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THE BETTING MODEL: THREE COMPETING PREDICTIONS AND THE CORRUPTING INFLUENCE OF BOOKMAKERS.

Joop M. Houtkooper

Jan Swammerdam Institute, University of Amsterdam.

SUMMARY:

Hofstee and Nevels (1980) have proposed an apparently powerful methodological tool to settle scientific disputes, namely the betting model. A practical betting example with three participants is given, from which the conclusion is drawn that for three or more participants the sizes of bets cannot be freely negotiable. Furthermore it is shown that a riskless profit is gained in the middle position, called the bookmaker's position. The rationale of the bookmaker's position is discussed. The influence of the participation of bookmakers on more serious participants is shown to corrupt the reproducingness of the scoring rule. Possible modifications of the scoring rule for more than two participants have been tried, but as yet without success. Therefore the betting model is not in working order for the case that more than two participants want to bet with each other.

Hofstee and Nevels (1980) have proposed an apparently powerful methodological tool to settle scientific disputes, namely the betting model. As I see it, any subjective belief is forced to declare itself either not seriously or seriously meant, in which case it is immediately exposable to the acid test.

In the following I will give a practical example with the purpose of showing that some extra rules will be necessary to regulate the betting behavior between more than two participants.

As it was, the three of us, B, D and J, were together, discussing a rather speculative phenomenon and after long talks and a lot of hedging, falling several times in the trap of infinite regression, we discovered we could agree on a rather simple experiment with two different outcomes, called Star and Not-Star. Now, each of us stated his subjective probability of Star: P_D=.55, P_B=.505 and P_J=.515.

With three participants, there are three betting positions between the pairs of participants and by simply adding them we get a pay-off matrix. We calculate the pay-off for both outcomes using the quadratic scoring rule and the subjectively expected values (SEV) for each bet and for each participant, shown in the following table.

TABLE:
Pay-offs and Subjectively Expected Values in an example with three participants.

Bet	Outcome Not-Star	Outcome Star	SEV
B-D	.047475	-.042525	.002025
B-J	.010200	-.009800	.000100
J-D	.037275	-.032725	.001225

Participant			SEV' *)
B	.057675	-.052325	.002125
D	-.084750	.075250	.003250
J	.027075	-.022925	.001325

*) SEV for the participants if bets have equal sizes (=1).

So far, nothing seems wrong, but being people of flesh and blood as well as betting addicts, we decided to bet with money. But how much?

At this point, B said: "Why should I bet with you at all, J? I get better odds with D!" And I answered after some thought: "I have to take up position with one of the extremes to gain some money."

In the following analysis we assume everybody behaves rationally, that is, each participant tries to maximize his gains according to his own beliefs. Now we can explore what should happen when participants have a certain freedom to chose the size of their bets. This freedom can for example be limited by the maximum loss (M) one allows oneself.

First we consider what a consistently successful bookmaker does. After a little arithmetic on the above pay-off matrix, we calculate the pay-off for J if the size, H_{BJ}, of his bet with B is 3.5 times the size, H_{DJ}, of his bet with D. Size is here used as the multiplier of the standard pay-off: $V_{AB}=q_B^2-q_A^2$ (Hofstee and Nevels 1980).

The result is: V_J= .001575 H_{DJ}, independent of the outcome of the experiment! This is the bookmaker's ideal of balancing his bets against each other.

Secondly, we turn to B's selfish outcry mentioned above. For a given maximum possible loss M, B can balance his bets with J and D in such a way to maximize his SEV. This results in the following ratio of bets: the size of his bet with D: H_{BD}=M/.042525 and the size of his bet with J: H_{BJ}=0 indeed! So it is true that he gains nothing by betting with a coward, as long as someone who is more stupid is available. The same goes for D of course, except that his maximum allowable bet is M/.047475, slightly lower than B's optimum (assuming that B and D act with the same value for M). This means that B cannot bet as much as he wants with D, so that he will expect a small gain by agreeing on a small bet with J, within the limit of his maximum possible loss M.

Thirdly, what can J do to stay in the game, or rather to increase his SEV? The value of his small bet with B is only .00106 M. If I choose to side with B, my SEV becomes .001125. If D is rational enough to split his betting capability equally between B and me, the value of my bet becomes:
SEV'_J= .001125 * .5 * M/.047475 = .01185 M, a more than ten-fold increase of my expectation over the case where I behaved according to belief in the reproducingness of the quadratic rule. I can do even better by outbidding B, but then he will do the same, up to the point where my SEV becomes zero.

At that point, it is still profitable for B to outbid me. The result is B calling at about .48, but possibly only as long as D remains unaware of our competition. Anyway, to cut a long story short, in the case of three participants the reproducingness of the quadratic rule does not hold if the sizes of bets can be freely negotiated.

If all bets have fixed size it is again possible to choose the bookmaker's position. In this case, by picking a position in the middle one guarantees a gain, independent of the outcome of the experiment. To demonstrate why the existence of the bookmaker's position means that reproducingness is endangered, we have to look further than the case of three participants. First, it is shown in appendix 1 that the optimal position of a third bettor is exactly equal to his subjective belief, in accordance with the reproducingness found for the case of two participants. This means that the third bettor would generally be wrong to chose the bookmaker's position.

However the existence of a profitable bookmaker's position implies that everyone is participating in every bet. Whether one has a better opinion than the bookmaker is a secondary question. Therefore, due to human ignorance, laziness and unwillingness to take a risk, one can be sure that for every issue there will be many bookmakers. How will these influence the willingness of more outspoken people to become extreme bettors? Of course, the gains of the bookmakers have to come from the more outspoken bettors.

In appendix 2 it is shown that in such a situation the optimum position of an extreme bettor is not equal to his own subjective belief. The assumptions in this situation are one opponent with a fixed position and a given number of bookmakers. The extreme bettor is forced towards the middle and therefore the scoring rule is not reproducing.

The difference between the situations in appendices 1 and 2 is in the degree of reactivity of the participants, with respect to the behavior of the others. I believe that the assumption of such reactive behavior as shown by the bookmakers in appendix 2 is justified by the occurrence of such factors as ignorance, laziness and the unwillingness to take risks, which can be relied upon, perhaps like nothing else in the social sciences. Moreover, in some cases the unwillingness to take a risk may even constitute rational behavior (e.g. see Molenaar, 1980).

The conclusion at this point must be that the betting model requires additional rules for more than two participants, to insure reproducingness, so that the biggest gain is obtained when calls reflect the best estimate for each of the participants, independent of the behavior of the others.

As we have seen, the problems we noted earlier with negotiable sizes of bets are not solved by equal sizes of bets.

Neither does the exclusion of bettors with a non-extreme position work, because of the tendency to take up an extreme position, by outbidding.

The possibility of a restriction on reactive behavior, e.g. by inviting tenders for an experiment, is quite contrary to the idea of a scientific statement being a public statement as well as an offer of a bet. This way of solving the problem is to be left as a very last resort.

I tried to correct the effect of the bookmaker by subtracting from each participant's gain the bookmaker's gain and redistributing this amount proportional to the squared distance from the bookmaker's position. This however leads to an amplification of the differences in opinion, which is another kind of failure of reproducingness. Although other functions than a squared distance appear rather counterintuitive, different ways to redistribute the taxed bookmaker gain might be examined.

Alternatively there may be a solution in some kind of 'two party system', prescribing non-extreme bettors to pick the nearest extreme position with the size of their bet related to their relative position between the extremes. It is however a complicated task to clarify what the consequences of any such rules are. Also we must take account of the possibility of cooperation between two or more rational, but of course unscrupulous, participants. E.g. for the two party system with unweighted bets such cooperative behavior would lead to the introduction of stooges who spoil reproducingness.

Another suggestion would be to prescribe the balancing of extreme positions against each other. Participants would then be paired according to their rank from each end of the scale of betting positions. For this rule, like for the exclusion of non-extreme bettors, outbidding may cause some problems.

Other rules might well be developed by scrutinizing real-life situations, such as the stock-market, where somewhat similar processes occur.

The conclusion must be that the betting model is not in working order yet. Here, we have to realize that science is the business of not merely two people but of the whole scientific community. Also, the value of the betting model stems from its application to this situation, rather than to the two-person case. Therefore the validity of the betting model as a usable methodological tool (and consequently as a methodological model) is called into question.

In this paper, several ways of saving the betting model have been cursorily examined. None of these has yielded a reproducing set of rules for the multi-participant case, but the possibilities have certainly not been exhausted yet.

Meanwhile, let the naive betting addict beware when against more than one opponent!

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APPENDIX 1:

We want to show that the optimum position for a third bettor is equal to his subjective belief.

Given are p_a and p_b, the positions of two bettors and P_c, the subjective belief of the third.

The pay-offs are $V_{ij}=q_j^2-q_i^2$

The subjective expected value of C is the weighted sum of his pay-offs for both possible outcomes:

$$SEV_c=P_c(q_a{}^2+q_b{}^2-2q_c{}^2)+Q_c(p_a{}^2+p_b{}^2-2p_c{}^2)$$
$$=P_c\{(1-p_a)^2+(1-p_b)^2-2(1-p_c)^2\}+(1-P_c)(p_a{}^2+p_b{}^2-2p_c{}^2)$$
$$=P_c(-2p_a-2p_b+4p_c)+p_a{}^2+p_b{}^2-2p_c{}^2$$

Differentiation shows that SEV_c has its maximum for $p_c=P_c$.

APPENDIX 2:

We disprove reproducingness of the quadratic rule in the following way: For a serious bettor we calculate his optimum position in the presence of one serious opponent with a fixed position and N bookmakers, who are always chosing the middle position between the other two.

Let p_a and p_b be the positions of the serious bettors A and B, so that the N bookmakers chose $p_0=(p_a+p_b)/2$. The subjective belief of B is P_b. Then:

$$SEV_b=P_b(q_a{}^2-q_b{}^2+Nq_0{}^2-Nq_b{}^2)+Q_b(p_a{}^2-p_b{}^2+Np_0{}^2-Np_b{}^2)$$
$$=q_b{}^2(-3N/4-1)+q_b\{q_aN/2+Q_b(N+2)\}+q_a{}^2(1+N/4)-q_aQ_b(N+2)$$

SEV_b has its maximum for $q_b=q_b'$, where:

$$q_b'=\frac{q_aN+Q_b(2N+4)}{3N+4}$$

Since generally q_a is not equal Q_b, for N>0 this gives q_b not equal to Q_b, so that the quadratic scoring rule is not reproducing.

For N=0, we are in the situation of two participants, where $q_b'=Q_b$.

In the limit of infinite N, $q_b'=q_a/3 + 2Q_b/3$. In practice, this means that both serious bettors are forced towards each other, so that their difference of opinion will diminish.