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INSTATIONARY DOLPHIN FLIGHT: THE OPTIMAL ENERGY EXCHANGE BETWEEN A SAILPLANE AND VERTICAL CURRENTS IN THE ATMOSPHERE

by

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Abstract

Sailplanes, when launched, need to extract their energy from vertical motion of the air in the atmosphere. The usual procedure is to circle and climb in vertical currents of limited spatial dimension ("thermals") under cumulus clouds and thereafter to exchange the gained height into distance by gliding out to the next cumulus cloud. While doing so sailplane pilots will often encounter larger scale regions where the atmosphere moves in vertical direction. Upto recent years the optimal strategy in such situations was assumed to be to fly slower through regions with upward moving air and faster through regions with downward moving air. Only a few years ago it was discovered both in theory, from energy considerations, as well as in practice, by contest pilots, that in some circumstances more energy could be extracted from the atmosphere when the loadfactor, i.e. the total aerodynamic lifting force on the wings divided by the weight, was varied. The research reported in this paper was set up to investigate this phenomenon. To that end a simple dynamic model was assumed for the sailplane and the optimization problem was formulated as an optimal control problem with terminal constraints. This optimal control problem was solved numerically for a number of different situations as regards to the extent as well as to the strength of the vertical currents encountered. The computer program used for this purpose was a rather general continuous optimal control program based on the use of (conjugate) gradients in function space and a projection operation to account for the terminal constraints. Some preliminary results are presented

Keywords

Aerospace trajectories, Optimal control, Numerical methods, Energy control.

1. Introduction

The optimization of sailplane flight trajectories has long been the exclusive domain of theoretically inclined sailplane pilots. Wellknown is the so-called MacCready theory made popular by 1956 World Champion Paul B. MacCready and described very well in the book [R-1] of the 1970-1974-1978 World Champion Helmut Reichmann. This theory deals with the selection of the optimal speed between the columns of rising air ("thermals") which are often found in summer under cumulus clouds and which are the main locations for glider pilots to regain the altitude lost by gliding out. The simple solution to this problem has led to a number of special instruments and in-flight computing aids with which the realization of the optimal solution is made into a very simple matter.

Only very recently sailplane trajectory optimization problems have also drawn the attention of optimization specialists, who, among others became very much interested in the optimal dynamic trajectories of sailplanes (eg. [P-1] [P-2] [P-3] and [P-4]). The main reason for this interest was the fact that the dynamic behaviour of sailplanes can be described by some simple nonlinear differential equations and as such constitutes an interesting example for trying out modern dynamic optimization techniques. Recently, it turned out that the optimal trajectories of sailplanes through vertically moving air masses were even more interesting than expected: Two completely different optimal solutions were found in problem situations which were not very different from each other. It is with the second, more or less unexpected optimal solution that this paper is mainly concerned with.

The first type of optimal sailplane trajectory that was found can easily be explained. It is based on the logic that a sailplane should fly fast through regions where the motion of the air is directed downwards and should fly slow through regions with rising air. In the every day situations where regions with upward moving air usually alternate with regions with downward moving air, the resulting optimal trajectory is a wavy trajectory, that, for similarity reasons, is called a <u>dolphin flight trajectory</u>. This type of trajectory is actually being practiced regularly these days by many sailplane pilots. To distinguish this first type of optimal trajectory from the second to be discussed below, we will call it the <u>quasi-stationary</u> dolphin flight trajectory.

The second type of optimal trajectory that was found differs from the first in that the motion of the sailplane is much more brusque and involves strongly varying normal loads: Through regions of downward directed air the sailplane is flown at very small or even negative normal loads. In regions of rising air the sailplane is accelerated until very near the point of maximal vertical atmospheric velocity where at high normal loads the sailplane is forced to climb. The basic reason for the optimality of this type of trajectory is the fact that high normal loads in combination with a vertical velocity yield a fast and favorable (potential) energy increase of the sailplane. In order to distinguish this type of optimal trajectory from the former, it will be called the <u>instationary</u> dolphin flight trajectory.

The possibility of improving the energy transfer from the moving atmosphere to the sailplane by varying the normal load on the sailplane has first been considered by some theoreticians under the glider pilots, notably by Joszef Gedeon [G-1] and by Wolfram Gorisch [G-2]. Both used computer simulations to investigate the phenomenon, neither of both, however, made use of dynamic optimization techniques. Pierson and Chen [P-3] encountered the second type of optimal trajectory when they applied dynamic optimization techniques to the problem of determining the minimum altitude loss trajectory through a prescribed vertical wind distribution. Their finding, which they could not explain at that time, provided the motive for the research reported on in this paper. This research has as final goal te derive practical strategies, if possible, by which the optimal instationary dolphin flight can be realized. Similar research is being carried out at the Technical University of Braunschweig by G. Schänzer [S-1] .

In the present paper first the energy transfer phenomenon is discussed in Section 2. Next an optimal control problem is formulated in Section 3 which fits in with the usual optimization of sailplane flights and which has, depending on the data, either of both optimal dolphin flight trajectories as solution. The main characteristics of the optimal control program used to derive the first results form the topic of Section 4 whereas the first results themselves are presented in Section 5. A short summary of some preliminary conclusions concludes the paper.

2. The energy transfer

The equations for the transfer of energy from the moving air to the sailplane may be derived from simple mechanical considerations. To be considered to that end are the forces that act on the sailplane and the absolute velocities of the sailplane relative to an inertial reference frame. Assuming the flight of the sailplane restricted to the vertical plane and the gravity acceleration $\vec{g}$ constant relative to a flat earth, the main forces acting on the sailplane are as sketched in Figure 1. They are the sailplane's weight $\vec{W} = m\vec{g}$ and the aerodynamic force $\vec{R}$. The latter is usually thought of to be sum of a component $\vec{L}$, the lift, perpendicular to the velocity relative to the air and a component $\vec{D}$, the drag, in the direction of the velocity relative to the air. The velocity of the sailplane relative to an inertial reference frame is the vector sum of the velocity $\vec{v}$ relative to the air and the velocity $\vec{u}$ of the atmosphere, which is assumed to have only a vertical component. The equation of motion of the sailplane in terms of the coordinate $\vec{s}$ of the inertial reference frame thus becomes

$$m\ddot{\vec{s}} = \vec{W} + \vec{L} + \vec{D}$$

where an arrow $\rightharpoonup$ is used to denote vectors and where

$$\dot{\vec{s}} = \vec{v} + \vec{u} \quad .$$

The increase of the kinetic energy per unit of time is given by the innerproduct of the forces acting on the sailplane and the absolute velocity

$$\mathring{T} = \langle m\ddot{\vec{s}}\,,\, \dot{\vec{s}}\rangle = \langle \vec{L} + \vec{D} + \vec{W}\,,\, \vec{v} + \vec{u}\rangle$$

where the notation $\langle\,,\,\rangle$ is used to denote the innerproduct. The increase in potential energy is similarly given by the expression

$$\mathring{U} = -\langle \vec{W}\,,\, \vec{v} + \vec{u}\rangle \ .$$

The total energy change of the sailplane is the sum of the changes in the kinetic and the potential energy i.e.

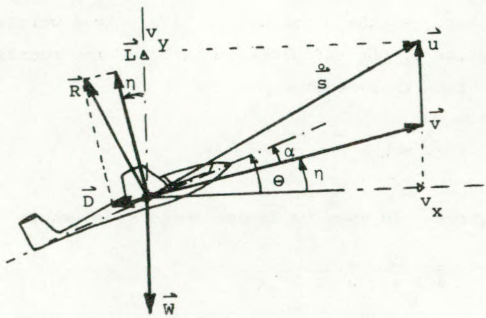

Fig. 1. Forces, velocities and angles.

$$\mathring{E} = \mathring{T} + \mathring{U} = \langle \vec{L} + \vec{D} \ , \ \vec{v} + \vec{u} \rangle$$

or, equivalently, as $\vec{L}$ is perpendicular to $\vec{v}$

$$\mathring{E} = (\vec{L} + \vec{D} \ , \ \vec{u}) + (\vec{D} \ , \ \vec{v}) \ .$$

If η (see Figure 1) is the angle between the lift vector and the vertical and use is made of the wellknown formulas for the lift and the drag

$$L = C_L \cdot \frac{1}{2} \rho v^2 S$$

$$D = C_D \cdot \frac{1}{2} \rho v^2 S$$

where C_L and C_D are coefficients, ρ is the density of the air and S is area of the wing, then the energy increase of the sailplane may be written out as

$$\mathring{E} = \frac{1}{2} \rho v^2 S \, (C_L \, u \cos \eta - C_D \, u \sin \eta - C_D v) \ .$$

Of interest for the present discussion is the normal acceleration of the sailplane, which is usually expressed in terms of the number of times n that the normal acceleration is greater than the weight of the sailplane, i.e.

$$n = \frac{L}{mg} = \frac{C_L \cdot \frac{1}{2} \rho v^2 S}{mg} \ .$$

With this definition the equation for the energy increase becomes

$$\mathring{E} = n \cdot mg \ (u \cos \eta - \frac{C_D}{C_L} (n,v) \ \sin \eta - \frac{C_D}{C_L} (n,v) \ v)$$

where the notation $\frac{C_D}{C_L}$ (n,v) is used to emphasize that the drag-lift ratio corresponds to a situation (or to be precise an angle of attack), where at a velocity v the lift force L equals n times the weight. In this case it makes sense to define a hypothetical rate of climb

$$w_p(n,v) = - \frac{C_D}{C_L} (n,v) \ v$$

which is the rate of climb that a sailplane that weighs n times as much as the actual sailplane would have in equilibrium flight at the velocity v.

With this definition the energy increase per unit time of the sailplane is given by the illustrative expression

$$\frac{\mathring{E}}{mg} = n(u\cos\eta - \frac{C_D}{C_L}(n,v)\,u\sin\eta + w_p(n,v))$$

From the expression it is obvious that energy transfer from the moving atmosphere to the sailplane is directly proportional by the normal acceleration of the sailplane, the magnitude of which the pilot can select from a relatively broad range. From the expression it also immediately follows that it is advantageous to make use of high or low normal accelerations depending on whether the vertical velocity within the brackets in the expression is positive or not, i.e. whenever the following inequality is satisfied

$$u\cos\eta + w_p(n,v) \geq \frac{C_D}{C_L}(n,v)\; u\,\sin\eta \approx 0\;.$$

This conclusion holds for any instant of time during a flight of the sailplane through an atmosphere with a variable vertical velocity. The optimal trajectory in such circumstances is the subject of the next section.

3. The optimal control problem

In order to get some insight in the problem of the optimal energy transfer from the moving atmosphere to a sailplane, the problem of the best strategy to fly a sailplane through a (model) thermal was considered for different values of the vertical atmospheric velocity and for different horizontal dimensions of the thermal. This problem can be formulated as an optimal control problem: An appropriate choice for the quantity to be optimized is the relative time, which is defined as the sum of the actual time to fly the horizontal distance specified and the time necessary to recover the lost height in the next thermal. This relative time thus is dependent on - and that explains the adjective "relative" - the value of the rate of climb in the next thermal. By this choice of object function the problem formulation fits in very well with the usual practice in sailplane trajectory optimization. (cf. de Jong [J-1], [J-2]).

The equations of motion that present themselves are the usual simplified equations for motion in the vertical plane. They apply to the case where the sailplane is considered to be a point mass subject to only the force of gravity and to an aerodynamic force, the components of which are given by the wellknown lift- and dragformulas. Assuming that the relation that exists between the lift- and the drag coefficients in stationary flight continues to hold in instationary situations, the lift coefficient may be selected as the control variable. As initial and final conditions for the velocity of the sailplane those values may be chosen which would have been optimal if only isolated thermals with the hypothesized rate of climb would have been present.

In formula form, the optimal control problem thus may be formulated as follows:

"For all $t \in [t_b, t_f]$ select $C_L(t)$ so as to minimize

$$\int_{t_b}^{t_f} 1 \, dt - \frac{1}{z} \int_{t_b}^{t_f} v_y \, dt$$

subject to the equations of motion

$$\overset{\circ}{x} = v_x \qquad \overset{\circ}{y} = v_y$$

$$\overset{\circ}{v}_x = [- L(C_L, v) \sin\eta - D(C_L, v) \cos\eta \,] \,/\, m$$

$$\overset{\circ}{v}_y = [L(C_L, v) \cos\eta - D(C_L, v) \sin\eta \; - mg] \,/\, m$$

where

$$L(C_L, v) = C_L \frac{\rho S}{2} v^2 \qquad D(C_L, v) = C_D(C_L) \frac{\rho S}{2} v^2$$

$$v = (v_x^2 + (v_y^2 - u_a(x))^2)^{1/2}$$

$$\eta = \arctan\left(\frac{v_y - u_a(x)}{v_x}\right)$$

$$u_a(x) = u_{a,max}\, e^{-(\frac{x}{R} - 2.5)^2} [1 - (\frac{x}{R} - 2.5)^2]$$

$$C_D(C_L) = \sum_{i=0}^{p} k_i C_L^{\,i} \quad ,$$

subject to the initial and final conditions

$$x(t_b) = 0 \qquad x(t_f) = 5R$$

$$y(t_b) = 0 \qquad y(t_f) \text{ free}$$

$$v_x(t_b) = v_x(t_f) = v_{x,MC}(z)$$

$$v_y(t_b) = v_y(t_f) = v_{y,MC}(z)$$

and, finally, subject to the control constraints

$$C_{L,min} \le C_L \le C_{L,max}$$

It may be remarked that the specified distribution $u_a(x)$ for the vertical atmospheric velocity is equal to the distribution advocated by Gedeon [G-1] for a single thermal. The velocities $v_{x,MC}(z)$ and $v_{y,MC}(z)$ are the velocities that correspond to the optimal solution of the classical sailplane trajectory or MacCready problem for the case that there exist only isolated thermals in which a net rate of climb equal to z can be realized.

The optimal control problem as formulated above is a free final time problem which can be easily simplified for computing purposes by the choice of the horizontal coordinate x as new independent variable instead of the time t. Using the relation that exists between the derivatives with respect to t and x

$$a' = \frac{da}{dx} = \frac{da}{dt} \cdot \frac{dt}{dx} = \frac{1}{\mathring{x}} \mathring{a} = \frac{1}{v_x} \mathring{a}$$

the optimal control problem may be formulated in a similar way as before as a fixed terminal time problem with the object functional replaced by

$$\int_0^{5R} \frac{dx}{v_x} - \frac{1}{z} \int_0^{5R} \frac{v_y}{v_x} dx$$

and the equations of motion replaced by

$$x' = 1 \qquad y' = v_y/v_x$$

$$v_x' = [- L(C_L,v) \sin\eta - D(C_L,v) \cos\eta] \,/\, m v_x$$

$$v_y' = [L(C_L,v) \cos\eta - D(C_L,v) \sin\eta - mg] \,/\, m v_x \;,$$

where $L(C_L,v)$, $D(C_L,v)$, v, η, $u_a(x)$ and $C_D(C_L)$ are defined as before. The initial and final conditions as well as the control constraints remain unchanged in this formulation.

4. The computer program

The optimal control problem formulated in the preceding section was numerically solved for a number of different cases on the Burroughs B7700 computer of the Eindhoven University of Technology with the aid of the computer program OPTCONTROL. This program is a rather general program written in Algol 60 for solving numerical optimal control problems by means of a conjugate gradient algorithm in function space. The program is a further development of the program that was designed for the determination of the optimal strategies for a controlled World model and that as such was described by de Jong and Dercksen [J-3] . Since then the program has been expanded, the main expansion being that terminal constraints can now also be taken care of. This is realized by the use of a projected gradient in function space in combination with a restoration procedure. For the projection of the gradient use is made of the modified Gram-Schmidt algorithm. The program has also been expanded to be able to handle free final time problems, but this facility was not used in the present case. The control constraints were taken care of by means of the clipping technique already described in the paper mentioned above.

In its present form the program OPTCONTROL has the nice feature of being quite generally applicable and rather easy to use. Its main disadvantage is its relatively poor efficiency: For the forward integration of the state equations and the backward integration of the adjoint equations use is made of a standard fourth order Runge-Kutta integration procedure with in most cases 50 integration steps. One integration of the simple system of the state or co-state equations costs in the order of 1 sec processing time. With in the order of 300 integrations (on the average) being not unusual for convergence, the total amount of computer time thus required is much too high for the research program being envisaged. Investigations are currently underway with the goal to obtain a more efficient procedure for the numerical solution of the optimal control problem at hand. As an aside at this point it may be remarked that the high average number of integrations required for convergence may be attributed for a large part to the poor performance of some individual subroutines, of which should be mentioned in particular the line-minimization procedure and the convergence test.

5. Numerical results

Solutions of the optimal control problem were obtained for an LS-3-sailplane, which is a modern 15m-racing-class sailplane, that from a computational point of view is specified by the following data

wing area	S	=	10.5	m^2
weight	W	=	346,5	kgf

and by the coefficients of the polynomial that approximates its C_L - C_D - relationship between $C_{L,min} = -1.4$ and $C_{L,max} = +1.4$

$$k_0 = +0.0118 \qquad k_1 = -0.0254$$
$$k_2 = +0.0770 \qquad k_3 = -0.0540$$
$$k_4 = +0.0166$$

The horizontal and the vertical velocities, that correspond for an LS-3 to the optimal solution of the MacCready problem and which served as initial and final conditions are, depending on the net rate of climb z assumed for the isolated thermals respectively, for z = 2m/s

$$v_{MC,x} = 41.631 \text{ m/s} \qquad v_{MC,y} = -1.344 \text{ m/s}$$

and for z = 4 m/s

$$v_{MC,x} = 48.708 \text{ m/s} \qquad v_{MC,y} = -2.064 \text{ m/s} .$$

As first step in the investigation the optimal control problem was for comparison purposes solved for 4 x 2 x 2 = 16 different combinations:

4 ranges (or, equivalently, 4 radii of the model thermal)

500 m (R = 100) , 1000 m (R = 200 m)

2000 m (R = 400) , 4000 m (R = 800 m)

2 maximum atmospheric velocities, $u_{a,max}$

2.5 m/s , 5 m/s

2 hypothetical net rate of climb , z

2 m/s , 4 m/s .

A survey of the most relevant data on the effects of the optimization in these cases is presented in Table 1. To be noted there is that the resulting type of the optimal trajectory is conform the expectations: for large ranges (or large termal radii) and comparatively low (initial and final) velocities the quasistationary dolphin flight trajectory ("type I-trajectory" according to Pierson and Chen [P-3]) is optimal, for short ranges (small thermal radii) and high initial velocities the instationary dolphin flight trajectory ("type II-trajectory") is optimal. For the intermediate range the type of the trajectory was not completely clear ("type I/II"). Of interest to note is that the effects of the optimization differ depending on the type of optimal trajectory. In case of a quasistationary dolphin flight (type I) trajectory

Combinations	Constant velocity trajectory			Optimal trajectory					
Range/$u_{a,max}$/z	h(m)	t(sec)	t_{rel}(sec)	type	h(m)	t(sec)	t_{rel}(sec)	Δt_{rel}(sec)	Δt_{rel}(%)
500/2.5/2	- 10.815	12.001	17.408	II	- 8.426	11.616	15.829	1.579	9.07
500/2.5/4	- 16.646	10.258	14.419	II	- 14.799	9.962	13.662	0.757	5.25
500/5 /2	- 5.482	12.000	14.741	II	+ 7.152	11.524	7.948	6.793	46.08
500/5 /4	- 12.110	10.256	13.283	II	- 0.267	9.941	10.008	3.275	24.66
1000/2.5/2	- 21.588	24.013	34.807	II	- 17.826	24.831	33.744	1.063	3.05
1000/2.5/4	- 33.336	20.512	28.846	II	- 29.838	20.755	28.214	0.632	2.19
1000/5 /2	- 10.800	24.013	29.413	II	+ 0.909	25.142	24.688	4.725	16.06
1000/5 /4	- 24.325	20.495	26.576	II	- 11.621	20.960	23.865	2.711	10.20
2000/2.5/2	- 43.146	48.056	69.629	I	- 27.372	55.107	68.793	0.836	1.20
2000/2.5/4	- 66.330	41.088	57.670	I/II	- 60.436	42.366	57.475	0.195	0.34
2000/5 /2	- 21.133	47.899	58.465	I	+ 24.161	60.943	48.863	9.602	16.42
2000/5 /4	- 47.802	41.124	53.074	I/II	- 34.934	43.577	43.577	0.764	1.44
4000/2.5/2	- 86.368	96.082	139.266	I	- 56.542	107.286	135.557	3.709	2.66
4000/2.5/4	-132.940	82.128	115.363	I	-116.711	85.645	114.823	0.540	0.47
4000/5 /2	- 43.595	96.087	117.884	I	+ 59.020	121.539	92.029	25.855	21.93
4000/5 /4	- 96.390	82.141	106.238	I	- 23.739	96.349	102.284	3.954	3.72

Table 1. Comparison of the effects of the optimization on the trajectory data.

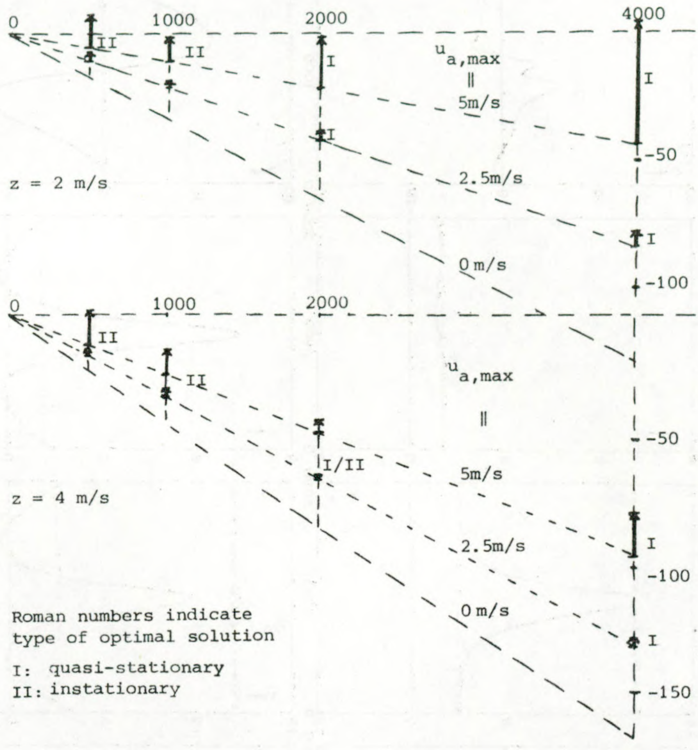

Fig. 2. Effect of model thermal and optimization on the relative height. Arrows represent gain due to optimization.

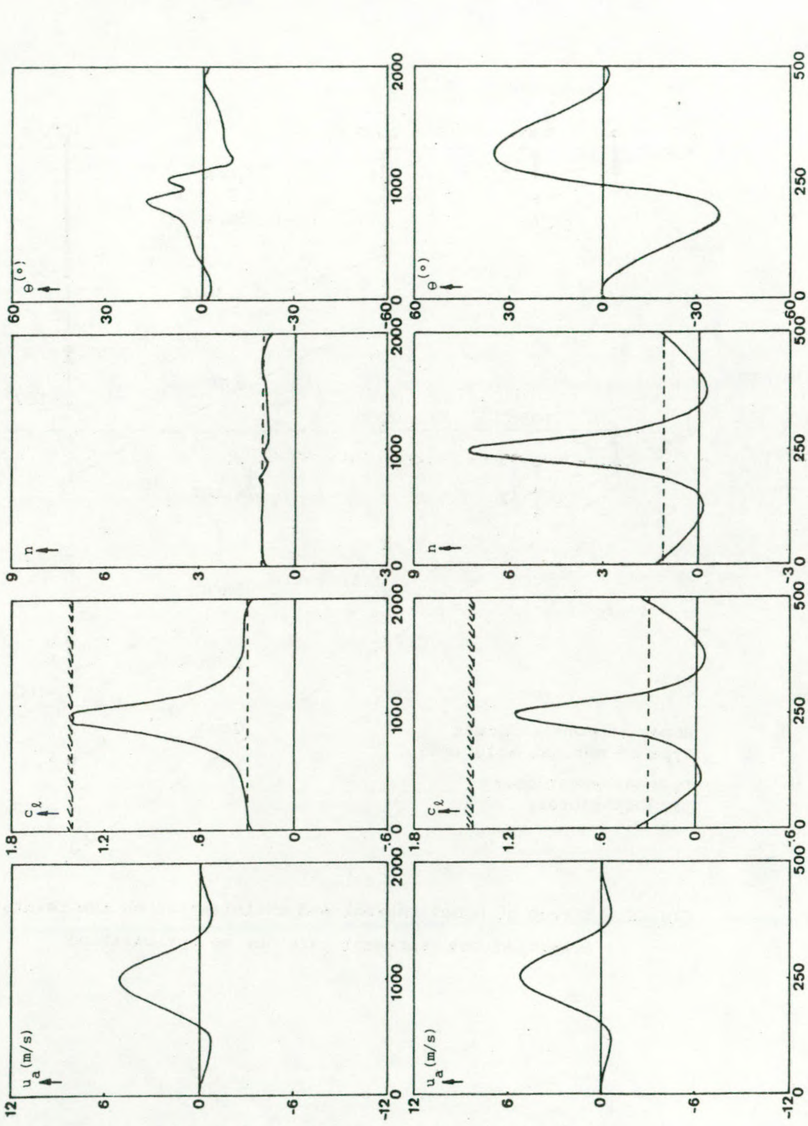

Figure 3a: Timehistories for the optimal solutions

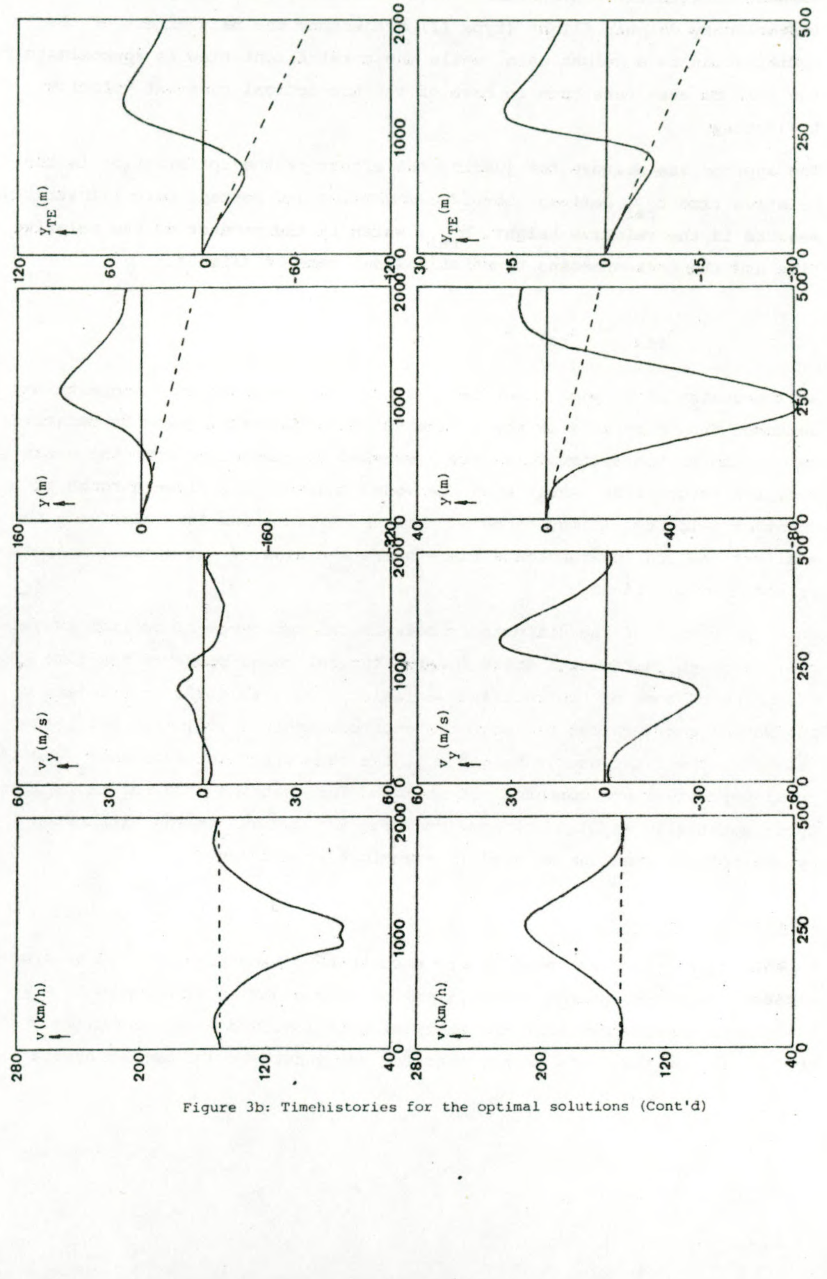

Figure 3b: Timehistories for the optimal solutions (Cont'd)

the actual time of flight as well as the actual height gain are large in comparison with the non-optimal constant velocity trajectory. In case of instationary dolphin flight (type II) trajectory the main effect of the optimization is a height gain, while the total flight time is approximately the same or even less then in case of the non-optimal constant velocity trajectory.

The appropriate measure for judging the effect of the optimization is the relative time t_{rel} defined above. An equivalent and perhaps more illustrative measure is the relative height, h_{rel}, which is the product of the relative time and the corresponding hypothetical net rate of climb z

$$h_{rel} = t_{rel} \cdot z \quad .$$

An impression of to what practical results the optimization amounts to may be gathered from Figure 2, in which schematically the extra gains in relative height due to the optimization are presented in comparison with the gains in relative height that result when the model thermals are flown through at constant velocity. It may be noted that the extra gains are relatively the smallest for the intermedidate range where the type of the optimal trajectory is not clearly defined.

An illustration of the differences between the two types of optimal trajectories is given in Figure 3 where for two typical cases plots of the time histories of some of the relevant variables, i.e., the lift coefficient C_L, the normal acceleration factor n, the attitude angle Θ (cf. Figure 1), the height h, the total energy height h_{TE}, the absolute rate of climb v_y and the total velocity v are compared. It is hoped for that the study of plots like these eventually may lead to some strategy for optimal energy extraction by sailplanes that can be used in practical situations.

6. Concluding remarks

In this paper the first preliminary results have been presented of an investigation into the optimal energy transfer from a moving atmosphere to a sailplane. Consideration of the physical laws involved leads immediately to the conclusion that this energy transfer may indeed be influenced by the pilot who has the option to change the normal load factor of the aerodynamic forces on the sailplane. Varying the loadfactor, however, also influences the trajectory and the determination of the optimal energy transfer thus turns out to be equivalent to the determination of the optimal trajectory, provided that the appropriate object functional is used. The particular choice of the relative time for object functional fits in with the usual quasistationary sailplane trajectory optimization and as such seems to be the best starting point for a search for a practical optimal strategy.

Using a model for a vertical atmospheric velocity distribution that has generally been accepted as a realistic model for the actual atmosphere, it was found that the two completely different types of optimal solutions encountered by earlier investigators also result in case of this realistic model. In fact, the parameter values for which either of both types of optimal trajectories result are quite common values which are quite likely to occur in practice. A further search for a strategy that selects and generates the appropirate optimal trajectory seems therefore to be quite worth while from a practical point of view.

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