y follows from

$$\frac{\partial U(y)}{\partial y} = - a - Ay = 0 \tag{2.4}$$

The vector $\tilde{y}$ that solves this equation may be compared with the value y^0 that is provided by the parties in the inquiry. As y^0 may be considered only approximately optimal we may write

$$y^0 = \tilde{y} - \delta \tag{2.5}$$

with δ having some error distribution e.g. $N(0,\Delta)$. Working backwards we may use (2.4) together with (2.5) as a restriction on the estimation problem (2.3). Substitution of (2.5) in (2.4) gives

$$a + Ay^0 + v = 0$$

where $v = A\delta$. For n=2 this gives with diagonal A

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} y_1^0 \\ y_2^0 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

which after some rearranging becomes

$$\begin{bmatrix} 0 & 1 & 0 & 2y_1^0 & 0 \\ 0 & 0 & 1 & 0 & 2y_2^0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \frac{1}{2}a_{11} \\ \frac{1}{2}a_{22} \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \tag{2.6}$$

or

$$R\beta + v = 0 \tag{2.7}$$

Note that after removal of the off-diagonal elements of A the vector β

in (2.2) equals that in (2.6). Hence the vector β in (2.7) is the same as that in (2.3) and contains in general K+1=2n+1 components. Combining (2.7) and (2.3) we have

$$\begin{bmatrix} U \\ 0 \end{bmatrix} = \begin{bmatrix} Y \\ R \end{bmatrix} \beta + \begin{bmatrix} \varepsilon \\ v \end{bmatrix} \tag{2.8}$$

If we assume zero expectations and independence of ε and v the variance-covariance matrix of the error may be written as

$$\begin{bmatrix} \varepsilon \\ v \end{bmatrix} = \begin{bmatrix} \sigma^2 V & 0 \\ 0 & \lambda\sigma^2 V_0 \end{bmatrix} \tag{2.9}$$

where λ is an indicator that relates the a priori information to the information contained in the sample.
The Theil-Goldberger (1961) estimator of β is

$$\beta = [Y'V^{-1}Y + \frac{1}{\lambda} R'V_0^{-1}R]^{-1}Y'V^{-1}U \tag{2.10}$$

Special cases are:

$\lambda \to 0$ the optimum $\tilde{y}$ equals y^0; (2.7) becomes deterministic and constitutes together with (2.3) a problem of estimation under exact restrictions; the dependence of the estimated parameters on the information in table 2 is very limited.

$\lambda \to \infty$ the optimal character of y^0 does not provide useful information; (2.10) reduces to a common G.L.S. estimator and the estimated parameters as well as the optimal value $\tilde{y}$ depend exclusively on the evaluations U^i and the programs y^i given in table 2.

To use (2.10) for the estimation of the parameters of (2.1) we must find appropriate values for the matrices V and V_0. The value of V is discussed in section 3. For V_0 we will assume:

$$V_0 = \begin{bmatrix} \sigma_1^2 & & & & \\ & \sigma_2^2 & & & \\ & & \sigma_3^2 & & \\ & & & \sigma_4^2 & \\ & & & & \sigma_5^2 \end{bmatrix} \tag{2.11}$$

where σ_i^2 is the variance of the variable y_i over the past. The reason for choosing (2.11) is that one may expect that the more volatile a variable y_i has been in the past, the more difficult it will be to specify its optimal value y_i^0. Note that (2.11) simultaneously solves the problem of the different dimensions of the components of the vector y. Note also the implicit assumption

$$\Delta = E\delta\delta' = EA^{-1}vv'(A^{-1})' = A^{-1}(\lambda\sigma^2 V_0)A^{-1};$$

which implies that the error δ has a larger variance for greater σ^2. This is reasonable because the accuracy with which table 2 is completed will certainly bear some relation to the accuracy with which y^0 is specified. Note finally that Δ is diagonal if A is. This implies that the optimum for variables with nearly linear utility-profiles and hence smaller values of a_{ii} are assumed to be more difficult to evaluate. The diagonality of A, which we have assumed for reasons of parsimony, becomes more convincing the more "basic" the five key-variables are i.e. the less they play the role of intermediaries with some final goals in the background. Of course it would have been possible to assume A to be an unrestricted symmetric matrix, if sufficient observations would have been available. As pointed out before it is usually not possible to obtain that many observations.

3. The assumption of heteroskedasticity.

3.1. Two types of error.

To compute (2.10) we still need to specify the variance-covariance matrix of the error term ε or more specifically the matrix V that is introduced in (2.9). We will do so in this section.

As a first step assume that the overall preference function of a political party is separable with respect to the set of variables y_1 to y_5 on the one hand and to the set of all other more secondary target variables on the other hand. Then the error term ε consists of two components ε_1 and ε_2. The former represents the quadratic utility function of the neglected secondary targets and ε_2 is an error of measurement. In this paper we will concentrate on the latter and therefore assume that the ε_{1i} are independent drawings from some symmetric distribution with constant variance and constant expectation $\bar{u}$. This may be a bold assumption at least for some points y^i, but it would require a separate analysis to incorporate this aspect. At the end of this section we will devote a few words to it.

3.2. The error of measurement and heteroskedasticity.

To illustrate the role of ε_2 consider again the case n=2, for instance a vector y with only unemployment (y_1) and inflation (y_2) as components. Assume that the programs in these two variables and their evaluations are such as given in fig. 1.

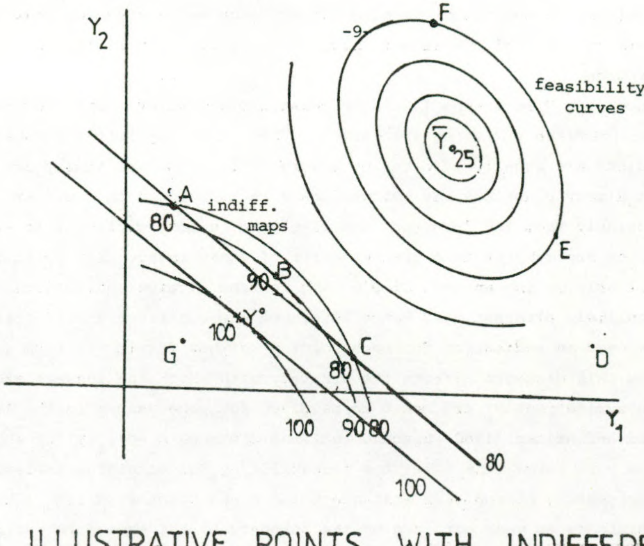

FIG.1 ILLUSTRATIVE POINTS WITH INDIFFERENCE MAPS AND FEASIBILITY CURVES

Figure 1 shows the optimal value y^0 to which a utility-index 100 is attached and the "present" situation $\bar{y}$, which we have given an index of 25. Apart from these also the points A, B, C,... have indices and the problem is to construct an indifference map. From the straight lines it is clear that in this fiugre a linear indifference map and hence a linear utility function would not work as point B with index 90 would be on the "wrong" side of the indifference curve 80 passing through A and C. Quadratic indifference curves would do here, but it is clear that in practice such a neat situation will only hold approximately. For instance, if point C had carried an index 75 we would have been obliged to construct the same indifference map - assuming that the index U_c bears an error of 5 - rather that taking refuge in utility-functions of higher order.

In regressing U on Y -see (2.3)- we construct a quadratic indifference map by minimizing the sum of squared errors. Then it is implicitly assumed that all points are equally affected by errors. This homoskedasticity assumption is not a very plausible one. Situations such as point D that differ considerably from the "present" are likely to be more difficult to evaluate and hence are subject to a greater error of measurement. This is illustrated most clearly by the answers of D'66 who refused to give evaluations on some unlikely alternatives. Hence before we can construct an indifference map we need an indication for each point y^i of how far it is from $\bar{y}$ and how this distance affects the accuracy with which the answers are given. As the evaluations U_i are meant to be given for some year τ in the future, Merkies and Nijman (1980) assumed that the distance $d_i=d(y_i,\bar{y})$ of some point y^i from $\bar{y}$ is larger the lower the probability p_i of attaining at least y_i in the forecasting period. For instance point E and point F in fig. 1 have the same distance to $\bar{y}$ as both are on the boundary of the same forecasting interval. This means that the probability p_E of attaining or surpassing point E in the forecasting period is equal to the probability p_F of attaining or surpassing point F.

Two questions now remain. First we have to show how the distance affects the accuracy of the answers and secondly we must indicate how the forecasting intervals are constructed. As the variance of the error term ε_i is assumed to be an increasing function of the distance it is a decreasing function of the probability p_i.

For convenience we have chosen

$$\text{var}\ (\varepsilon_i) = f(p_i) = p_i^{-2} \tag{3.1}$$

This together with the assumption of independence of the evaluation error leads to

$$E\ \varepsilon\varepsilon' = \sigma^2 V = \sigma^2\ \text{diag}\{p_1^{-2}, p_2^{-2}, \ldots, p_N^{-2}\} \tag{3.2}$$

-where V is the matrix in (2.9)- and therefore

$$V = (P'P)^{-1} \tag{3.3}$$

The transformations $T=PU$, $G=PY$ and $u=P\varepsilon$ now lead through (2.3) to the equation

$$T = G\beta + u$$

which -apart from restriction (2.7)- can be estimated by ordinary least squares as $Euu' = \sigma^2 I$.

This means that as soon as we dispose over a suitable measure of p_i we can remove the problem of the varying accuracy with which the various programs are evaluated by multiplying each point y^i and the corresponding index U^i by p_i. We may clarify this again by figure 1. First we construct a set of feasibility curves around $\bar{y}$, each curve connected with some probability p. Subsequently each point y^i in fig. 1 is "deflated" by multiplying its utility index U^i with the probability p of the feasibility curve on which it is lying.

3.3. The construction of the feasibility curves.

The question which remains is the construction of the feasibility curves. These depend on the planning horizon τ and on the particular forecasting model used. To acquire an adequate forecasting model this should be tested against some reference period in the past, but however well the model fits the developments of this reference period this does not guarantee that it will also adequately describe future developments. This depends upon the generality of the model and in our context on the possibilities of the policy makers to check developments if they want to do so. Of course, they always may overestimate their power in this respect and it is exactly the subjective views on future developments that we are concerned with. To deal with these considerations Merkies and Nijman (1980) assumed that the variance of the error term in the forecasting period will be, say, μ times higher than the variance of the estimated model in the past. The specific value of μ depends on the generality of the model adopted and the subjective feelings of the politicians that the developments patterns will change in the future.

In their study Merkies and Nijman used simply linear trends as forecasting formulae and adopted correspondingly the rather high value $\mu=15$.

More formally we have the following forecasting model

$$y_{it} = \delta_i + \gamma_i t + \eta_{it} \qquad i=1,2,\ldots,5 \tag{3.5}$$

with $\eta_t \approx IN(0,\Sigma)$.

This is estimated over the reference period 1962-1976 indicated by $t = -15,-14,\ldots,-1$, which gives

$$\begin{bmatrix} y_{i,-15} \\ y_{i,-14} \\ \cdot \\ \cdot \\ y_{i,-1} \end{bmatrix} = \begin{bmatrix} 1 & -15 \\ 1 & -14 \\ \cdot & \\ \cdot & \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \delta_i \\ \gamma_i \end{bmatrix} + \begin{bmatrix} \eta_{i,-15} \\ \eta_{i,-14} \\ \cdot \\ \cdot \\ \eta_{i,-1} \end{bmatrix} \qquad i = 1,2,\ldots,5 \tag{3.6}$$

or in matrix form

$$Y_v = X\Pi + H \tag{3.7}$$

Estimating Π by $(X'X)^{-1}X'Y_v$ and using as a forecasting period 1977, which corresponds with $t = \tau = 0$ we obtain the following forecasting interval (see Hooper and Zellner (1961)).

$$\frac{T-k-m+1}{(T-k)m} E_\tau' \hat{\Omega}^{-1} E_\tau = F_\alpha(m,T-k-m+1) \tag{3.8}$$

under the condition that ε is distributed $N(0,\Omega)$. In (3.8) E_τ is the vector with the five forecasting errors in year $t=\tau=0$, k equals 2 for a linear trend, m equals the number of targets (=5) and

$$\hat{\Omega} = (\mu+q_\tau)S \tag{3.9}$$

with S the estimate of Σ, $q_\tau = X_\tau(X'X)^{-1}X_\tau'$ with $X_\tau=[1,\tau]=[1,0]$ as $\tau=0$ and μ the ratio of the variance in the future to that in the past, referred to before. For n=2 the forecasting intervals (3.8) can be drawn for various probability limits α (see fig. 1) and for each point y^i we can compute the probability p_i, connected with the forecasting interval on which it is lying. For n=5 graphical illustration is not possible but the computations can be made using (3.8) and the computed errors.

For instance for program y^1 we compute $e^1_\tau = \hat{y}_\tau - y^1$, where for $\tau=0$ $\hat{y}_\tau = X_\tau\hat{\Pi} = [1\ 0].\hat{\Pi}$ is the first column of $\hat{\Pi}$

$$e^1_\tau = \begin{bmatrix} 4.51 \\ 10.33 \\ 93.33 \\ 1.36 \\ 58.08 \end{bmatrix} - \begin{bmatrix} 4 \\ 7 \\ 92 \\ 1 \\ 57 \end{bmatrix} = \begin{bmatrix} .51 \\ 3.33 \\ 1.33 \\ .36 \\ 1.08 \end{bmatrix}$$

With the first factor of (3.8) being equal to 1/7 and the estimated $\hat{\Omega}$ being by (3.9) with $\mu=15$ (for $\mu=1$, $\hat{\Omega}$ is given in Merkies and Nijman (1980)) we obtain the probabilities as presented in table 3.

Table 3. Probabilities of attaining or surpassing target vector y^i in 1977

(μ = 15)

i	p_i	i	p_i	i	p_i	i=opt	p_i
1	.9954	6	.9967	11	.0925	PvdA	.65
2	.9876	7	.8111	12	.9710	CDA	.64
3	.9969	8	.9786	13	.0112	VVD	.94
4	.9961	9	.9791	14	.0202	D'66	.66
5	.8293	10	.9976	15	1.000	PPR	.11
						GPV	.86
						SGP	.13

The alternatives y^i referred to are given in table 2 for $i\neq 0$ and for $i=0$ (=opt.) in table 5.

The feasibility-curves (3.8) are not only useful for removing heteroskedasticity but also both for constructing a proper sample design if the inquiry is going to be repeated and for the evaluation of the optimal policies y^0_g that are given by the various parties. Repetition of the inquiry is indeed envisaged and at the time of this writing already under way. A first evaluation of the parties' optima is given in Merkies and Vermaat [1980] and [1981]. This was based on the last part of table 3. An alternative suggested by the present article, would be to base such an evaluation on the estimates of $\tilde{y}_g$ instead of using y^0_g.

3.4. The specification errors.

We end this section by devoting some words to the neglect of the error ε_1, the part of the preference function that refers to the secondary policy targets. We start again in fig. 1. Each point y^i that is evaluated is in fact a projection of a point z^i in all relevant dimensions, secondary targets (as well as instruments if these have values of their own) included. In attaching a utility index u^i to a point y^i therefore implies the parties questioned have implicitly evaluated all vectors z^i that are consistent with projection y^i. A particular projection may lead to quite different evaluations. For example, in fig. 2 we represent the projection of table 2 with the evaluations of the PvdA in the domain of y_1 and y_2. If the socialists would have been confronted with the programs in these two variables alone they would certainly have given different indices. More formally we could have written (2.3) as

$$U = Z\theta + \varepsilon_2 = Y\beta + X\gamma + \varepsilon_2 = Y\beta + \varepsilon_1 + \varepsilon_2 \qquad (3.10)$$

where Y contains the targets of table 2, X all other ("secondary") targets and ε_2 as before the measurement error in U connected with the evaluation of the targets specified. Implicitly we have assumed that this measurement error is not affected by the neglect of specifying the X values. But apart from this as long as X is not specified the respondent must follow his own imagination on what reasonable values of X are thought of. He may have followed a forecasting procedure for $z^i = [y^i, x^i]$ as we have done in par. 3.3 for y^i only. Various possible values of x^i could thus have been thought of. If we take the mathematical expectation of these values conditional upon the specified values of y we get the following model for ε of (2.3)

$$\begin{aligned} \varepsilon_i &= \varepsilon_{1i} + \varepsilon_{2i} \\ &= E[x^i | y^i]\gamma + \varepsilon_{2i} \end{aligned} \qquad (3.11)$$

Hence the mathematical expectation of ε_i is not necessarily zero. If $E\varepsilon_{1i}$ is positive it adds to the utility index U_i given. If it is negative a lower utility index U_i results.

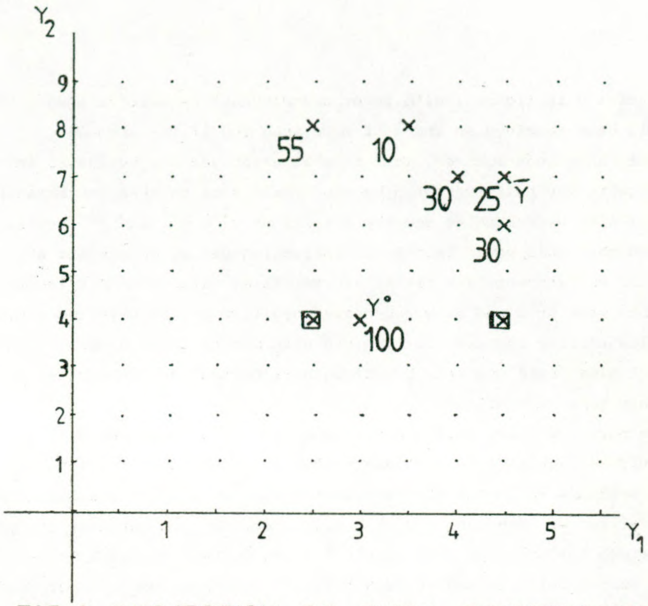

FIG.2 PROJECTION OF PVDA PREFERENCES ON Y_1 AND Y_2

Y_1 = UNEMPLOYMENT Y_2 = INCREASE OF PRICE INDEX

⊠ = REVOLUTION

For instance point G in figure 1 with lower unemployment as well as lower inflation would have received an index of more than 100 if the secondary objectives could have been ignored. That some alternatives are evaluated in the way (3.11) suggests may be illustrated by the comments of the PPR who remarked that they would like very much to see the situations y^{11}, y^{12} and y^{13} arise, but they feared that this would lead to an increasing use of energy and a violation of the environment. The latter are variables which were not incorporated in y byt as they were in z and $E\varepsilon$ was declared negative in the case, we could have expected the low utility indices that the PPR attached to these programs. Other parties did not give these comments but from their optimal programs y^0 we may assume that they behaved similarly.

To take this aspect explicity into account numerical information about $E[x^i|y^i]\beta_2$ would be required. In most cases such information is completely lacking, but sometimes as in the PPR-example the sign of this expression is known. As mentioned earlier we have not tried to incorporate this in our analysis yet, but simply assumed $E[x^i|y^i]\gamma$ to be a constant $\bar{u}$. In further research we may try to decompose the sample into subsamples each having a different constant or dummy.

4. The results

If we insert the probabilities of table 3 in (3.2) to obtain the matrix V and estimate σ_i^2 in (2.11) from the same date of the period 1962-1967 as used for the estimation of the probabilities p we are ready to compute (2.10) for each party. We adopted a value $\lambda=1$ for each, thus attributing equal value to both sources of information. This leads to the preference functions of table 4.

Table 4. Quadratic preference functions of Dutch political parties

party	linear part a_1	a_2	a_3	a_4	a_5	quadratic part $\frac{1}{2}A_{11}$	$\frac{1}{2}A_{22}$	$\frac{1}{2}A_{33}$	$\frac{1}{2}A_{44}$	$\frac{1}{2}A_{55}$
PvdA	11.3	3.0	4.0	4.4	-8.8	3.0	-0.11*	0.1	-1.0*	0.3
	(7.3)	(3.2)	(1.3)	(4.3)	(1.2)	(2.8)	(1.2)	(0.3)	(1.5)	(0.5)
CDA	31.7	3.7	4.6	13.4	1.8	10.5	0.5	0.25	4.9	1.1
	(5.9)	(3.2)	(1.0)	(4.3)	(1.1)	(2.3)	(0.6)	(0.1)	(2.2)	(0.3)
VVD	1.3	17.3	5.4	25.0	7.7	1.0	3.0	0.5	8.1	0.7
	(21.5)	(11.2)	(3.7)	(14.3)	(4.0)	(8.3)	(3.5)	(0.7)	(6.3)	(1.3)
D'66	-1.6*	5.4	3.4	8.5	2.4	-0.2*	0.9	0.2	4.2	0.5
	(8.9)	(5.5)	(1.8)	(13.5)	(1.9)	(2.1)	(1.1)	(0.4)	(11.0)	(0.5)
PPR	63.0	18.7	3.9	16.6	0.7	12.7	4.9	1.1	8.3	0.1
	(5.6)	(4.6)	(1.1)	(8.5)	(1.1)	(1.3)	(1.8)	(0.6)	(6.8)	(0.3)
SGP	32.7	6.2	1.6	7.6	1.2	10.9	0.7	0.1	2.3	0.1
	(4.9)	(2.7)	(0.8)	(3.5)	(0.9)	(1.9)	(0.5)	(0.1)	(1.7)	(0.1)
GPV	12.3	6.6	1.9	9.3	5.0	2.1	1.0	-0.2*	-0.5*	1.0
	(8.0)	(4.9)	(1.6)	(6.6)	(1.7)	(1.9)	(1.1)	(0.3)	(3.5)	(0.6)

* denotes non significant wrong sign. Standard errors are between brackets.

The standard errors given in table 4, show that the wrong signs of the coefficients marked with an asterisk are not significantly different from zero. As the (non-significant) positive coefficient a_1 for D'66 in connection with the (non-significant) positive sign of A_{11} implies that this party is in favour of unemployment, we can hardly accept this result as being final. But the small number of observations for this party hardly allows meaningful inference.

The validity of the preference-functions can also be judged from the estimates of the optima $\tilde{y}_g$ for each party. These are given in table 5.

Table 5. Estimated optima $\tilde{\tilde{y}}_g$ (λ=1) and reported optima y_g^o (λ=0).

	y_1		y_2		y_3		y_4		y_5	
λ:	1	0	1	0	1	0	1	0	1	0
PvdA	2.59 (.81)	3	20.55 (155.46)	4	76.14 (39.16)	89	3.20 (5.27)	-2	73.40 (30.48)	59
CDA	2.99 (.13)	3	2.98 (3.64)	2.5	82.75 (4.11)	82	-0.37 (.31)	0	56.2 (.48)	57
VVD	3.79 (6.09)	3	4.10 (2.26)	4	86.65 (5.75)	85	-0.54 (.63)	-1	51.63 (8.98)	55
D'66	0.45 (25.23)	2	3.89 (2.47)	2.5	81.44 (24.03)	85	0.0 (1.19)	0.0	54.64 (2.35)	52
PPR	2.02 (.11)	2	5.07 (.42)	5	90.20 (.98)	90	-0.0 (.37)	0	51.21 (32.79)	62
SGP	3.00 (.10)	3	2.79 (2.13)	2.5	84.91 (5.40)	78	-0,64 (.61)	-0.5	50.24 (9.59)	48
GPV	1.52 (1.31)	2	-25.38 (341.18)	3	96.50 (9.04)	85	9.78 (62.93)	0	54.46 (1.47)	55

approximate standard errors are between brackets.
Unsatisfactory results are underlined.

The estimates of the "true" optima of the CDA and the VVD appear to be not very different from what they reported as optimal. The results of the other parties are less close to the figures of the inquiry.
For the PvdA the optimum values for y_2, y_3 and y_5 are very different from their respective y^o values. This is due to the fact that the preference function for this party is almost linear. Hence the optimum can hardly be estimated with a quadratic function. The unreliable coefficient for D'66 with respect to unemployment could be expected from our previous scepsis, although it is remarkable that a lower rather than a higher unemployment figure appears.
Large differences also appear for the collective share (y_5) for the PPR and somewhat less with respect to $\tilde{y}_3$ and y_3^o for the SGP.
Finally as the function of the GPV from table 4 may be taken as completely

linear, it is no surprise that its optimum can hardly be estimated with a quadratic function. Only y_1 and y_5 show reasonable values.
A better insight is obtained if we take the limiting standard error of $\hat{\tilde{y}}_g$ into account. This can be approximated from the standard errors and the coefficients of table 4 by means of formula (4.1) which is derived from a formula given by Cramèr [1946, pp. 353-354]:

$$\frac{\text{var }\hat{\tilde{y}}_i}{(\hat{\tilde{y}}_i)^2} = \frac{\text{var }a_i^o}{a_i^2} + \frac{\text{var }A_{ii}}{A_{ii}^2} - \frac{2\text{ cov }a_i A_{ii}}{a_i\ A_{ii}} \tag{4.1}$$

The standard-errors $(\text{var }\hat{\tilde{y}}_i)^{\frac{1}{2}}$ obtained by this formula are given in table 5 in brackets below the estimated optimal values. These allow the computation of approximate confidence intervals for the optima $\tilde{y}$ of the various parties. The reported optima thus appear to be for each party largely within the approximate 95%-confidence interval.
All these results depend on the a priori chosen value $\lambda=1$. For $\lambda \to 0$ all values $\hat{\tilde{y}}_g$ tend to y_g^0 but the convergence is not monotone. We found that for computed optima with broad intervals the results are very sensitive with respect to λ. For parties such as the CDA and the VVD for whom computed optima have small intervals the results are more robust with respect to λ -see table 6-, although for $\lambda \to \infty$ the functions lose their significance, as could be expected.

Table 6. Sensitivity of $\hat{\tilde{y}}_g$ with respect to λ.

CDA	$\lambda=0$	0.5	1	10	100	1000
y_1	3	3.0	3.0	3.0	3.0	3.1
y_2	2.5	2.8	3.0	4.3	5.0	5.0
y_3	82	82.7	82.8	81.2	76.8	75.2
y_4	0	-0.4	-0.4	-0.5	-0.6	-0.8
y_5	57	56.2	56.2	56.2	56.2	56.2
VVD						
y_1	3	5.0	3.8	3.6	3.7	0.9
y_2	4	4.1	4.1	-0.4	8.4	7.8
y_3	85	86.3	86.7	88.0	89	89.3
y_4	-1	-0.6	-0.5	-0.3	-0.2	-0.1
y_5	55	52.8	51.6	45.3	9.4	-190.3

As a final check on the significance of the results we may compute the preferernce matrix $U_g(y_h^{opt.})$. It would be most appropriate to use the estimated value $\hat{\hat{y}}$ for the required $y_h^{opt.}$, but some of the components of $\hat{\hat{y}}_h$ are highly insignificant and do not make sense in a gain matrix. Hence it is safer to use y_h^0 for all parties h instead. The insignificant wrong signs of the quadratic preference may also prevent useful results. Hence we replaced these by zeroes in the computations but without reestimating the functions under these zero restrictions. With these amended functions and the reported optima y_h^0 we obtain in table 7.

Table 7. Preference matrix $U_g(y_h^0)$ for Dutch political parties

g \ h	PvdA	CDA	VVD	D'66	PPR	SGP	GPV
PvdA	87.6	85.8	60.0	26.7	<u>91.7</u>	-9.1	57.9
CDA	69.7	<u>99.5</u>	96.4	69.6	38.7	21.5	87.6
VVD	52.0	69.5	<u>99.9</u>	97.1	21.1	58.0	92.8
D'66	73.2	100.2	100.9	98.1	67.3	74.4	<u>102.5</u>
PPR	105.2	40.3	104.5	97.9	<u>153.8</u>	-53.6	109.4
SGP	100.4	109.9	<u>111.5</u>	102.8	83.2	110.3	101.2
GPV	68.0	85.5	86.3	83.7	9.3	61.7	<u>86.8</u>

It is somewhat odd to find that the preference matrix is not dominated by its diagonal. This means that e.g. D'66 might have preferred after all the optimum values of the CDA, the VVD and the GPV over the alternative which they had themselves indicated as optimal in the inquiry. And the PvdA and the SGP might after all have considered the optima as described by the PPR and the VVD, as superior to their own.
Neither of these differences, though is significant. Hence further analysis is required, see e.g. Van Daal and Merkies (1981).

5. Conclusions.

In this paper it is shown to be possible to describe policy objectives of various political groups in mathematical terms, although the results in itself are to be amended in various ways before they actually become adequate. The provisional functions mentioned in table 4 are of some value for the Christian Democrats (CDA) and the Liberals (VVD). The more confidence one has in these results, the more one should replace the reported optima y^0 by our computed value $\ddot{y}$. There is ample room for further analysis. First we may try to introduce asymmetries that were found to be of relevance in earlier studies into the quadratic functions. Then the aspect of biased indices U^i when secondary objectives are not negligible as mentioned at the end of section 3 needs further attention. The aim should be to find functions that pass the various tests of sign and counterchecking by preference-matrices or ranking-test as shown in Van Daal and Merkies (1981). If such functions can be obtained they may facilitate collective choice with respect to issues of short-term economic policy and add empirical content to collective choice theory.

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